Mathematical Literacy

Learner's Workbook

Grade 11

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Department of Education

learning channel
defining *your* destiny.

Contents

How to use the Learning Channel Mathematical Literacy programme for Grade 11

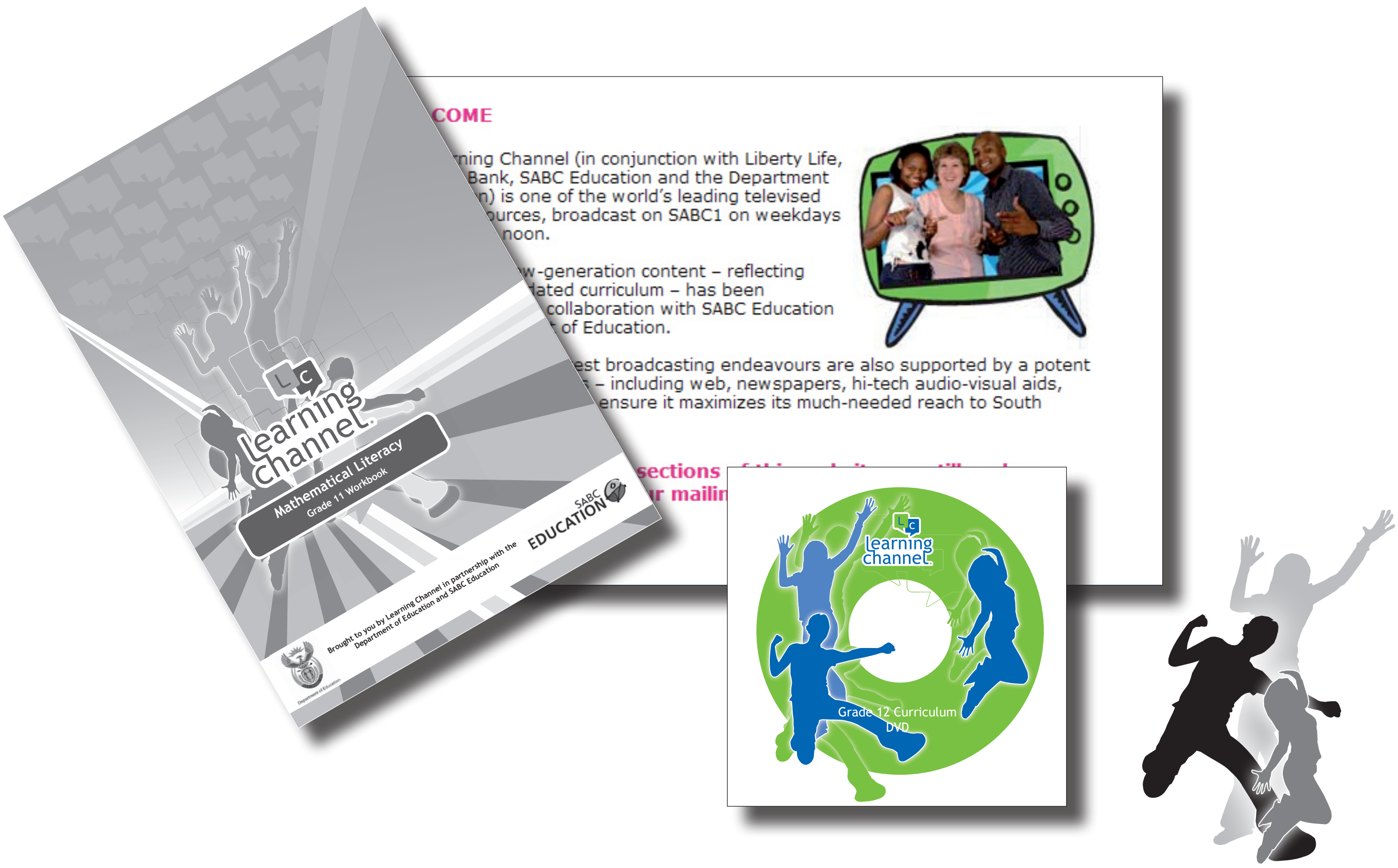
Congratulations and thank you for choosing this Learning Channel Mathematical Literacy Grade 11 programme.

This Mathematical Literacy programme is comprehensive and covers all the Learning Outcomes, Assessment Standards, knowledge, key concepts and skills for this subject as stated in the National Curriculum Statement – everything you need to make a success of your world. However, it does not replace your teacher or textbook!

This Learning Channel programme is for everyone … you may be using this at home or in your classroom with your teacher and classmates. You may have chosen this programme because you are struggling with Mathematical Literacy and as a result you're not achieving the grades you know you deserve. Or, you may be using it because it will help you earn the distinction you've set as your goal. Wherever you are and whatever your reason, this programme will give you the head start you need.

The Learning Channel programme consists of three components:

- Lessons to watch on DVD;
- A learner workbook, with exercises and activities for you to complete; and
- If you are connected to the Internet, the Learning Channel website.



Here are some tips on how to make the most of this programme

Before sitting down to study, make sure you have the following to hand:

- The Learning Channel Mathematical Literacy for Grade 11 DVD;
- The Learning Channel Mathematical Literacy for Grade 11 Workbook;
- Pen and paper; and
- Your DVD remote control – if you are watching this on a DVD player.

➢ Insert the Learning Channel Mathematical Literacy for Grade 11 DVD disc into your computer or DVD player. Press play.

➢ The subject name and grade will appear, followed by the title of the lesson, the lesson number and the duration of the lesson.

➢ Next, you will be told what lesson to turn to in your workbook.

➢ The Learning Outcomes and Assessment Standards will appear, followed by the lesson overview. This will tell you exactly what you will be expected to do by the end of the lesson.

We suggest that you watch the entire lesson before working in the workbook. While watching the lesson you can stop the DVD when you need to review or refresh what has been said or if you want to take down notes.

While watching the lesson you will also see the PAUSE icon. This alerts you to an activity you can complete in the workbook. If you feel that you are ready to try this concept or skill related activity, press the PAUSE button on your remote control, television or computer screen. Press PLAY once you have completed the activity.

At the end of the lesson you will see a summary of the key concepts covered, if you've been taking notes you can jot these down or find them in your workbook.

➢ All the exercises and activities are designed so that you can complete them on your own. Some activities, however, can also be completed with a partner, in a group or as a class. These opportunities are clearly indicated with icons (see page vii).

➢ Check your answers against the solutions provided at the end of the workbook. Errors may indicate that you have missed or not understood key concepts. Watch the lesson again, refer to any notes you have made and redo the activities you did not master.

The Learning Channel website offers extra features like subject glossaries, past exam papers, study tips and the National Curriculum Statement. Visit the website to make use of the extra features.

If you are a teacher using this programme with your class, you will find teacher tips at the end of the Learner's Workbook. These tips will help you facilitate the use of the AV lesson and convey its content to your learners.

What the icons mean

DVD	DVD – watch the lesson
INDIVIDUAL	Individual work – do this activity on your own
PAIRS	Pair work – do this activity with a partner
GROUPS	Group work – do this activity in a group
CLASS	Class work – do this activity as a class
SELF ASSESSMENT	Self-assessment – assess yourself
BASELINE ASSESSMENT	Baseline assessment – what I know before starting (prior knowledge)
FORMATIVE ASSESSMENT	Formative assessment – how I am progressing
SUMMATIVE ASSESSMENT	Summative assessment – a check of what I know
PROJECT	Project – a project to research and present
COLLECTION OF EVIDENCE	Collection of evidence activity – include in your portfolio

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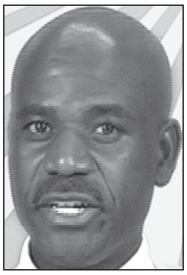
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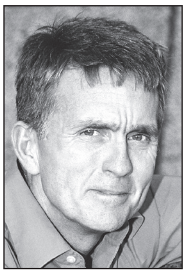
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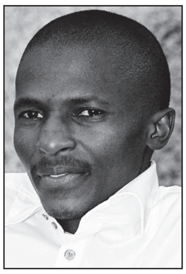
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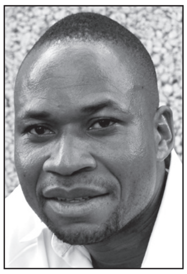
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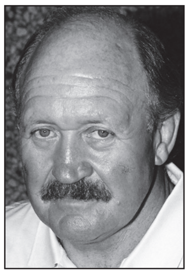
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Other Learning Channel products

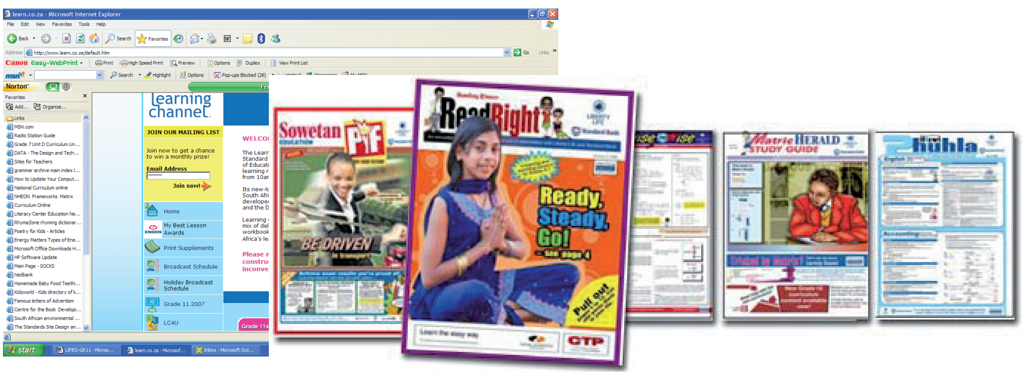
Other products in the Learning Channel Grade 11 series

- Learning Channel Physical Sciences for Grade 11 DVD lessons and Learner's Workbook
- Learning Channel Mathematics for Grade 11 DVD lessons and Learner's Workbook
- Learning Channel English Home Language for Grade 11 DVD lessons and Learner's Workbook
- Learning Channel Life Sciences for Grade 11 DVD lessons and Learner's Workbook
- Learning Channel Life Orientation for Grade 11 DVD lessons and Learner's Workbook
- Learning Channel Business Studies for Grade 11 DVD lessons and Learner's Workbook
- Afrikaans First Additional Language for Grade 11 DVD lessons and Learner's Workbook
- Learning Channel Accounting for Grade 11 DVD lessons and Learner's Workbook
- Learning Channel English First Additional Language for Grade 11 DVD lessons and Learner's Workbook

The Learning Channel (in conjunction with Liberty Life, Standard Bank, SABC Education and the Department of Education) is one of the world's leading televised learning resources, broadcast on SABC1 on Lessondays from 10am to noon.

Its new-look, new-generation content – reflecting South Africa's updated curriculum – has been developed in close collaboration with SABC Education and the Department of Education.

Learning Channel's latest broadcasting endeavours are also supported by a potent mix of delivery platforms – including web, newspapers, hi-tech audio-visual aids, workbooks and SMS – to ensure it maximizes its much-needed reach to South Africa's learners.



Learning Channel offers an extensive range of educational material on video or DVD. You can order 15-20 hours of interactive learning with a tutor, accompanied with a workbook to be used in the privacy of your own home or school.

CDs with digitised video lessons are also available.

To order your Learning Channel CDs, DVDs, videos and workbooks, please contact Takalani. E-mail: info@learn.co.za Phone: (011) 639-0170

PERCENTAGE

An introduction

Lesson 1

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and operations in context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues
Assessment standard
AS 11.1.1
In a variety of contexts, find ways to explore and analyse situations that are numerically based by:
- Estimating efficiently
- Working with formulae by hand and with a calculator
- Showing awareness of the significance of digits
- Checking statements and results by doing relevant calculations

(The range of problem types includes percentage, ratio, rate and proportion.)

