Mathematics

Trigonometry


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Main Menu

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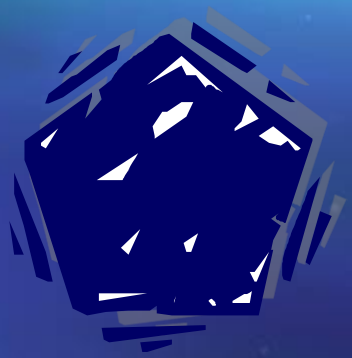
Reduction formula



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Trigonometric equations

Trigonometric functions

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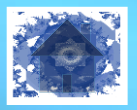


Trigonometric Functions

Introduction

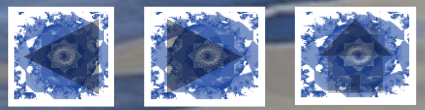
Example 1

Example 2

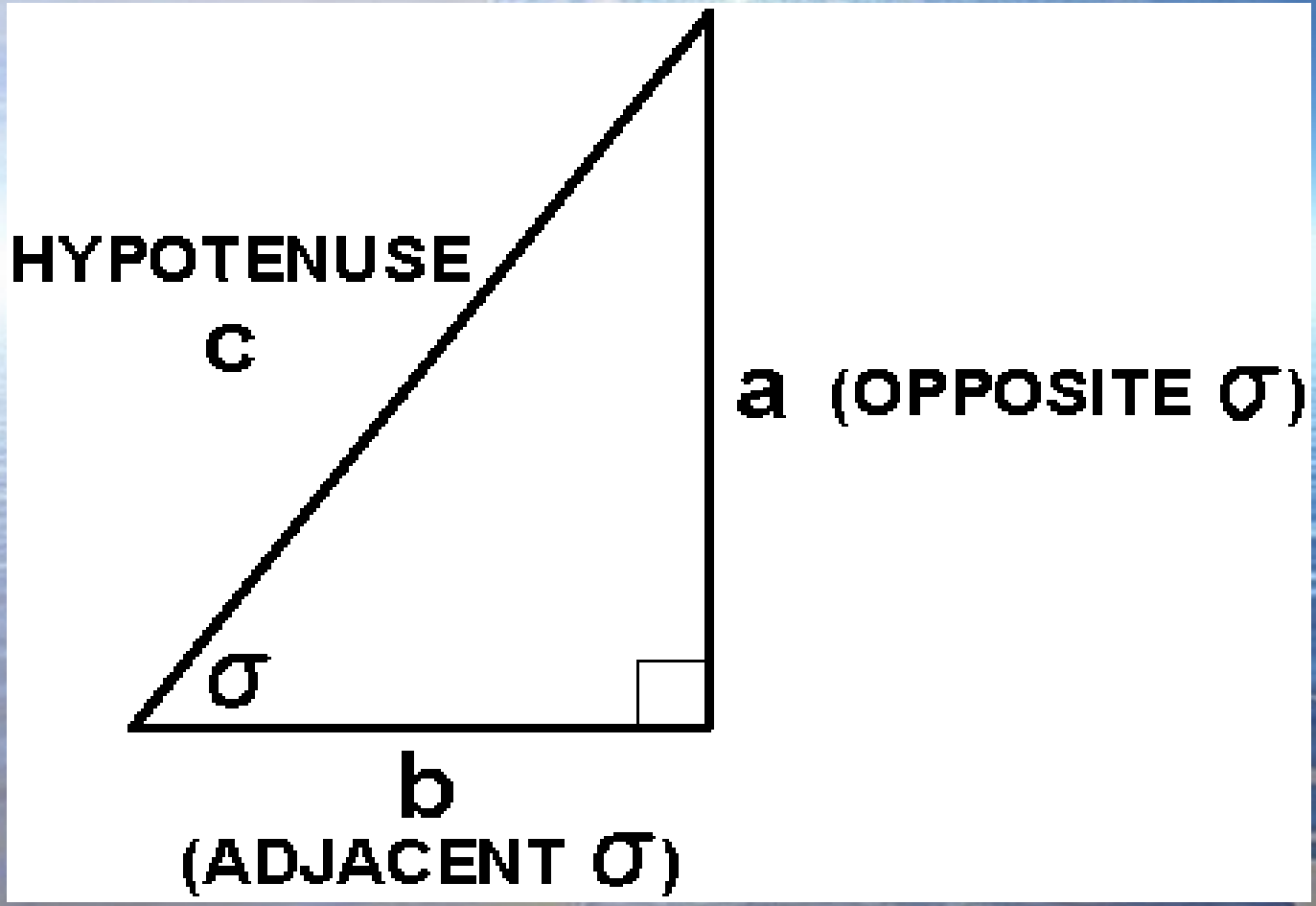
Trigonometric functions of acute angle

Trigonometric functions of acute angle: sine, cosine, tangent, cotangent, secant, cosecant.

Exact values of trigonometric functions for some most used acute angles



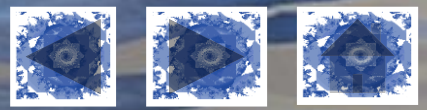
Expressing trigonometric functions of an acute angle σ in terms of x and y coordinates



$$\sin(\sigma) = \frac{\text{opp } \sigma}{\text{hyp}} \quad ; \quad \csc(\sigma) = \frac{\text{hyp}}{\text{opp } \sigma}$$

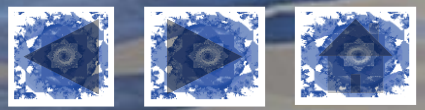
$$\cos(\sigma) = \frac{\text{adj } \sigma}{\text{hyp}} \quad ; \quad \sec(\sigma) = \frac{\text{hyp}}{\text{adj } \sigma}$$

$$\tan(\sigma) = \frac{\text{opp } \sigma}{\text{adj } \sigma} \quad ; \quad \cot(\sigma) = \frac{\text{adj } \sigma}{\text{opp } \sigma}$$



Or in words…

- *Sine:* sin σ = *a / c*
  - (*a ratio of an opposite leg to a hypotenuse)*
- *Cosine:* cos σ = *b / c*
  - *(a ratio of an adjacent leg to a hypotenuse)*
- *Tangent:* tan σ = *a / b*
  - *(a ratio of an opposite leg to an adjacent leg)*
- *Cotangent:* cot σ = *b / a*
  - *(a ratio of an adjacent leg to an opposite leg)*
- *Secant:* sec σ = *c / b*
  - *(a ratio of a hypotenuse to an adjacent leg)*
- *Cosecant:* cosec σ = *c / a*
  - *(a ratio of a hypotenuse to an opposite leg)*



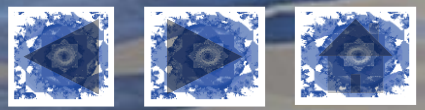
Trigonometric Functions

Introduction

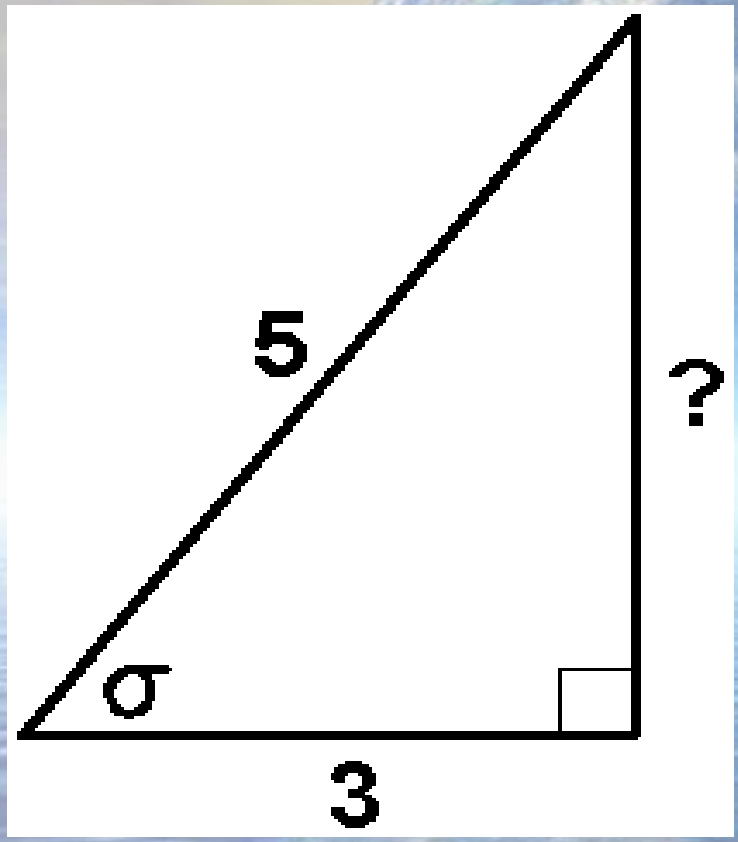
Example 1

Example 2

Example 7



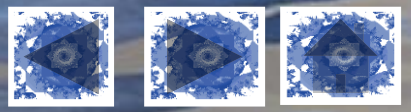
What are the six trigonometric ratios for σ?