Overview

In this lesson we will focus on working with percentage. We will define percentage, learn how to change fractions into percentages and consider the advantages and disadvantages of using percentages.

Lesson

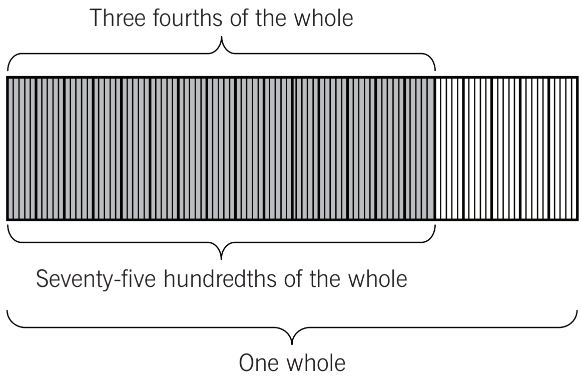
A percentage is a fraction. It is the part of the whole expressed in hundredths.

Additional example 1

Consider the fraction three-quarters – it describes three parts of a whole that has been divided into four equal parts. See diagram below.

To change three-quarters into a percentage, we need to divide the whole into one hundred equal parts instead of four. Because 100 divided by 4 is 25, we need to divide each of the fourths into 25 equal parts.

Additional example 2



It follows that three-quarters corresponds to 75 of the 100 equal parts or 75%. Three-quarters and seventy-five hundredths are equivalent fractions. This means that three-quarters is 75%.

Methods and worked examples

Expressing a fraction as a percentage is useful because we have a better understanding or “feel” of what a percentage is than we have of the equivalent fraction. For example**:**

A water tank has a capacity of 5 500 litres. This means that it can contain 5 500 litres of water. If there is only 1 950 litres in the tank then the tank is $\frac{1\,950}{5\,500}$ full. This is clumsy and one gets a better “feel” of how full the tank is if we say that it is 35% full.

Another reason why percentages are useful is that it allows us to express a fraction as a whole number. For example, 35% is a lot easier to read and write than twenty-one sixtieths or 0,35.

One can calculate a percentage using a calculator. For example:

If seven of the eight girls in a class have boyfriends, what percentage of the girls has a boyfriend?

Use your calculator and type the following: 7 ÷ 8 and then the % key. This should give an answer of 87,5%.

The following example shows how we can use calculators to compare situations. It also serves to show how we can lose some the actual quantities when we use percentage.

If a dam with a capacity of 1,625 Ml has 1,072 Ml in it, how full is the dam?

Using the calculator, we get about 66%.

If a second dam with a capacity of 13,488 Ml has 8,902 Ml in it, how full is this dam?

Using the calculator, we get about 66%.

The answer to both of these problems is 66%, but one of the dams has a lot more water in it. When we use percentage, we lose the actual values.

LC INDIVIDUAL

LC PAIRS

LC SUMMATIVE ASSESSMENT

Activity

Practice examples

1. Out of a class of 37 learners, 25 are boys.
   1.1 What percentage of the class are boys?
   1.2 What percentage of the class are girls? Show two ways of getting this answer.
2. There are 153 cars parked in a carpark. The carpark has a capacity of 180 cars. Express how full the carpark is as a percentage.
3. Grace wrote two mathematics tests. For the first test she got $\frac{23}{30}$ and for the second test she got $\frac{25}{40}$. In which of the two tests did she do better? Compare the two tests by converting each mark to a percentage.
4. The teacher said that if 85% of the class remembered to bring their train fare, she would take them on an outing. Did the learners go on the outing if 35 out of a class of 42 learners remembered to bring their money?
5. A restaurant has two seating areas, one outside and one inside. The inside area can seat 60 people and the outside area can seat 30 people.

2

5.1 There are 45 people eating in the inside area at a particular time. Express how full the inside area is as a percentage.

5.2 At the same time there are 24 people in the outside area. Express how full the outside area is as a percentage.

5.3 Percentage-wise, which is fuller? Which area has more people to be served?

5.4 Express how full the whole restaurant is as a percentage.

6. A local rock band played in a concert on Friday and Saturday night.

6.1 On Friday night 4 613 out of the crowd of 6 500 were under 18. What percentage of the crowd was under 18?

6.2 On Saturday night 5 023 out of a crowd of 7 256 were under 18. What percentage of the crowd was under 18?

6.3 What percentage of the crowd for both nights together were under 18?

6.4 The concert venue can accommodate 7 500 people. Express how full the venue was on Friday night as a percentage.

6.5 Express how full the venue was on Saturday night as a percentage.

6.6 On which night would you have preferred to go and why?

Lesson 2

PERCENTAGE

Adding a percentage of an amount to an amount

Learning Outcomes and Assessment Standards

Learning Outcome 1

Number and operations in context

The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues

Assessment Standard

AS 11.1.1

In a variety of contexts find ways to explore and analyse situations that are numerically based by:

- Estimating efficiently.
- Working with formulae by hand and with a calculator.
- Showing awareness of the significance of digits.
- Checking statements and results by doing relevant calculations.
- (The range of problem types includes percentage, ratio, rate and proportion.)

Overview

In the previous lesson we converted a fraction or ratio into a percentage. In this lesson we will consider two further calculations.

Lesson

LC DVD

We will:

- Calculate a percentage of an amount; and
- Add a percentage of an amount to an amount. In other words we will increase an amount by a particular percentage.

These calculations are used in the following situations: when we add a tip to a restaurant bill; when we get a salary increase; and when shop owners want to add VAT to the goods they sell.

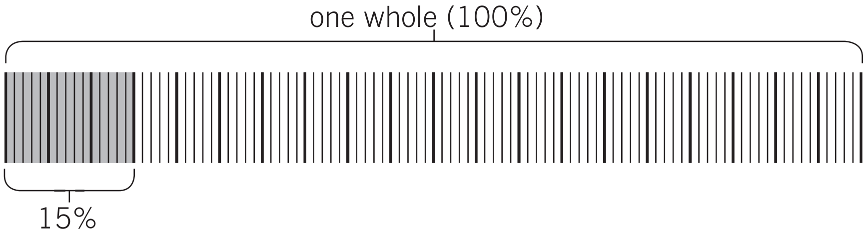
(The range of problem types we will cover includes percentage, ratio, rate and proportion.)

Methods and worked examples

The following example shows you how to find a percentage of an amount first and to add it to an amount.

Worked example:

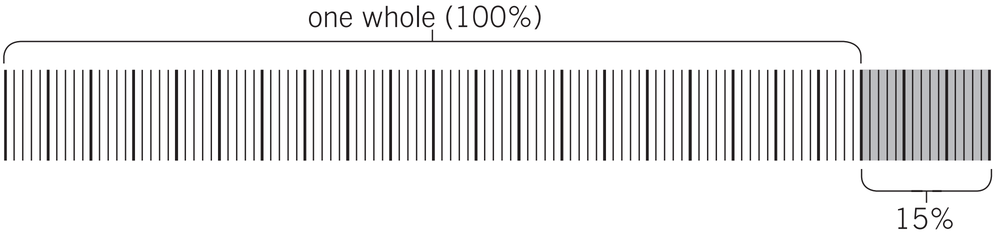
How much must you pay the waitress if you decide to add 15% to a bill of R75?



There are two steps to this problem. First you need to find 15% of the whole – in this case R75.

To do so on a calculator, you enter **75.** into your calculator and then press **×15%.** This will give you an answer of R11,25. This is the tip you will give the waitress – 15% of the amount of the bill.

Next you need to add the 15% to the whole.



In the case of the bill you must add R11,25 to R75. On your calculator, you continue by simply pressing **+** and **=** and it will give you the answer of R86,25.

If you do not necessarily want to know how much the waitress's tip is, you can do the calculation in one step as follows:

Enter **75** and **+** (because you want to add a percentage) and then type **15%**. This gives you the answer of R86,25 straight away.

Consider the following additional examples:

Additional example 1

1. If Frank earns R2 500 a month and gets a 7% increase, how much will he earn in the future?

Additional example 2

2. If Veronica earns R7 500 a month and gets a 3% increase, how much will she earn in the future?

Solutions:

(1) Doing the calculation in two steps we can see that Frank will earn R175 more a month (calculator keys: enter **2500×** and then **7%**). Now you must add this to the original amount. Therefore Frank will earn R2 500 + R175 = R2 675 (calculator keys: press **+** and then **=**). So Frank earns R175 more a month and earns a total of R2 675.

To do the calculation in one step only, you need to enter **2500+7%** and you get the answer of R2 675.

(2) Repeat the same sequence for Veronica. Veronica gets R225 extra a month, which means she now earns R7 725 a month.

There is an interesting twist to these examples. Frank got the larger percentage increase. He got a 7% increase, whereas Veronica got only

a 3% increase. But Veronica now gets R225 extra a month, whereas Frank gets only R175 extra a month. What this illustrates is that, although percentage is powerful and useful, unless we also know the actual amounts we are dealing with, we can sometimes miss important information.

INDIVIDUAL

SUMMATIVE ASSESSMENT

Activity

Practice examples

1 How much did the waitron get as a tip if you gave him a 15% tip and your bill came to R235?

2 The newspaper says petrol is going to go up by 7,5%. What is the new petrol price, if petrol costs R6,50 per litre at the moment?

3.1

3.2 Siswe earns R6 000 and Belulwe earns R3 500. They are to receive an annual increase of 5,5% this year. Calculate their new salaries.

They get the same percentage increase, but who gets more money added to his/her salary? Why does this happen?

4 If a new car costs R189 000 without VAT, what will it cost with 14% VAT? (VAT stands for value added tax and is added to all goods and services in South Africa.)

5 If the price of milk increases by 8%, how much will a litre of milk cost you if it costs R4,25 now?

6 Rebecca earns R5 500 at the moment, but is getting a 6,25% increase. Ishmael earns R9 000 but is getting only a 5% increase.

6.1 Calculate how much extra each person will earn.

6.2 Compare their percentage increase and their actual increase in terms of money.

7\. Zahrah got $\frac{23}{30}$ for her first mathematics test. What would her mark be out of 30 if her marks improved by 15% in her next test?

PERCENTAGE

Subtracting a percentage of an amount from an amount

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and operations in context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues
Assessment standard
AS 11.1.1
In a variety of contexts find ways to explore and analyse situations that are numerically based by:

- Estimating efficiently.
- Working with formulae by hand and with a calculator.
- Showing awareness of the significance of digits.
- Checking statements and results by doing relevant calculations.

(The range of problem types includes percentage, ratio, rate and proportion.)

Overview

In this activity we will extend our work on percentage to subtract a percentage of an amount from an amount. In other words, we will decrease an amount by a particular percentage.

Lesson

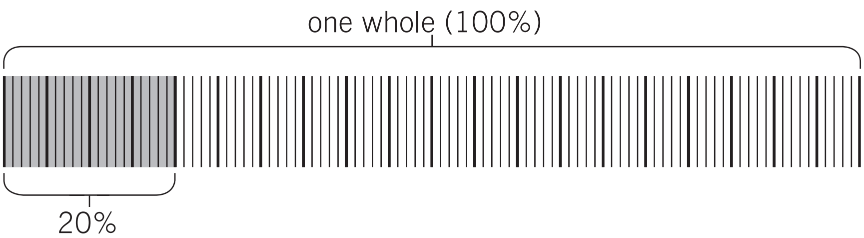
Such calculations are used when we: calculate discounts; and determine salary deductions, such as taxes and so on.

Methods and worked examples

The following example shows you how to find out a percentage of an amount and to subtract it from an amount.

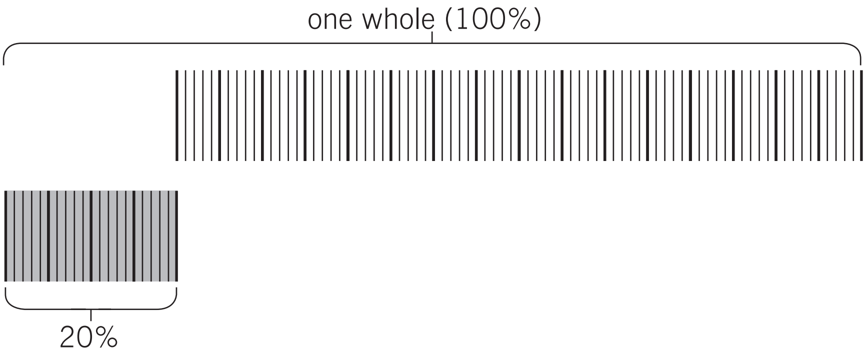
A skirt costs R250. If the shopkeeper gives you a 20% discount, how much will the skirt cost you?

The first step is to calculate the percentage of the whole – in this example 20%. To do so, we divide the whole into 100 equal pieces (percentages) and then determine the percentage we want. See the diagram below.



On your calculator you enter 250 and then press × and 20%.
This will give you an answer of R50. This is the discount the shop will give you.

The next step is to deduct the 20% from the whole, as depicted in the diagram on the next page.



On your calculator you can continue where you left off earlier and simply press **–** and **=** and it will give you the answer of R200.

Other approaches:

If you do not necessarily want to know how much discount you will get, you can do the calculation in one step as follows.

Enter 250 and **–** (because you want to decrease or subtract a percentage) and then type 20%. This gives you the answer of R200 straight away.

There is another way to work out this problem. Recall that, in our problem, R250 is the whole or 100%. If you subtract 20% from the whole then you are left with 80% of the whole. In other words when you subtract 20% of an amount from an amount, you are left with 80% of the amount and you can simply calculate what 80% of the amount is.

Check this by finding 80% of R250. The calculator strokes are as follows: enter 250 and ×80%. This will give you an answer of R200.

Consider the following additional examples:

INDIVIDUAL

FORMATIVE ASSESSMENT

Additional example 1

1. How much of your salary will you take home if you earn R5 000 a month and must pay 15% tax?

Solution:

We will do the solution in three ways.

Method 1: Use this method if you want to know how much actual tax you are paying.

Find 15% of R5 000 on your calculator by entering 5000 and ×15%. This gives an answer of R750. This is the actual amount of tax that you will pay.

To find out what amount you will take home, you need to subtract this amount from R5 000. Enter **–** and then **=**

You will take home R4 250.

Method 2: Use this method if you are interested only in the final value – it takes only one step.

Enter 5000 and **–** followed by 15%.

You press minus 15% because you want to subtract or deduct 15% from R5 000. This gives an answer of R4 250.

Method 3: You subtract 15% from the whole, which means you find 100% – 15% = 85% of the whole.

Enter 5000 and ×85%.

This gives an answer of R4 250.

You can do the calculation: 100% – 15% = 85% on your calculator first and the rest of the problem by using the memory key. The key strokes are as follows.

100-15=. This gives an answer of 85. Ask the calculator to remember this amount for you by pressing the M-key. A little M appears on the screen to show that the calculator is keeping a value in its memory.

Continue as before: Enter 5000 and ×L%. The answer R4,250 will appear.

The MRC key is the memory recall and memory clear key.

Additional example 2

INDIVIDUAL

FORMATIVE ASSESSMENT

2. *Which is the better deal for a jacket:*

Cost price R240 and a 22,5% discount?

OR

Cost price R200 and a 6% discount?

Solution:

We will do the first part of the problem: 22,5% discount on the cost price of R240 using all three methods.

Method 1: Find 22,5% of R240 on your calculator by entering 240 and ×25,5%. This gives an answer of R54. This is the actual discount.

Now, to find out what you will pay for the jacket, you need to subtract this amount from R240. Enter - and then =

You will pay R186.

Method 2: You can do the above problem in one step.

Enter 240 and - followed by 22,5%

You minus 22,5% because you want to subtract or deduct 22,5% from R240. This gives an answer of R186.

Method 3: You subtract 22,5% from the whole, which means that you find 100% – 22,5% = 77,5% of the whole.

Enter 240 and ×77,5%

This gives an answer of R186.

Alternate key strokes: 100-22,5=M followed by 240 and ×L%. The answer is R186.

Now we will do the second part of the problem: 6% discount on the cost price of R200 using all three methods and you will find that the answer is R188.

Note: There is an interesting twist to this problem. The discount on the more expensive coat was huge: 22,5%, while for the less expensive coat it was far less, only 6%. The actual discount of R54 for the more expensive coat was much more than the actual discount of R12 for the other coat and yet the two final prices were very similar. Once again we make the point that, although percentage is powerful and useful, unless we also know the actual amounts we are dealing with, we can sometimes be misled.

INDIVIDUAL

SUMMATIVE ASSESSMENT

Activity

Practice examples

1. A clothes shop advertises a 33,3% discount on all goods. Calculate what you would pay for the following:
   1.1 A pair of jeans costing R350
   1.2 A shirt costing R199
   1.3 A pair of shoes costing R259
2. Fred went on an outdoor survivor course and complained that he had lost 7% of his body mass. He weighed 85 kg before he went on the course.
   2.1 How much weight did he lose?
   2.2 What does he weigh now?
3. Viwe got $\frac{27}{40}$ for his first mathematics test. What mark did he get if his mark decreased by 12% in his second test?
4. A group of friends decide to start saving 5% of their salary each month.

   Johan earns R5 500

   Nomakhazi earns R7 500

   Zahrah earns R4 200

   For each person, calculate:
   4.1 How much money must he/she deposit into the bank?
   4.2 How much money does he/she have left over?
   4.3 Who saves the most money?
5. Two shops advertise the same type of running shoes. Footloose advertises that it is having a sale and will give a 15% discount on all running shoes. Its shoes usually cost R650. Run Wear has running shoes costing R600 and advertises that it will give a 10% discount on all purchases.
   5.1 Calculate the sale price for each pair of shoes.
   5.2 Which shop has the cheaper shoes?

PERCENTAGE

Calculating percentage change

Lesson 4

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues
Assessment Standard
AS 11.1.1
In a variety of contexts find ways to explore and analyse situations that are numerically based, by:
- Estimating efficiently
- Working with formulae by hand and with a calculator
- Showing awareness of the significance of digits
- Checking statements and results by doing relevant calculations

(The range of problem types includes percentage, ratio, rate and proportion.)

Overview

In the previous lessons we have looked at the following calculations:
- Changed fractions or ratios into percentages;
- Calculated a percentage of an amount;
- Added a percentage of an amount to an amount; and
- Subtracted a percentage of an amount from an amount.

In this lesson we will consider another type of percentage calculation. We will:
- Calculate a percentage change.

Lesson

A common example of percentage change is the year-on-year inflation. If inflation is said to be 6%, this gives an indication of the percentage change in the price of a basket of goods from one year to the next. This means that the price of the goods has increased by 6%.The way that inflation is measured is as follows:

Every month, let's say January 2006, statisticians find out the cost of a list of 1 500 goods and services. For our example let us say this comes to R5 000.

January 2006

R5,000

January 2007

R5,650

One year later, in January 2007, statisticians find out the cost of that same list of 1 500 different goods and services. This time they find that the cost is R5 650.

Note: they use the same list of goods and services each time to ensure that they are comparing the same things with one another.

The basket of goods and services has increased by R650.

This increase, expressed as a percentage of the original amount, is called the year-on-year inflation or percentage change.

Note: when we added a percentage of an amount to an amount, in a previous lesson, we knew the percentage and had to find the actual amount to add it on. In this case we know the amount that has been added so we need to express it as a percentage of the original amount.

Methods and worked examples

For the inflation problem above we can write down the steps on how to calculate percentage change as follows:

Step 1: find the difference between the final value and the original value.

Final value – original value = R5 650 – R5 000
= R650

Step 2: find what percentage this is of the original amount.

This takes us back to the first lesson when we learnt to express a fraction as a percentage. The calculator keys are: 650/5000%

This gives an answer of 13%

We can write the above two steps as a formula:

$$\% \text{ change} = \frac{\text{final value} - \text{original value}}{\text{original value}} \text{ (expressed as a \%)}$$

The inflation example was an example of where the amount had increased. The following example is an example of where the amount decreases.

Worked example:

Last year 150 incidents of theft were reported at our school. This year 117 incidents were reported. What is the percentage change?

Solution:

$$\begin{aligned}\% \text{ change} &= \frac{\text{final value} - \text{original value}}{\text{original value}} && \text{(expressed as a \%)}\\ &= \frac{117 - 150}{150} && \text{(expressed as a \%)}\\ &= -\frac{33}{150} && \text{(expressed as a \%)}\\ &= -22\%\end{aligned}$$

The key strokes on the calculator are:

117-150/150%

The negative sign in the answer shows that the original amount decreased or got smaller.

Note: this calculation reminds us of subtracting an amount from an amount. In this example we know the whole, i.e. the original amount. We worked out what was subtracted, i.e. the difference between the final and the original amounts, and we express this as a percentage of the whole.

In the case of subtracting an amount from an amount, we knew the percentage change so we first calculated what the amount to be subtracted was and then we subtracted it from the amount. In this case we know the amount that was subtracted and our job is to calculate what percentage it is of the original amount.

Consider the following additional examples:

Additional example 1

1. What is the percentage change in the price of a can of cold drink if the price went from R2,50 to R2,75?

Additional example 2

2. What is the percentage change in the volume of water in a tank that goes from 1 600 l of water to 1 150l of water?

Solutions:

(3) $\% \text{ change} = \frac{\text{final value} - \text{original value}}{\text{original value}}$ (expressed as a %)

$= \frac{2{,}75 - 2{,}50}{2{,}50}$ (expressed as a %)

$= \frac{0{,}25}{2{,}50}$ (expressed as a %)

$= 10\%$

The answer is positive because there was an increase in the price of the cold drink.

(4) % change = final value - original valueoriginal value (expressed as a %)

= 1 150 – 1 600/1 600 (expressed as a %)

= −450/1 600 (expressed as a %)

= –28,125% ≈ –28%

The answer is negative because there was a decrease in the volume of water.

Activity

INDIVIDUAL

SUMMATIVE ASSESSMENT

Practice examples

1 Calculate the percentage increase in the price of bread if the price increased from R5,20 to R5,90.

2 Calculate the percentage of VAT on goods and services if a pair of sunglasses costs R250, excluding VAT, and R285, including VAT. (VAT: value added tax)

3 The table below shows how the population of South Africa grew from 1996 to 2001.

1996	40,5 million
2001	44,8 million

Calculate the percentage growth in population from 1996 to 2001.

4 The table below shows how the urban population of four centres in South Africa has grown from 1991 to 2007.

	1991 (census)	**1996 (census)**	**2007 (estimate)**
Johannesburg	4 180 897	4 508 915	7 372 314
Cape Town	2 175 951	2 775 677	4 709 212

George	63,895	94,121	197,056
Vryheid	25,099	53,228	175,119

Information from *World Gazetter*

4.1 Calculate the percentage increase in population for Johannesburg from 1991 to 1996.

4.2 Calculate the percentage increase in population for Johannesburg from 1996 to 2007.

4.3 Calculate the percentage increase in population for Cape Town from 1996 to 2007.

4.4 Calculate the percentage increase in population for George from 1996 to 2007.

4.5 Calculate the percentage increase in population for Vryheid from 1996 to 2007.

4.6 Which of the four centres had the biggest percentage growth in urban population?

4.7 Which of the four centres had the most people moving to the centre since 1996?

4.8 Calculate the percentage increase in population for Vryheid from 1991 to 1996.

5 The following advertisement appeared in the local newspaper:

One-Year Business Computer

Diploma

EARN WHILE YOU LEARN!!