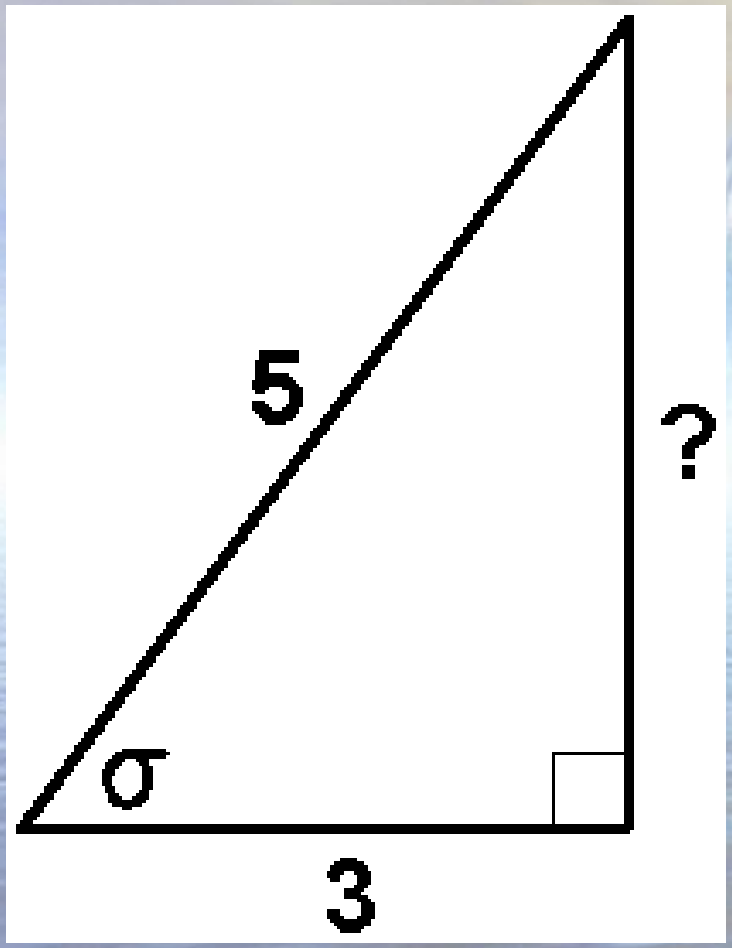
NOTE!!!

We need the length of at least one of the legs of our right triangle.

STEP 1



Use the Pythagorean Theorem . . .

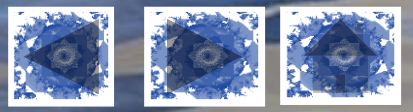


$$3^2 + ?^2 = 5^2$$

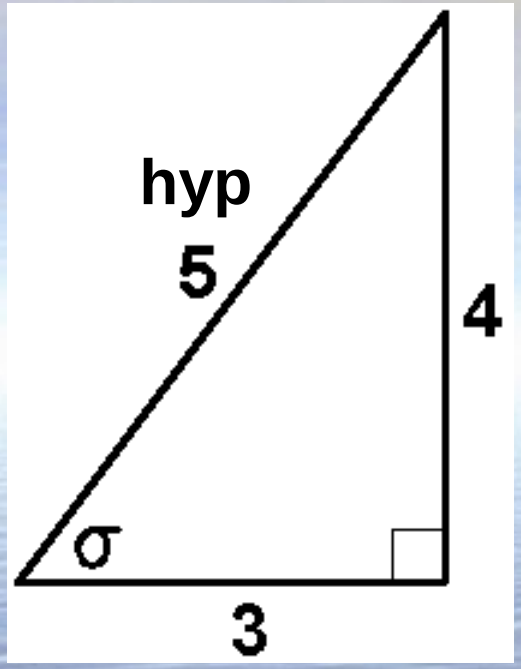
$$9 + ?^2 = 25$$

$$?^2 = 16 \; ; \; ? = \pm 4 \; ; \; ? = 4$$

STEP 2



Substitute your answer into the ratios:



$$\sin(\sigma) = \frac{4}{5} \; ; \quad \csc(\sigma) = \frac{5}{4}$$

$$\cos(\sigma) = \frac{3}{5} \; ; \quad \sec(\sigma) = \frac{5}{3}$$

$$\tan(\sigma) = \frac{4}{3} \; ; \quad \cot(\sigma) = \frac{3}{4}$$

Trigonometric Functions

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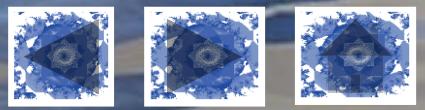
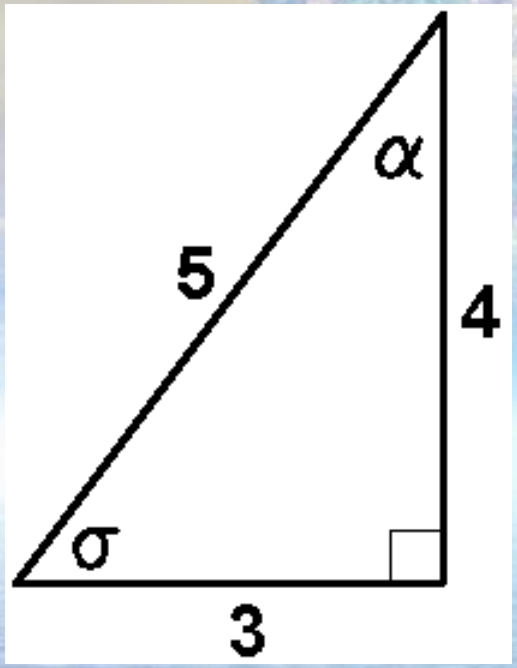
Example 1