Was R18 999 now only R11 999

What percentage discount is this company offering for this course?

6 A sheep farmer had 78 new-born lambs. After unexpected cold weather he had only 56 lambs left because many of them died. What percentage of his flock of lambs did he lose?

7.1 Rashid’s salary went up from R4 500 a month to R4 800 a month. What is his percentage increase?

7.2 Tshapelo’s earns R5 000 a month and his salary also went up by R300 a month. What percentage increase did he get?

PERCENTAGE

Calculating an original amount after a percentage change

Lesson 5

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues.
Assessment Standard
AS 11.1.1
In a variety of contexts find ways to explore and analyse situations that are numerically based, by:
- Estimating efficiently.
- Working with formulae by hand and with a calculator.
- Showing awareness of the significance of digits.
- Checking statements and results by doing relevant calculations.

(The range of problem types includes percentage, ratio, rate and proportion.)
AS 11.1.1
Relate calculated answers correctly and appropriately to the problem situation by:
- Interpreting answers in terms of the context.
- Reworking a problem if the first answer is not sensible or if the initial conditions change.
- Interpreting calculated answers logically in relation to the problem and communicating processes and results.

Overview

In the previous lessons we looked at the following calculations:

- Changed fractions or ratios into percentages;
- Calculated a percentage of an amount;
- Added a percentage of an amount to an amount
- Subtracted a percentage of an amount from an amount; and
- Calculated a percentage change.

Lesson

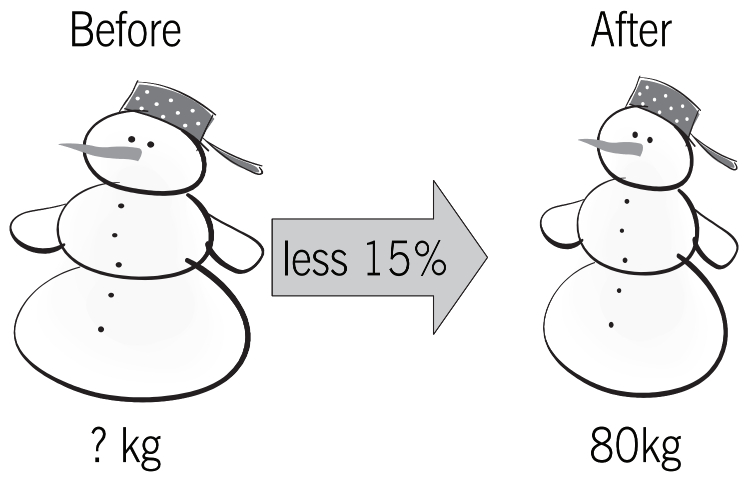
LC DVD

In this lesson we will consider the last type of percentage calculation. We will:

Calculate an original amount after a percentage change.

An example of where you would need to do this type of calculation is the following:

A man went on a diet and lost 15% of his body weight. He weighs 80 kg now and you want to know how much he weighed originally. See the following diagram:



Methods and worked examples

It is important in this example to establish where the 100% is. We start off with a man, i.e. 100% of him, his original weight. He goes on diet and loses 15% of his weight; therefore we must subtract 15% from this 100%. We end up with 85% of the man, which we know to be 80 kg.

To determine how much the man weighed originally, we recognise that:

80 kg corresponds to 100% – 15% = 85%

Therefore $\frac{80}{85}$ will give us how many kilograms correspond to 1%.

Which implies that 0,9412… kg = 1%
(calculator key strokes **80/85%**)

And so 100% will be 0,941 × 100 = 94,12 kg

The man originally weighed 94,12 kg.

It is easy and important to check this answer – you can do this by using the skills you developed in lesson 3 on subtracting an amount from an amount.

Subtract 15% from 94,12 kg.

Key strokes: **94,12-15%**

This gives an answer of 80,002 kg (close enough to 80!), which shows that our answer of 94,12 kg was correct. It is important to check a solution to a problem.

Consider the following additional examples:

LC INDIVIDUAL

Additional example 1

1. What is the VAT exclusive cost of an item that costs R13,68? VAT is 14%.

LC FORMATIVE ASSESSMENT

Additional example 2

2. The price of petrol increased by 7% last night and now costs R6,38 a litre. What was the original cost per litre?

Solutions:

(1) The VAT exclusive price is 100%.

Therefore the VAT inclusive price is 100% + 14% = 114%.

R13,68 corresponds to 114%

Therefore $\frac{13,68}{114}$ will give us how much corresponds to 1%.

Therefore R0,12 = 1%

Therefore 100% will be 0,12 × 100 = R12

So the VAT exclusive price is R12.

(2) The original cost per litre is 100%

Therefore R6,38 corresponds to 100% + 7% = 107%

Therefore $\frac{6,38}{107}$ will give us how much corresponds to 1%.

Therefore 0,05962... = 1%

Therefore 100% will be 0,0596 × 100 = R5,96

So the original price is R5,96.

Activity

INDIVIDUAL

SUMMATIVE ASSESSMENT

Practice examples

1 A car salesman offers you a 12,5% discount if you take a demonstration model. He said the price of the car would be R174,563. What would it cost you if you preferred to buy a brand new car?

2 The VAT inclusive price of a CD is R120. If VAT is 14%, how much of the R120 does the music shop keep?

3 A water tank has 2,75 kl of water left after losing 40% of its water due to a leak. How much water was there in the tank originally?

4 A farmer increased his number of cattle by 15% and now has a herd of 230 cattle. What was the size of his original herd?

5 The owner of a restaurant promises to pay you R75 a shift. He claims that this is 20% more than he was paying his waiters last holiday season. What was he paying his waiters before?

6

Jackets – all colours and sizes

Only R245

30% DISCOUNT

Calculate the original price of the jackets.

7 You are travelling from Johannesburg to Cape Town. You stop at Bloemfontein and complain that the journey is so long. Your friend laughs at you and says you have travelled only 30% of the way and that you still have about 1 000 km to go. Calculate the distance from Johannesburg to Cape Town.

8 You and a group of friends go to a restaurant. When the bill comes you notice that the owner has added in a 10% tip for the waitress already because your group had more than six people in it. If the bill came to R930,12, what was the price of the actual meal without the tip?

9 The dimensions of a newly renovated room are 9 m × 4,5 m.

9.1 What is the area of this room?

9.2 Calculate the area of the original room if the area has increased by 44,64%.

9.3 Suggest what the dimensions of the old room could have been.

General examples from all five lessons on percentage.

10 A shop advertises that all shoes can be bought at a 25% discount.

10.1 What is the sale price of a pair of shoes that cost R299?

10.2 What was the original price of a pair of shoes, if the sale price is R187,50?

11 The distance from Durban to East London is 674 km and from Durban to Port Elizabeth is 984 km. If you are travelling from Durban to Port Elizabeth and have reached East London, what percentage of the journey have you completed?

12 Your bus fare of R6,20 goes up by 5%. What will a bus trip cost you now?

13 What is the percentage increase in the price of beef if it increases from R39,99 per kg to R52,99 per kg?

RATIO

An introduction

Lesson 6

Learning Outcome 1
Number and Operations in Context:
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues
Assessment Standard
AS 11.1.1
In a variety of contexts, find ways to explore and analyse situations that are numerically based, by:

- Estimating efficiently
- Working with formulae by hand and with a calculator
- Showing awareness of the significance of digits
- Checking statements and results by doing relevant calculations

(The range of problem types includes percentage, ratio, rate and proportion.)
AS 11.1.1
Relate calculated answers correctly and appropriately to the problem situation by:

- Interpreting answers in terms of the context
- Reworking a problem if the first answer is not sensible or if the initial conditions change

Interpreting calculated answers logically in relation to the problem and communicating processes and results

Overview

In this lesson we start on a new topic: ratio. We will look at what a ratio is and how we use it in daily activities.

Lesson

LC DVD

Ratio is often used in the same breath as proportion and rate. We will deal with proportion and rate in later lessons.

A ratio is a comparison of two (or more) numbers called the terms of a ratio.

A ratio can be expressed in different ways:

- In words a to b
- With a colon $a : b$
- As a fraction $\frac{a}{b}$

Ratios do not have units as we are comparing quantities of the same kind. This will become clearer in the following example.

The example that we are going to focus on is when we make cold drink by mixing water and concentrate. The instructions on side of the bottle of concentrate say that cold drink is made by mixing concentrate to water as follows: 1 : 3 – this is a ratio written with a colon.

It is important to know the order of the ratio i.e. one unit of concentrate to 3 units of water and not the other way round.

Note: the ratio has no units leaving us to choose our own units.

This means we can mix:

1 cup of concentrate with 3 cups of water

or

1 bottle of concentrate with 3 bottles of water

or

1 bucket of concentrate with 3 buckets of water

The mixtures would all taste the same. The only difference would be in the quantity of juice that we have made.

Methods and worked examples

A builder uses ratio when he mixes concrete.

He has to mix three different components when mixing concrete. He needs to mix gravel, sand and cement. He needs to mix these quantities in a particular ratio in order to get the right quality concrete.

There are two different grades of concrete: lower strength concrete and medium strength concrete. The difference between the two grades is in the ratio of the components. See table below.

	gravel	**sand**	**cement**
Low strength	6	3	1
Medium strength	4	2	1

The ratios are given with no units. The builder will therefore choose a unit that suits him. He could mix a low strength concrete by using 6 wheelbarrows of gravel; 3 wheelbarrows of sand; and 1 wheelbarrow of concrete or if he wanted only a small amount, he could mix 6 buckets of gravel; 3 buckets of sand; and 1 bucket of cement.

Note: We will ignore the water that is also used in making concrete as that is not an issue here. Water is only used to mix these components and activate the cement. As the mixture sets, the water evaporates out of it, so the amount of water used by the builder affects only how easy the product is to work with and how long it will take to get hard.

Worked example

Calculate the quantities of gravel and sand that the builder must use to make a medium strength cement, if he has 2 bags of cement.

Solution: The quantity of cement has been doubled. Therefore the amount of gravel and sand must also double in order to keep the ratio the same.

gravel	**:**	**sand**	**:**	**cement**	
4	**:**	**2**	**:**	**1**	**Mixing ratio**
×2 ↓		**×2 ↓**		**×2 ↓**	
8	**:**	**4**	**:**	**2**	**Actual number of bags**

Therefore the builder will need 8 bags of gravel and 4 bags of sand. The size of the bags must all be the same.

These two ratios: 4 : 2 : 1 and 8 : 4 : 2 are called **equivalent ratios**.

4 : 2 : 1 is the **simplest form** of the ratio.

Consider the following additional examples:

Additional example 1a and 1b

INDIVIDUAL

FORMATIVE ASSESSMENT

1. Concrete for the foundations of a house is mixed in the following ratio by volume:

   gravel : sand : cement = 4 : 2 : 1.

   Determine the missing amounts in the following mixtures:

   a 4 bags of cement

   b 12 buckets of gravel

Solutions

(a)

gravel	**:**	**sand**	**:**	**cement**	
4	**:**	**2**	**:**	**1**	**Mixing ratio**
×4 ↓		**×4 ↓**		**×4 ↓**	
16	**:**	**8**	**:**	**4**	**Actual number of bags**

The builder needs 16 bags of gravel and 2 bags of sand.

(b)

gravel	**:**	**sand**	**:**	**cement**	
4	**:**	**2**	**:**	**1**	**Mixing ratio**
×3 ↓		**×3 ↓**		**×3 ↓**	
12	**:**	**6**	**:**	**3**	**Actual number of buckets**

Note: (1) You must use the same unit for each component i.e. all bags or all buckets.

(2) You can multiply any of the terms in the ratio by a number as long as you multiply all of the terms by the same number and the components of the mixture will be in the same ratio.

INDIVIDUAL

SUMMATIVE ASSESSMENT

Activity

Practice examples

Work on your own/with a partner/in groups

1 Khutso is 12 years old and her brother, Vuyani, is 8 years old. Their father says that he will pay them pocket money in the ratio of their ages.

1.1 Write down the ages of Khutso and Vuyani as a ratio. Remember to specify in which order you are writing the ratio.

1.2 Write this ratio in its simplest form.

1.3 How much pocket money will Vuyani get, if Khutso gets R60.

2.1 What will the teacher : learner ratio be if a school, A, of 945 learners has 21 teachers?

2.2 Write the ratio in its simplest form.

2.3 Another school, B, has 1 050 learners and 25 teachers. What is its teacher : learner ratio in simplest form?

2.4 Which has the better teacher : learner ratio school A or school B? Give a reason for your answer.

3 A new grey colour of paint is made by mixing 3 parts of black paint with 5 parts of white.

3.1 Write down the ratio of black : white paint.

3.2 How much white paint do you need to add to the black paint if you have 1,5 l of black paint?

3.3 How many litres of grey paint will you have once you have mixed the black and white paint together?

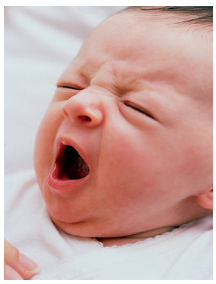
4 A salad dressing is made by mixing vinegar and salad oil in the ratio 2 : 3.

4.1 Explain what this ratio means.

4.2 How much salad oil will I need to make up the dressing if I have 20 ml of vinegar?

4.3 I am catering for a big function and want to make up a large quantity of dressing. How much vinegar will I need to make up the dressing if I have 750 ml of salad oil?

5 Sex ratio is the ratio of males to females in a population. In humans at the time of birth the sex ratio is said to be: 105 : 100



5.1 Explain what is meant by this ratio.
5.2 Write this ratio in its simplest form.
5.3 What is the estimated number of boys that were born if 5 000 girls were born?
5.4 This sex ratio is said to be “at the time of birth”. Do you think this ratio is the same for a population of over 70 year olds? Explain your answer.

Lesson 7

RATIO

Dividing an amount in a given ratio

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues
Assessment Standard
AS 11.1.1
In a variety of contexts, find ways to explore and analyse situations that are numerically based, by:

- Estimating efficiently
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- Checking statements and results by doing relevant calculations

(The range of problem types includes percentage, ratio, rate and proportion.)
AS 11.1.2
Relate calculated answers correctly and appropriately to the problem situation by:

- Interpreting answers in terms of the context
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- Interpreting calculated answers logically in relation to the problem and communicating processes and results

Overview

In this lesson we look at sharing an amount in a given ratio.

Lesson

An example of where this type of calculation would need to be done is the following:

A hairdresser needs to make up a 60 ml mixture of tint and hydrogen peroxide. The ratio of tint : peroxide is 1 : 2. The hairdresser will need to determine how many millilitres of tint and how many millilitres of peroxide to mix to make up the 60 ml mixture.

Methods and worked examples

The important step to solve these problems is to establish how many equal parts the mixture will consist of. In the tint-peroxide problem the mixture will consist of three equal parts.

mixture = 1 part tint + 2 parts peroxide = 3 parts.

It now follows that each equal part is 60 ÷ 3 = 20 ml and:

Tint needed = 1 × 20 ml = 20 ml

Peroxide needed = 2 × 20 ml = 40 ml

Check: 20 ml + 40 ml = 60 ml – the correct total

20 : 40 = 1 : 2 – the correct ratio.

Consider the following additional example:

Additional example

INDIVIDUAL

FORMATIVE ASSESSMENT

Eric invests R2 000 in a business and Aarnout invests R1 500. They agree to share their profits in the same ratio as their investments. How much must each person get if they make a profit of R1 575?

Solution:

The investment ratio is: Eric : Aarnout = 2 000 : 1 500

$$\downarrow \div 500 \downarrow$$

= 4 : 3 (simplified)

If the profit is divided in this ratio then Eric will get four equal parts of the profit and Aarnout three equal parts. Therefore, the profit needs to be divided into seven equal parts (4 + 3 = 7).

1 575 ÷ 7 = 225, therefore, each equal part is equal to R225.

Eric gets: 4 × R225 = R900

Aarnout gets: 3 × R225 = R675

Check: R900 + R675 = R1 575 – the correct total

900 : 675 = 4 : 3 – the correct ratio.

Activity

INDIVIDUAL

SUMMATIVE ASSESSMENT

Practice examples

1. Soso has four children and Zoleka has five children. They divide a box of apples in the same ratio as the number of children they have. If there are 108 apples in the box, how many apples does each of the mothers take home?
2. The ratio of boys to girls at Winfield Primary School is 3 : 2. How many boys attend the school if there are 550 learners at the school?
3. Jason and Lee worked at the soccer stadium and were given R150 to share at the end of the day. Jason had worked for six hours and Lee had worked for three hours. How much did each boy earn if they shared the money in the same ratio as the number of hours they worked?
4. Xoliswa wants to make up $1\frac{1}{2}$ litres of orange juice. The instructions on the bottle of concentrate are:

   mix concentrate : water in the ratio 1 : 3.

   Calculate the volume of concentrate and water needed.
5. Ismail (12), Darshan (7) and Zahrah (5) received R1,200 from their grandmother. They are told to share it in the ratio of their ages. Calculate how much each child receives.
6. Concrete for the foundations of a house are mixed in the following ratio by volume:

   gravel : sand : cement = 4 : 2 : 1.

   A builder needs to mix 6 kl of concrete. Calculate the quantities of each substance that he needs.
7. Samson invests R5 000 in a business and Edward invests R1 500.

   They agree to share their profits in the same ratio as their investments.

   How much must each person get if they make a profit of R3 250?

Lesson 8

PROPORTION

Direct proportion

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues
Assessment Standard
AS 11.1.1
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(The range of problem types includes percentage, ratio, rate and proportion.)
AS 11.1.2
Relate calculated answers correctly and appropriately to the problem situation by:
- Interpreting answers in terms of the context.
- Reworking a problem if the first answer is not sensible or if the initial conditions change.
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AS 11.2
Draw graphs (by hand and/or by means of technology, where available) as required by the situations and problems being investigated.
AS 11.2.3
Critically interpret tables and graphs in a variety of real-life and simulated situations by:
- Estimating input and output values.

Overview

In this lesson we continue working with ratios and begin to focus on a related concept – proportion. In particular we will focus on direct proportion.

Lesson

When two ratios are equal, for example if a : b = c : d, we say that the four quantities, a, b, c and d form a proportion.

In addition, we say that two variables are in direct proportion to each other if they are always in the same ratio.

In direct proportion situations one quantity increases as the other quantity increases and any pair of the two quantities is always in the same ratio as any other pair of the same quantities.

Examples of direct proportion include:

- Scales on plans or maps;
- Buying recipe ingredients for a certain number of people;
- The relationship between travelling time at constant speed and distance; and
- The relationship between the length of time your electric lights are on and the amount of electricity you use.

Methods and worked examples

Consider the scale on a house plan, which is given as 1 : 50. This means that every one unit on the plan corresponds to 50 units on the ground.

1 cm on the plan corresponds to 50 cm on the ground, similarly 1 mm on the plan corresponds to 50 mm in the ground and 3 cm in the plan corresponds to 3×50 cm $=$ 150 cm on the ground. The length on the plan is in direct proportion to the length on the ground.

Worked example:

If the width of a bedroom on the plan is 6 cm, calculate the actual width of the bedroom (i.e. the width of the bedroom on the ground).

plan : ground

1 : 50

↓ ÷6 ↓

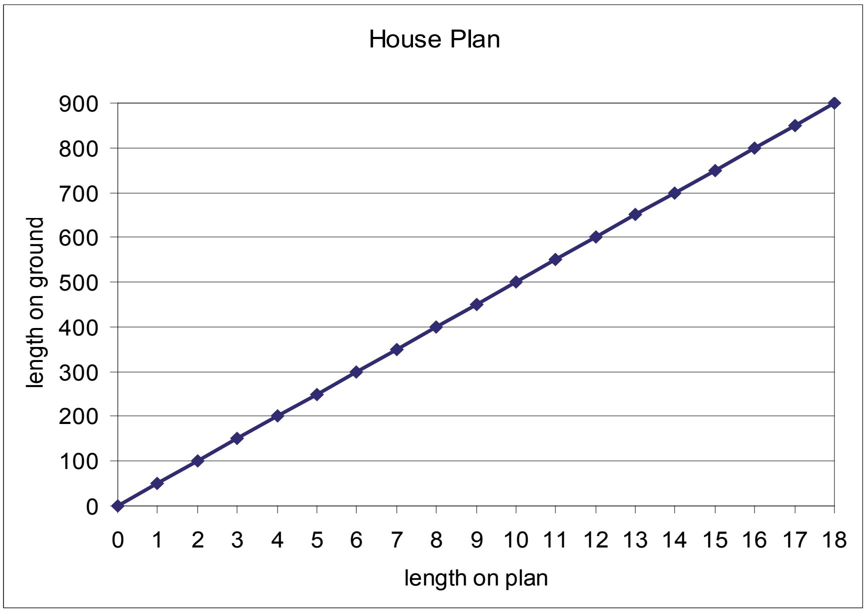
6 : 300

plan : ground

You could calculate all the dimensions on a plan in this way or you could draw a table of values and read off the answers. See below.

Length on plan (cm)	1	2	3	4	5	6	7	8	9	10
Length on ground (cm)	50	100	150	200	250	300	350	400	450	500
(m)	0,5	1,0	1,5	2,0	2,5	3,0	3,5	4,0	4,5	5,0

If you drew up a table like the one above, you could just read off the answer: 6 cm on the plan corresponds to 300 cm on the ground. However, this table would not be much use if the distance on the plan was say 3,7 cm. In this case a graph might be more useful, allowing you to read off the value from the graph. See below.



Note how:

- As the length on the plan increases, the length on the ground also increases.
- The graph is a straight line and passes through the origin (0;0).

These are characteristics of direct proportion relationships. Direct proportion relationships can be represented in a number of ways.

(1) In symbols:

distance (ground) α distance (plan)

(2) As an equation:

distance (ground) $= 50 \times$ distance (plan)

This is called the proportionality constant

(3) As a ratio:

distance (ground) : distance (plan) = 50 : 1

Consider the following additional example:

INDIVIDUAL

FORMATIVE ASSESSMENT

Additional example 1

If five bags of sweets contain 120 sweets in total, how many sweets will there be in seven bags?

Solution:

bags : sweets

5 : 120

$\downarrow \div 5 \downarrow$

1 : 24

$\downarrow \times 7 \downarrow$

7 : 168

These are all equivalent ratios (a characteristic of direct proportion relationships):

5 : 120 = 1 : 24 = 7 : 168

There will be a total of 168 sweets in seven bags of sweets.

Activity

INDIVIDUAL

SUMMATIVE ASSESSMENT

Practice examples

1 The following recipe caters for six people:

600 g chicken fillets

250 g mushrooms

3 tomatoes

$\frac{1}{2}$ cup chicken stock

30 ml flour

1.1 Calculate the quantity of each of the ingredients if you want to make the recipe for 12 people.

1.2. Calculate the quantity of each of the ingredients if you want to make the recipe for eight people.

2. A map of South Africa has a scale 1 : 50 000.

2.1 What will the distance on the ground (in cm) be if the distance on the map is 2 cm?

2.2 What is this distance in kilometres?

2.2 What will the distance on the map be if the distance on the ground is 6 km?

3. Three chocolate bars cost R7,50.

3.1 Draw a table to show how the price of chocolate bars increases from one bar to eight bars.

3.2 Using the values in the table above, plot a graph and determine how much 12 bars will cost.

4. You need 6,28 m of fencing to enclose a circular flower bed with a diameter of 2 m. How much fencing do you need to enclose a flower bed with a diameter of 1,5 m?