Example 2

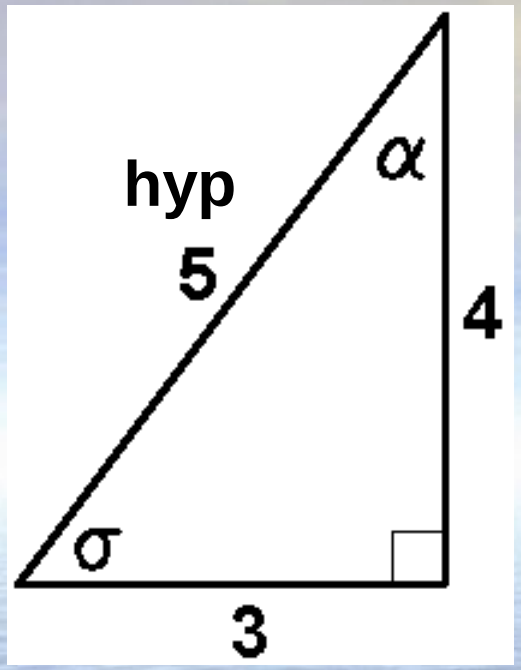
Example 2



Notice we have another angle at α.



We can obtain the six trigonometric ratios for α,



$$\sin(\alpha) = \frac{3}{5} \; ; \quad \csc(\alpha) = \frac{5}{3}$$

$$\cos(\alpha) = \frac{4}{5} \; ; \quad \sec(\alpha) = \frac{5}{4}$$

$$\tan(\alpha) = \frac{3}{4} \; ; \quad \cot(\alpha) = \frac{4}{3}$$

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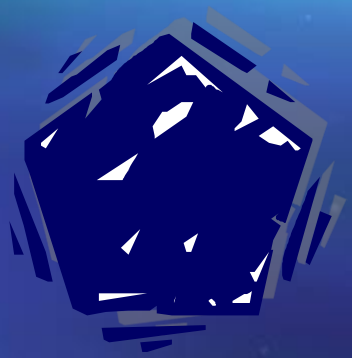
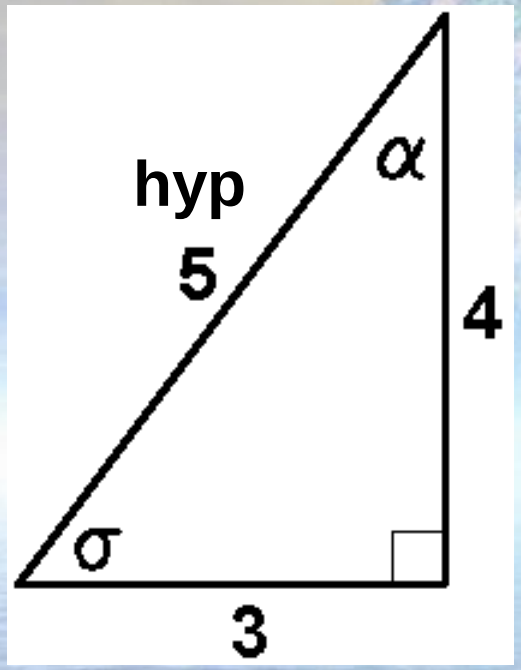
Trigonometric equations



Mathematical jokes

Together the model looks as follows.