5. If it typically takes you six hours to travel 540 km, how long will it take you to travel 700 km if you assume that you can travel at a constant speed?

6. My watch gains five minutes every eight hours. How many minutes will it gain in 24 hours?

7. A builder uses five pockets of cement to lay a floor of area 12 m^2. How many pockets of cement will he need to lay a floor of 20 m^2?

8. My car can travel 285 km on 30 litres of petrol. Estimate approximately how much petrol I will need to travel 650 km?

Lesson 9

PROPORTION

Inverse proportion

Learning Outcomes and Assessment Standards

Learning Outcome 1
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Relate calculated answers correctly and appropriately to the problem situation by:
- Interpreting answers in terms of the context.
- Reworking a problem if the first answer is not sensible or if the initial conditions change.
- Interpreting calculated answers logically in relation to the problem and communicating processes and results.

AS 11.2.2
Draw graphs (by hand and/or by means of technology, where available) as required by the situations and problems being investigated.
AS 11.2.3
Critically interpret tables and graphs in a variety of real-life and simulated situations by:
- Estimating input and output values.

Overview

In the previous lesson we looked at direct proportion: situations/relationships where one quantity increases as the other quantity increases AND any pair of the two quantities is always in the same ratio as any other pair of the same quantities.

In this lesson we focus on situations/relationships where one quantity decreases as the other quantity increases.

Lesson

LC DVD

Deciding on the number of wedding guests to invite when you have a fixed amount you want to spend on the catering is an example of an inverse proportion situation. As the cost per head increases, the number of guests you can invite decreases. Notice that the amount of money you have to spend stays the same.

Examples of inverse proportion include:

- The relationship between the length and width of a rectangle when the area of the rectangle is constant. As the length of the rectangle increases, the width decreases;
- The relationship between the speed at which you drive and the time taken to complete a journey if the distance of the trip is fixed. The faster you drive, the shorter time it will take you; and
- The relationship between the number of people needed to harvest grapes and

the time taken to harvest the grapes if the size of the vineyard is fixed. As the number of people increases, the time taken decreases. (Although it should be noted that this example breaks down as the number of people gets too large for the working conditions.)

Methods and worked examples

Consider keeping the area of a rectangle constant, while changing the lengths of the sides.

Worked example:

The width of a rectangle is 2 cm and its length is 10 cm. What will the length be if the width is increased to 4 cm and the area remains unchanged?

Area of the rectangle $= 2 \text{ cm} \times 10 \text{ cm} = 20 \text{ cm}^2$

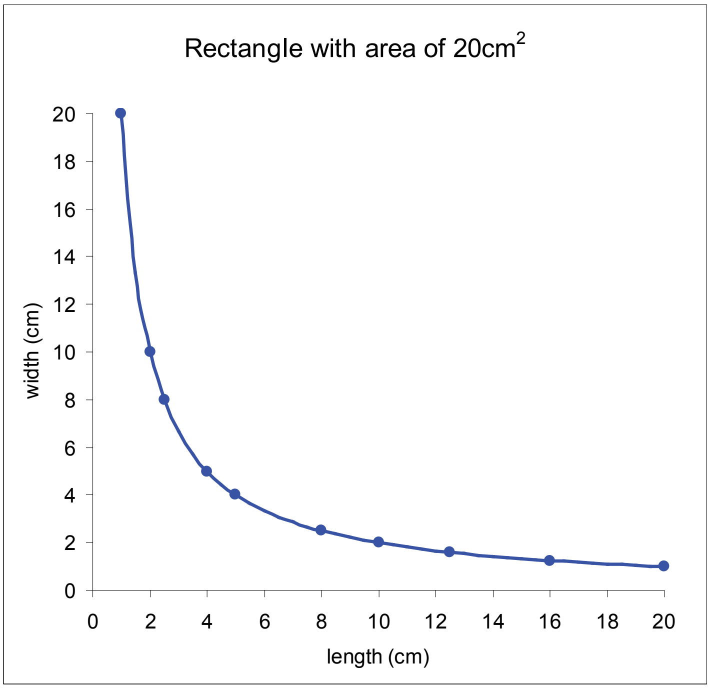
Since the area remains constant, the new length $= 20 \text{ cm}^2 \div 4 \text{ cm} = 5 \text{ cm}$.

You can find many more combinations of lengths and widths that will give an area of 20 cm^2 by drawing a table. See below.

Length of rectangle (cm)	1	1,25	1,6	2	2,5	4	5	8	10	20
Width of rectangle (cm)	20	16	12,5	10	8	5	4	2,5	2	1

Note: as the length increases, the width decreases and the area remains 20 cm^2.

Drawing a graph will give all possible combinations of lengths and widths to get an area of 20 cm^2. See below.



Note: the graph of an inverse proportion relationship is known as a rectangular hyperbola.

The above situation can be represented in a number of ways.

(1) In symbols:

$$\text{width } \alpha \frac{1}{\text{length}}$$

(2) As an equation:

width $= \frac{20}{\text{length}}$

or width $\times$ length $= 20$ (This is called the constant product)

Consider the following addtional examples:

INDIVIDUAL

FORMATIVE ASSESSMENT

Additional example 1a–b

1. How many bottles of water can you fill from a barrel with 200 litres if:
   - (a) The volume of each bottle is 5 litres?
   - (b) The volume of each bottle is 500 ml?

Solution:

For the 5 litre bottles: Number of bottles $= \frac{200 \text{ l}}{5 \text{ l}} = 40$ bottles.

For the 500 ml bottles: (500 ml = 0,5 litres): Number of bottles $= \frac{200 \text{ l}}{0{,}5 \text{ l}} = 400$ bottles.

Notice how as the volume of the bottles increases so the number of bottles that can be filled decreases and that the amount of water being used remains the same – the proportionality constant.

INDIVIDUAL

FORMATIVE ASSESSMENT

Additional example 2a–d

2. A piece of string, 30 cm long, is made into the shape of a rectangle.
   - (a) What will the length of the rectangle be if the width is 4 cm?
   - (b) Complete the following table:

Length of rectangle (cm)	1	2	3	4	5	7	8	11	12	13	14
Width of rectangle (cm)	14	13	12	11	10	8	7	4	3	2	1

   - (c) Does the width decrease as the length increases?
   - (d) Is this an inverse proportional problem? Justify your answer.

Solution:

(a) Since the perimeter of the rectangle is 30 cm, we get: 30 cm = 2 (4 cm + length) from which it follows that: length = 11 cm.

(b)

Length of rectangle (cm)	1	2	3	4	5	7	8	11	12	13	14
Width of rectangle (cm)	14	13	12	11	10	8	7	4	3	2	1

(c) Yes.

(d) No. The product of the width and the length is not constant. See below:

$1 \times 14 = 14$

$2 \times 13 = 26$

$3 \times 12 = 36$

$4 \times 11 = 44$

Activity

INDIVIDUAL

SUMMATIVE ASSESSMENT

Practice example

1. You are having a party for your 21st birthday. You budget R5 000 for the food.
   1.1 How many people can you invite if the caterer quotes R50 a head for a finger supper?
   1.2 How many people can you invite if the caterer quotes R125 a head for a three-course meal?
2. The recommended daily dosage of a particular vitamin is 1 000 mg.
   2.1 How many tablets of tablet A will you need to take if each tablet A contains 500 mg of that vitamin?
   2.2 Tablet B contains 250 mg of that vitamin. How many tablet Bs will you need to take to get the recommended daily allowance?
3. A carpenter can cut eight pieces of wood 30 cm long from a length of wood. How many pieces, each 60 cm long, will he be able to cut out of the same length of wood?
4. A rectangle has dimensions 30 cm x 10 cm. What will the new width be if the length changes to 25 cm and the area remains the same?
5. There are 45 children in a class. I have enough sweets to give each child four sweets. However, five children are absent on the day. How many sweets can each child receive now?

Lesson 10

RATE

An introduction

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues.
Assessment Standard
AS 11.1.1
In a variety of contexts find ways to explore and analyse situations that are numerically based, by:

- Estimating efficiently.
- Working with formulae by hand and with a calculator.
- Showing awareness of the significance of digits.
- Checking statements and results by doing relevant calculations.

(The range of problem types includes percentage, ratio, rate and proportion.)
AS 11.1.2
Relate calculated answers correctly and appropriately to the problem situation by:

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Overview

In this lesson we continue our study of ratios, but we will look at a special family of ratios called rate.

- A rate is a ratio where the two quantities being compared have different units.
- The unit of the rate is given by the ratio of the unit of the first quantity to the unit of the second quantity.
- A ratio has no units.

Lesson

L C DVD

In our first example of ratio we mixed cold drink in the ratio:

concentrate : water = 1 : 3

We could choose the unit we wanted to work with, for example we could use one cup of concentrate and three cups of water or one jug concentrate to three jugs water.

By contrast, a rate is a ratio with units. If we buy a 2 litre bottle of milk and pay R8,50 for it, we can write this as a comparison: cost : amount or quantity = R8,50 : 2 litres.

This time the units are important and we say "R8,50 per 2 litres". The unit of the rate of the is rands per litre or R/litre.

Supermarkets often give us the unit rate of groceries to help us compare prices of products so that we can get the best value for our money.

- Unit rates compare the first quantity to one unit of the second quantity.

Examples of rates include:

- Electricity rates (tariffs);

- Exchange rates;
- The petrol price is quoted in rands per litre;
- Crime rates; and
- Inflation rates.

Methods and worked examples

Which one of the following items gives better value for money:

5 kg of coffee at a cost of R165; or

2 kg of coffee at a cost of R58?

To be able to compare these two quantities we need to change the given rates to unit rates.

5 kg of coffee: Rate = $\frac{\text{R165}}{\text{5 kg}} = \frac{\text{R33}}{\text{1 kg}} =$ R33/kg

2 kg of coffee: Rate = $\frac{\text{R58}}{\text{2 kg}} =$ 291kg = R29/kg

The unit rate of the 2 kg tin of coffee is cheaper than the 5 kg unit rate and is, therefore, the better buy.

Consider the following additional example:

Additional example 1

LC INDIVIDUAL

LC FORMATIVE ASSESSMENT

How much must I pay for electricity if I use 255 kWh and the municipality charges 36,06c/kWh?

Solution:

Cost = consumption × rate

= 255kWh × 36.06c/kWh = 9,195.3c

Therefore you will pay R91,95.

Activity

LC INDIVIDUAL

LC SUMMATIVE ASSESSMENT

Practice examples

1 Tickets to a concert cost R35/child and R100/adult. Calculate how much it would cost a family consisting of two adults and three children.

2. Muffins at the local bakery cost R3,50 each, or you can buy a pack of four for R13.

   2.1 Which is the more economical buy?

   2.2 When would it not be wise to buy the more economical muffins?

3. Calculate the unit rate per kilogram of the following item in cents:

   5 kg of sugar at R10,99

4. You earn R60 for eight hours of work and your friend earns R70 for nine hours of work. Who gets the better hourly rate?

5. Calculate the petrol consumption of my car in km per litre if I used 50 litres of petrol to travel 520 km.

6. The rand/dollar exchange rate is R7,12/$.

6.1 Calculate the number of rands I would receive if I exchanged $35 for rands.

6.2 How many dollars can I buy for R8 000?

7. Which is the better buy:

500 g of butter @ R17,99 or 250 g of butter @ R11,49?

RATE

Average rate

Lesson 11

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues.
Assessment Standard
AS 11.1.1
In a variety of contexts find ways to explore and analyse situations that are numerically based, by:
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- Showing awareness of the significance of digits.
- Checking statements and results by doing relevant calculations.

(The range of problem types includes percentage, ratio, rate and proportion.)
AS 11.1.2
Relate calculated answers correctly and appropriately to the problem situation by:
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Overview

In this, and the next three lessons, we will focus on three kinds of rate:

- Average rate
- Constant rate; and
- Varying rate

Lesson

LC DVD

In this lesson we will consider average rate. Average speed is an example of average rate. A driver's speed varies as he drives because it is not possible to maintain the same speed throughout a journey. Sometimes he/she will go faster than at other times, but if he/she takes five hours to drive 500 km; we say that his/her average speed is 100 km/h.

Other examples of average include:

- The petrol consumption of a car – reported in litres per 100 km;
- The average rainfall for months of the year – reported in mm per day/week/month/year; and
- The run rate of a cricketer – reported in runs per inning.

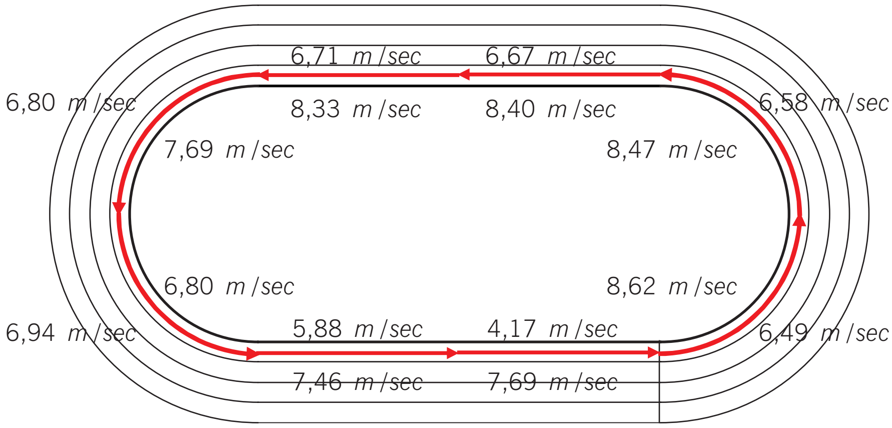
Methods and worked examples

Two athletes, Paul and Hendrick, compete in a 400 m running race. Paul starts out at a fast pace, but is unable to maintain his speed and slows down at the end of the 400 m. Hendrick runs a more even-paced lap and manages to achieve the same finishing time.

Worked example:

The diagram of the 400 m track below shows the speed of the two athletes for every 50 m. Paul's results are on the inside of the track and Hendrick's results are on the outside of the track. We can see that Paul (the results on the inside of the track) started the race at a faster pace than Hendrick, but finished at a slower pace. The athletes had exactly the same finishing time for the race and, if we calculate their average speed over the whole 400 m, we get the same average speed. We calculate their average speed as follows:

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$
$$= \frac{400 \text{ m}}{58 \text{ s}}$$
$$= 6{,}90 \text{m/s}$$



To summarise:

- Two athletes can have exactly the same finishing time for a race and, by having the same finishing time for the same distance, their average speed will be the same;
- The two runners might have run the race very differently;
- The average rate gives us an overall picture of the event and not the details; and
- The average rate does not imply the runners ran at that speed for the entire race.

Consider the following additional example:

Below is a record of the amount per month Soso spends on telephone calls. Calculate the average amount per month that she spends on her phone bill.

Month	**Amount**
January:	R342,75
February:	R267,53
March:	R453,65
April:	R245,98
May:	R320,56
June:	R403,76

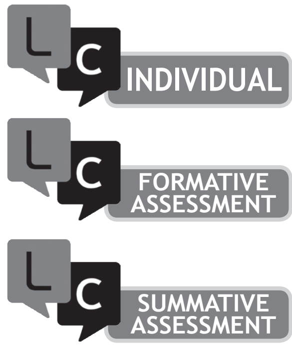
Solution:

Average rate = total amount/total number of months

$= \frac{\text{R2 034,23}}{\text{6 months}}$

= R339,04 per month

Activity



Practice examples

1 Jusaf works as a car guard. Below is a record of his earnings for one week.

Monday:	R48,00
Tuesday:	R56,50
Wednesday:	R43,60
Thursday:	R84,00
Friday:	R76,30
Saturday:	R104,60
Sunday:	R34,00

Calculate his average earning per day.

2 Rebecca travels 225 km to visit her friend. The graph below shows the time it takes her to travel certain distances.

2.1 Did Rebecca travel at the same speed for the entire journey? Explain your answer.

2.2 How long did the journey take in hours?

2.3 What happened after 30 minutes of travelling?

2.4 Calculate her average speed for the journey.

3. The average petrol consumption for Josie's car is 9,6 litres/100 km.

   3.1 Why is the petrol consumption of a car an example of average rate?

   3.2 Calculate approximately how many litres of petrol Josie will use for a journey of 780 km.

4. A family of five consumes on average 2,5 litres of milk a day.

   4.1 Does this mean that every day the family drinks exactly 2,5 litres of milk? Explain your answer.

   4.2 Why is it useful to calculate the average daily milk consumption of a family?

   4.3 The price of a 2 litre container of milk is R9.56. Calculate how much a family of five must budget for their monthly consumption of milk. Assume a month has 30 days.

5. You work as a waitron in a restaurant. You earn on average (including tips) R110 a day.

   5.1 Why is this an example of average rate?

   5.2 Calculate how much you could earn in a week (six working days).

RATE

Constant rate

Lesson 12

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues
Assessment Standard
AS 11.1.1
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- Checking statements and results by doing relevant calculations

(The range of problem types includes percentage, ratio, rate and proportion.)
AS 11.1
Relate calculated answers correctly and appropriately to the problem situation by:

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Overview

In this lesson we look at constant rate.

Lesson

DVD

We saw in the previous lesson that average rate does not tell you actually what happened at any one time, but gives you the overall effect of what happened. In constant rate situations the rate between the two quantities being compared is independent of the size of the quantities being compared.

An example of where it is important that the rate remains constant throughout the time period is when a patient is receiving medication via an intravenous drip.

Methods and worked examples

The intravenous drip sets used in hospitals are rates in terms of the number of drops that they deliver per ml. There are typically three types: 15 drops/ml; 20 drops/ml and 60 drops/ml (used with children). Nurses must calculate the number of drops per minute to which they must adjust the drip to ensure that the patient receives the dosage prescribed by the doctor.

Worked example:

To what rate must the 15 drops/ml drip be adjusted if 1 litre of solution must be administered over a period of six hours?

The rate of the solution dosage is:

Dosage rate: $= \frac{1\,000 \text{ ml}}{6 \times 60 \text{ min}}$

$= 2{,}77$ ml/min

Drip factor: $= 15$ drops/ml

Therefore the drip rate is: $= 2{,}77$ ml/min $\times$ 15 drops/ml

$= 41{,}66$ drops/min

≈ 42 drops/min (rounded up)

This means that patient will be receiving the medication at a rate of 42 drops per minute.

Consider the following additional example:

INDIVIDUAL

FORMATIVE ASSESSMENT

Additional example 1

A drip bag contains 450 ml of medication. The 20 drops/ml drip set is delivering 50 drops/min. How long will it be before the nurse must replace the bag?

Solution:

Flow rate $= 50$ drops/min

Drip rate $= 20$ drops/ml

Therefore the dosage rate is $= \frac{50 \text{ drops/min}}{20 \text{ drops/ml}}$

$= 2{,}5$ ml/min

Time taken to administer 450ml of the solution $= \frac{450 \text{ ml}}{2{,}5 \text{ ml/min}}$

$= 180$ minutes

$= 3$ hours

INDIVIDUAL

SUMMATIVE ASSESSMENT

Activity

Practice examples

1. A doctor prescribes 3 litres of fluid to be given to a patient over 24 hours. The type of drip to be used has a drip factor of 15 drops/ml. Calculate the drip rate.
2. A doctor prescribes 500 ml of fluid to be given to a patient over four hours. The type of drip to be used has a drip factor of 20 drops/ml. Calculate the drip rate.
3. A drip bag contains 450 ml of medication. The 15 drops/ml drip set is delivering 31 drops/min. How long will it be before the nurse must replace the bag?
4. A water tank has a capacity of 10 kl. A pump pumps water out at a constant rate of 25 l/min. How long will it take to empty the tank?

RATE

Varying rate

Lesson 13

Learning Outcomes and Assessment Standards

Learning Outcome 1
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The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues
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- Checking statements and results by doing relevant calculations

(The range of problem types includes percentage, ratio, rate and proportion.)
AS 11.1.2
Relate calculated answers correctly and appropriately to the problem situation by:

- Interpreting answers in terms of the context
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Overview

In the previous two lessons we looked at average rate – what the rate would have been if it had been constant and constant rate – where the rate between two quantities is independent of the size of the quantities being compared, i.e. it is constant. In this lesson we will focus on varying rate – where the rate changes all the time.

Lesson

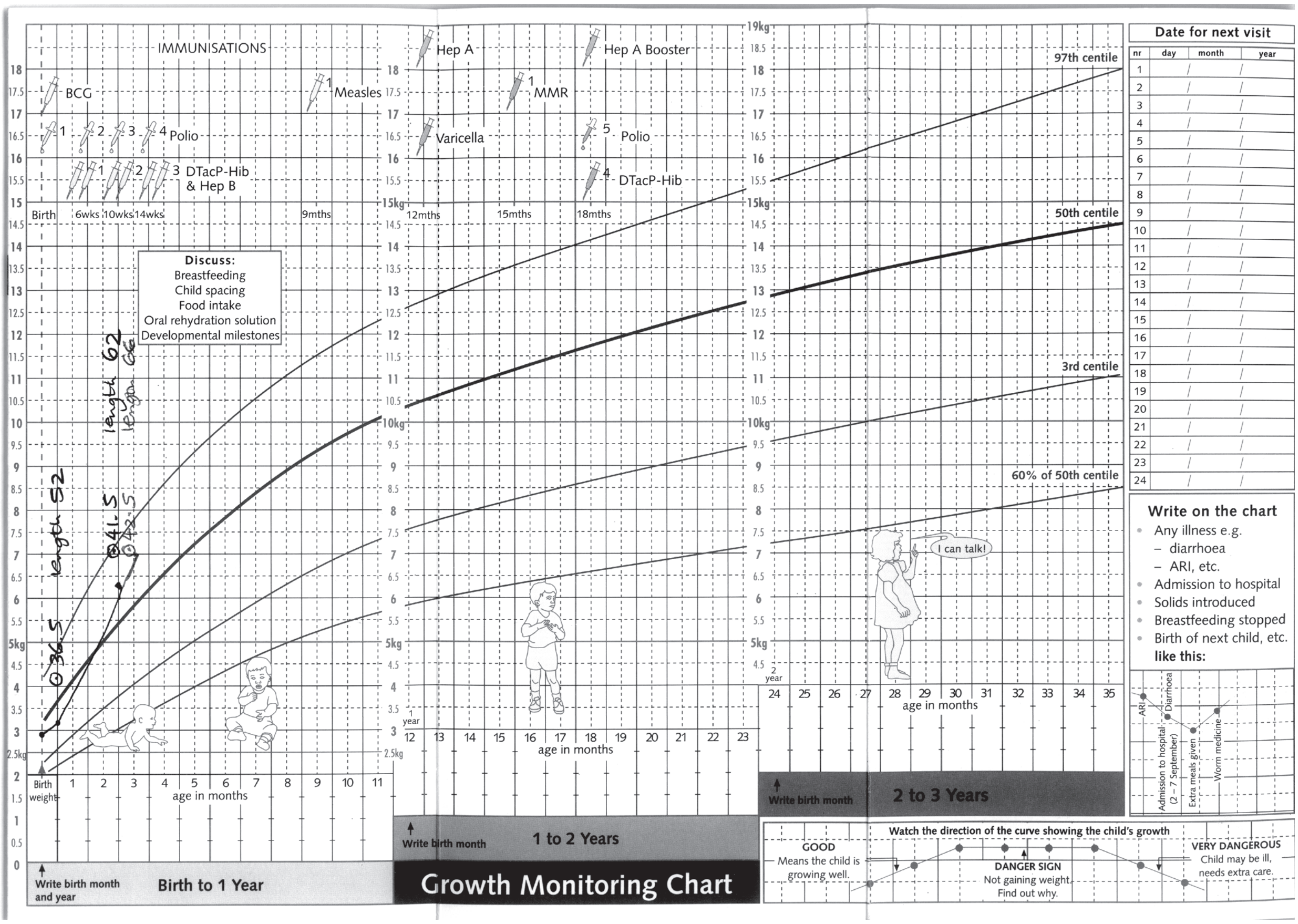
When working with varying rate, it is more important to consider the trend in the rate rather than the average rate.