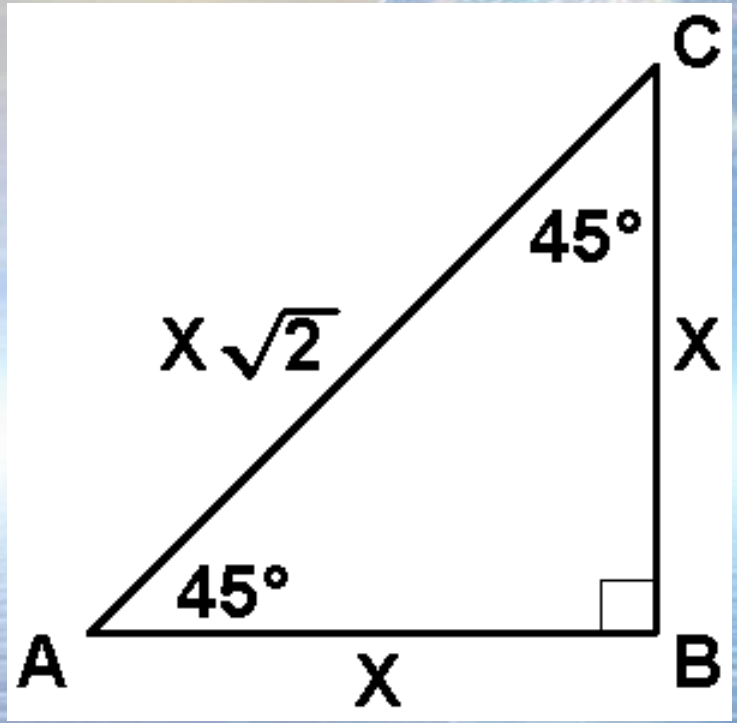


opp σ / adj α

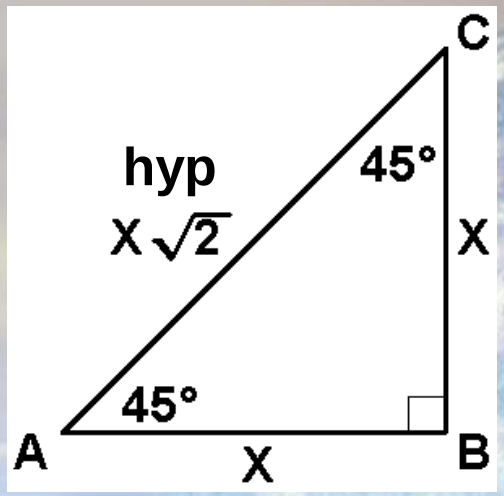
adj σ / opp α

With $\sigma + \alpha = 90°$

Recall the 45º - 45º - 90º Special Triangle.



What are the six trigonometric ratios for 45º?

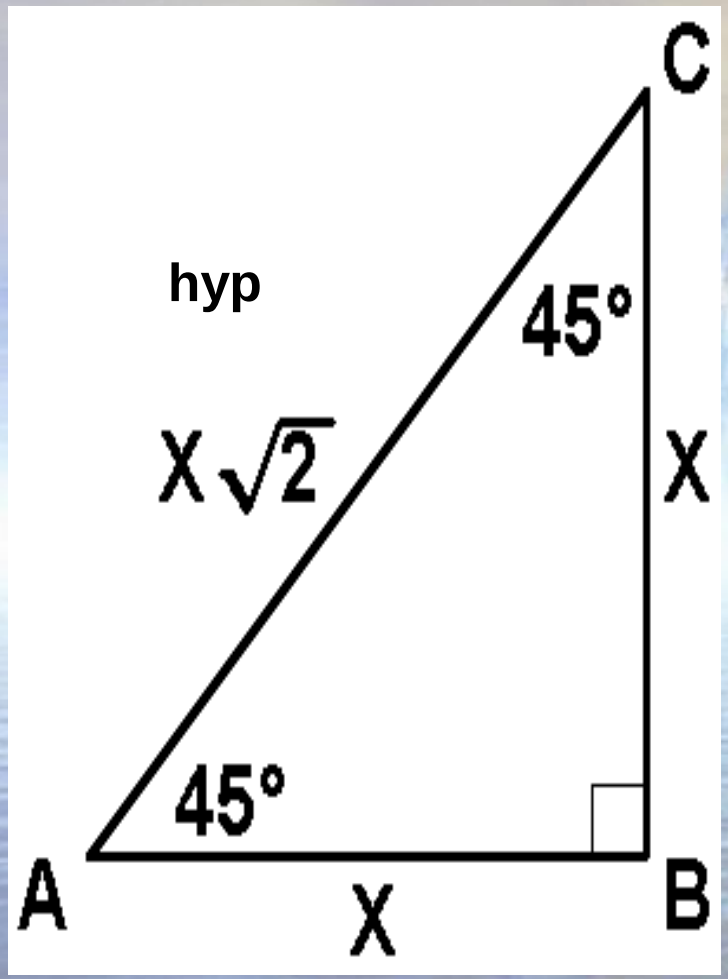


adj 45°

$$\sin(45^\circ) = \frac{\text{adj}}{\text{hyp}} ; = \frac{x}{x\sqrt{2}} ; = \frac{1}{\sqrt{2}} ; = \frac{\sqrt{2}}{2}$$

$$\cos(45^\circ) = \frac{\text{adj}}{\text{hyp}} ; = \frac{x}{x\sqrt{2}} ; = \frac{1}{\sqrt{2}} ; = \frac{\sqrt{2}}{2}$$

$$\tan(45^\circ) = \frac{\text{opp}}{\text{adj}} ; = \frac{x}{x} ; = 1$$



opp 45°

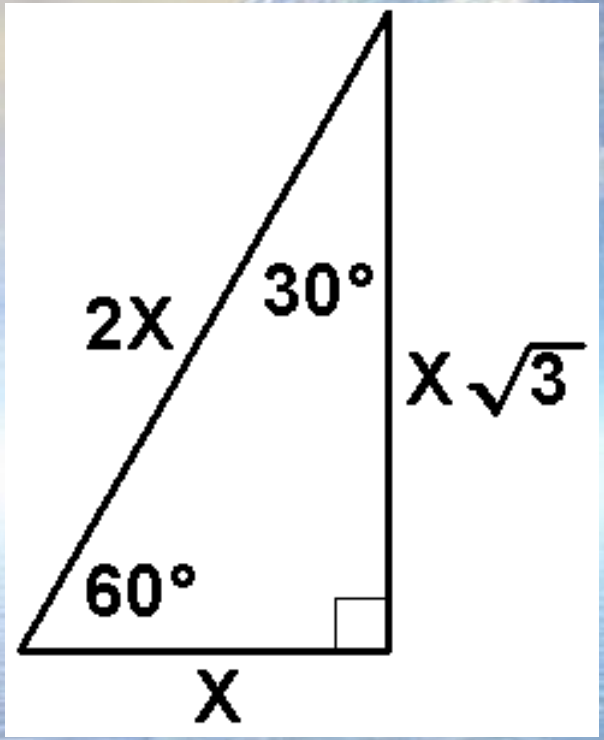
adj 45°

$$\csc(45^\circ) = \sqrt{2}$$

$$\sec(45^\circ) = \sqrt{2}$$

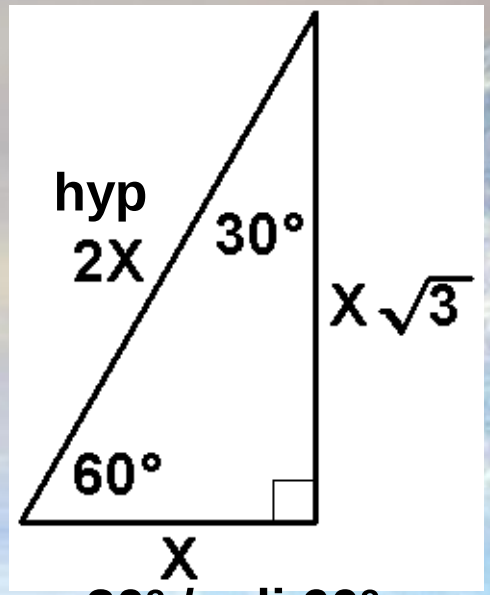
$$\cot(45^\circ) = 1$$

Recall the 30º - 60º - 90º special triangle.



What are the six trigonometric ratios for 60º ?

For 30º



opp 60º / adj 30º

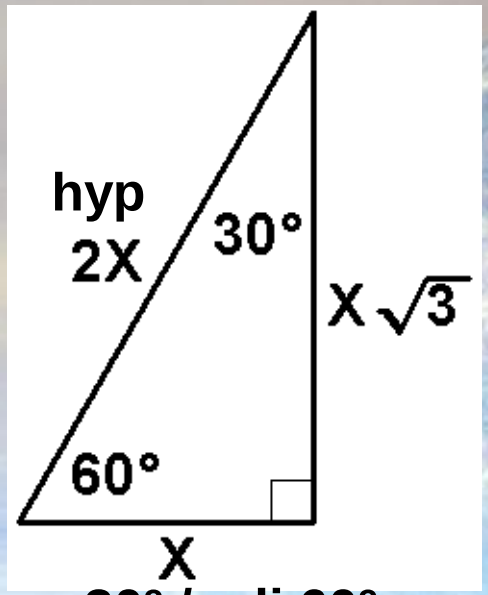
opp 30º / adj 60º

$$\sin(30^\circ) = \frac{\text{opp}}{\text{hyp}} \; ; \; = \frac{x}{2x} \; ; \; = \frac{1}{2}$$

$$\cos(30^\circ) = \frac{\text{adj}}{\text{hyp}} \; ; \; = \frac{x\sqrt{3}}{2x} \; ; \; = \frac{\sqrt{3}}{2}$$

$$\tan(30^\circ) = \frac{\text{opp}}{\text{adj}} \; ; \; = \frac{x}{x\sqrt{3}} \; ; \; = \frac{1}{\sqrt{3}} \; ; \; = \frac{\sqrt{3}}{3}$$

Thus,



opp 60º / adj 30º

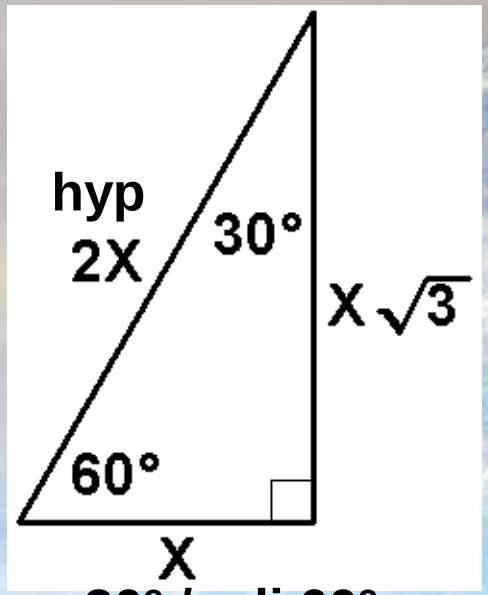
opp 30º / adj 60º

$$\csc(30^\circ) = 2$$

$$\sec(30^\circ) = \frac{2}{\sqrt{3}} \; ; \; = \frac{2\sqrt{3}}{3}$$

$$\cot(30^\circ) = \frac{3}{\sqrt{3}} \; ; \; = \sqrt{3}$$

For 60º



opp 60º / adj 30º

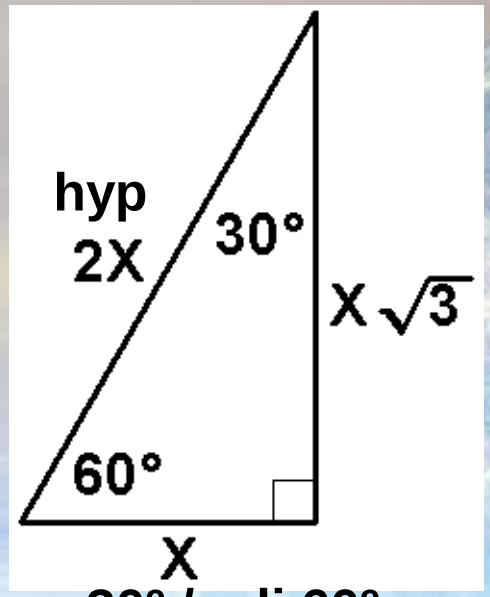
opp 30º / adj 60º

$$\sin(60^\circ) = \frac{\text{opp}}{\text{hyp}} \; ; \; = \frac{x\sqrt{3}}{2x} \; ; \; = \frac{\sqrt{3}}{2}$$

$$\cos(60^\circ) = \frac{\text{adj}}{\text{hyp}} \; ; \; = \frac{x}{2x} \; ; \; = \frac{1}{2}$$

$$\tan(60^\circ) = \frac{\text{opp}}{\text{adj}} \; ; \; = \frac{x\sqrt{3}}{x} \; ; \; = \sqrt{3}$$

Thus,



opp 60º / adj 30º

opp 30º / adj 60º

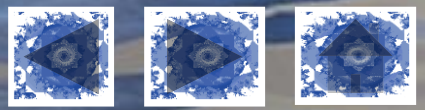
$$\csc(60^\circ) = \frac{2}{\sqrt{3}} \; ; \; = \frac{2\sqrt{3}}{3}$$

$$\sec(60^\circ) = 2$$

$$\cot(60^\circ) = \frac{1}{\sqrt{3}} \; ; \; = \frac{\sqrt{3}}{3}$$

Summary

σ	$\sin(\sigma)$	$\cos(\sigma)$	$\tan(\sigma)$
30^o	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
45^o	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60^o	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$

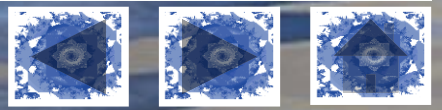


Angle σ	sin σ	cos σ	tan σ
Trigonometric ratios of 30º and 60º			
30º			
60º			
Trigonometric ratios of 0º, 45º and 90º			
0º			
45º			
90º			

Reduction formula of trigonometric functions of $90^o - \theta$

- These formulas permit:
  1) to find a numerical values of trigonometric functions of angles, greater than 90°;
  2) to execute transformations, leading to more simple expressions;
  3) to get rid of negative angles and angles, greater than 360°.

	sin	cos	tan	cot
$-\alpha$	$-\sin\alpha$	$+\cos\alpha$	$-\tan\alpha$	$-\cot\alpha$
$90^\circ-\alpha$	$+\cos\alpha$	$+\sin\alpha$	$+\cot\alpha$	$+\tan\alpha$
$90^\circ+\alpha$	$+\cos\alpha$	$-\sin\alpha$	$-\cot\alpha$	$-\tan\alpha$
$180^\circ-\alpha$	$+\sin\alpha$	$-\cos\alpha$	$-\tan\alpha$	$-\cot\alpha$
$180^\circ+\alpha$	$-\sin\alpha$	$-\cos\alpha$	$+\tan\alpha$	$+\cot\alpha$
$270^\circ-\alpha$	$-\cos\alpha$	$-\sin\alpha$	$+\cot\alpha$	$+\tan\alpha$
$270^\circ+\alpha$	$-\cos\alpha$	$+\sin\alpha$	$-\cot\alpha$	$-\tan\alpha$
$360^\circ k-\alpha$	$-\sin\alpha$	$+\cos\alpha$	$-\tan\alpha$	$-\cot\alpha$
$360^\circ k+\alpha$	$+\sin\alpha$	$+\cos\alpha$	$+\tan\alpha$	$+\cot\alpha$



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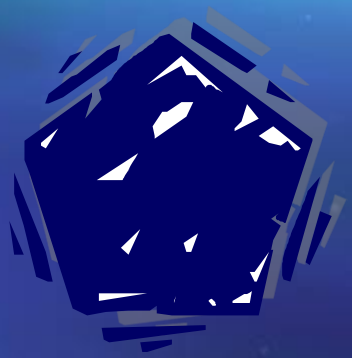
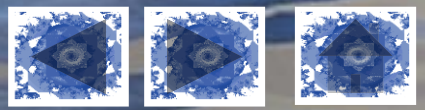


Mathematical jokes

Solving Trigonometric Equations

Example1



Solving Trigonometric Equations in the interval [0º;360º]

Step 1 : **Bring trigonometric equations into the form of sin x = number or cos x = number or tan x = number.**

If you have cosec x = number or sec x = number or cot x = number, then take the inverse of each side of the equation and bring into form of

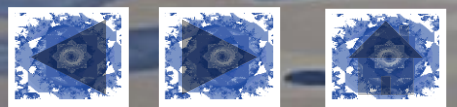
sin x = number or cos x = number or tan x = number.

Step 2 :

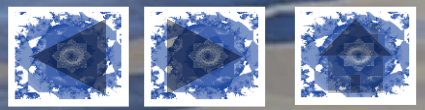
The sign of the trigonometric ratio, positive or minus, indicates in which quadrant the angle lies in

Step3: Obtain the reference angle from your calculator

Note: Always enter the trig ratio into the calculator as a positive value to obtain the reference angle.



EXAMPLE 7



- Solve for x in the given interval correct to one decimal place :

  $2 \sin x = 0{,}74 \quad ; \quad 0° < x < 360°$

- Step 1 : $(0° < x < 360°) 2 \sin x = 0{,}74 \sin x = 0{,}37$

- Step 2 : 1st quadrant

  2nd quadrant = [since sin x is positive]

  reference angle is 21,7°

- Step 3: $x = 21{,}7°$ or $x = 180° - 21.7°$

  $x = 158{,}3°$

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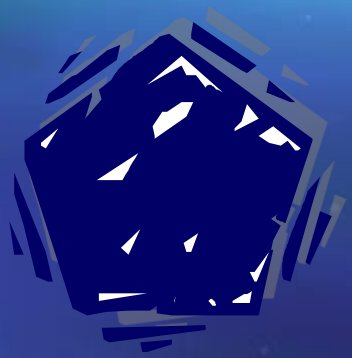


Mathematical jokes

Mathematical jokes

Einstein and telephone

Study and money

Birthdays

New in geometry

Two student-mathematicians, having birthdays on the same day, wished each other many happy returns on this day.

One of them said:

- **You'll have such birthday only in 11 years. The other answered him:**

- **- Okay, but you'll have such birthday only in 96 years.**

- **Both of them were satisfied with each other**

- **How old were they on this day ?**

Solution:

One of them was 25, and other 24.

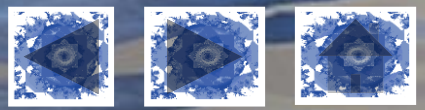
Why?

25=52, the next square is 62=36,

i.e. in 11 years.

24=4!, the next factorial is 5!=120,

i.e. in 96 years.



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- One woman asked Einstein to remember her telephone number: 361-343.
  Einstein answered:
  - It’s very easy. 19 squared and 7 cubed.

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New in geometry

- - How is the biggest side in a right-angled triangle called ?

  As all the pupils keep silent the teacher begins to help:

  - Hy-po-…
  - Hippopotamus !

Mathematical jokes

Einstein and telephone

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Birthdays

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Study and money

- Father writes a letter to his son-student:
  “Dear John!
  I send you 50 dollars, as you asked. By the way remember please that the number 50 is written with one zero, but not with two.”

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