Examples of varying rate include:

- Children's growth patterns as described in a Road to Health Chart (Department of Health, 2002);
- Exchange rates; and
- The gold price.

Methods and worked examples

Below is the **Road to Health Chart** (Department of Health, 2002).



The lines on the charts represent the percentiles. For example the 50th percentile (dark line on each of the charts) means that 50% of South African children have a weight less than this weight and 50% of South African children have a weight more than this weight. These lines do not tell you how a child's weight will change, but they allow you to compare a child's weight with other children of his/her age.

Worked example:

Consider the 50th percentile line. The amount by which a baby's weight changes each month is as follows:

Age	Weight	Change in weight
Birth	3,25 kg	
1 month	4,15 kg	0,90 kg
2 month	5 kg	0,85 kg
3 month	5,8 kg	0,80 kg
4 month	6,55 kg	0,75 kg
5 month	7,25 kg	0,70 kg

Note:

- The baby's weight increased each month;
- The amount by which the baby's weight increased was different every month; and
- The amount by which the baby's weight increased was less each month.

The growth rate of the baby changed each month. The first month was 0,9 kg/month, whereas the fifth month was 0,7 kg/month. Although the baby's weight was still increasing, the rate at which this was happening was decreasing.

If we go to 18 months the rate of change has decreased still further. See below:

Age	Weight	Change in weight
18 months	11,8 kg	
19 months	11,9 kg	0,15 kg
20 months	12,05 kg	0,10 kg

Note again:

- The baby's weight increased each month;
- The amount by which the baby's weight increased was different every month; and
- The amount by which the baby's weight increased was less each month.

We notice two trends in this example:

- The weight of the baby is increasing each month – growth rate is positive; and
- The growth rate is slowing down over time.

Of course we can calculate the average growth rate over 20 months as follows (we can calculate anything).

$$\text{Average growth rate} = \frac{12{,}05\text{ kg} - 3{,}25\text{ kg}}{20\text{ months}}$$
$$= 0{,}440\text{ kg/month}$$

However, average growth rate is neither useful nor meaningful the weight change varied through the months. Furthermore, there is nothing that could be done with the average weight change – a mother couldn't use it to monitor her childs growth etc. It is more important to be able to discern the trend in the growth rate from the graph than it is to calculate the average growth rate.

Activity

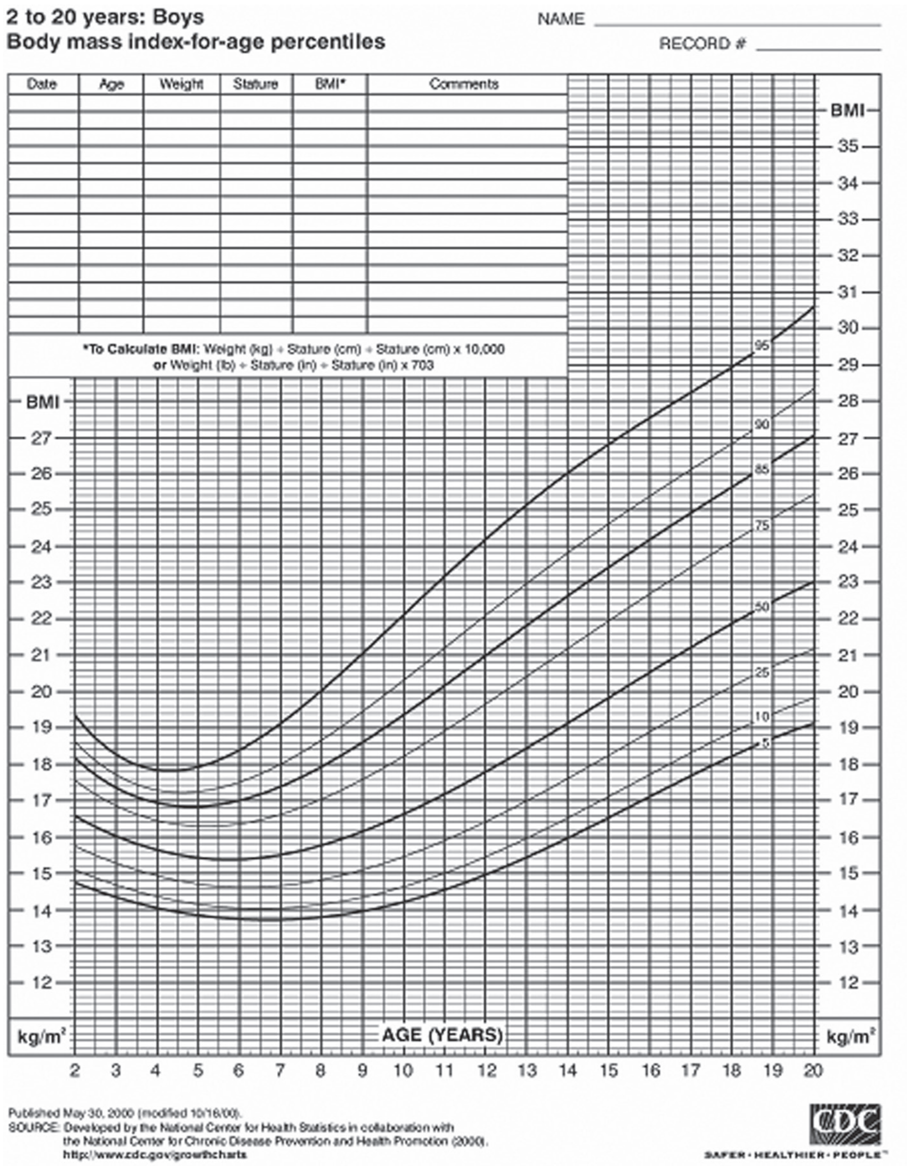
LC INDIVIDUAL

LC SUMMATIVE ASSESSMENT

Practice examples

1 On the next page is a chart showing the percentiles for the body mass index (BMI) for boys from two to 20 years.

BMI is a number calculated from a child's weight and height. It is an indicator of body fatness for most children and teens.



Use the chart to answer the following questions:

1.1 Using the 50th percentile, read off and record the BMI values for each year from two years of age to six years of age.

1.2 Find the amount by which the BMI is changing per year for these years.

1.3 Describe the trend in the BMI over the period from two to six years and the trend in the change of BMI per year.

1.4 Using the 50th percentile, read off and record the BMI values for each year from six years of age to 12 years of age.

1.5 Find the amount by which the BMI is changing per year for these years.

1.6 Describe the overall trend in the BMI over the period from six to 12 years and the trend in the change of BMI per year.

1.7 Explain why it would not be useful to calculate the average change of BMI per year over 20 years.

2. Below is a graph of the prime interest rate from the period January 1991 to December 2005.

Use the graph to answer the following questions:

2.1 In which month and year was the prime interest rate at its highest?

2.2 In which month and year was the prime interest rate at its lowest?

2.3 Describe what happened to the prime interest rate from March 1998 to September 2002.

2.4 Would it be useful to calculate the average change in prime interest rate per year for the 15-year period shown above? Explain your answer.

Lesson 14

FINANCE

Income and expenses

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues.
Assessment Standard
AS 11.1.1
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AS 11.1.3
Apply mathematical knowledge and skills to plan personal finances and investigate opportunities for entrepreneurship inclusive of:
- Specifying and calculating the value of income and expenditure items.
- Estimating and checking profit.

Overview

In this lesson we look at a new topic – finances. There are two key variables in most situations that involve money:
- Income – the money received by an individual or a company; and
- Expenses – the money spent by an individual or a company.

Lesson

These variables give rise to three important equations:
- income > expenses
- income = expenses
- income < expenses

The most desirable situation is for your income to exceed your expenses. This results in a surplus of money, which can be saved. If your income equals your expenses, we say that you are breaking even and if your income is less than your expenses you will need to borrow money to survive.

Methods and worked examples

To establish the right relationship between income and expenses requires planning. There are two key instruments to help you do this:
- Income and expenditure statements; and
- Budgets.

Income and expenditure statements are a record of the actual income and expenses for a given period of time.

Budgets show the projected income and expenses for a given period of time and are often based on previous income and expenditure statements.

Income typically includes the following:

- Salary/wages;
- Occasional income (part-time jobs);
- Interest from savings;
- Student or other loans; and
- Inheritance.

Expenses typically include the following:

- Rent/home loan repayments;
- Services (electricity, water …);
- Groceries (food, cleaning materials);
- Transport (car payments; petrol; bus fare);
- Telephone (landline and cellular);
- Loan repayments;
- Insurance;
- Clothing;
- Entertainment;
- Savings/investments; and
- Donations to charities.

As we develop and collect income and expenditure statements for a number of months, we will start to see patterns and these will assist us to plan ahead – help us to develop a budget. However, not all of the items will appear every time – there are two kinds of income and expenses:

- Fixed income and expenses – these are amounts that remain constant from one month to the next, e.g. salary and rent; and
- Variable income and expenses – these are amounts that vary from one month to the next, e.g. groceries and haircuts.

The challenge of budgeting is to do as much as possible with your income and this involves making choices. It is helpful to classify expenses into high-priority expenses, such as food, and lower-priority expenses, such as clothes. This classification is personal: what is a high priority for one person might not be for someone else.

Worked example:

The following statement shows some typical income and expenditure categories for a family. Each item has been classified as fixed or varying. The expenses have been classified as high or low priority.

INCOME		EXPENSES		
Salary	fixed	Home loan	fixed	high
Interest on savings	varying	Services (rates, water, electricity etc)	varying	high
Wages (part-time)	varying	Food	varying	high
		Transport	varying	high
		Education	fixed	high
		Telephone	varying	low
		Entertainment	varying	low
		Clothes	varying	low

INDIVIDUAL

PAIRS

Activity

Practice examples

1 Define the following terms in your own words:

1.1 Income and expenditure statement;

1.2 Budget;

1.3 Fixed income or expense;

1.4 Varying income or expense;

1.5 High-priority expense; and

1.6 Low-priority expense.

2.1 Draw up an income and expenditure statement for yourself or your family for the last three months. Try to use actual values for your income and expenses. Classify each item as fixed or varying and classify your expenses as either high or low priority.

2.2 Which of the following three equations best describes your situation:

- income > expenses
- income = expenses
- income < expenses?

FINANCE

Calculating the cost price of an item

Lesson 15

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues.
Assessment Standard
AS 11.1.1
In a variety of contexts find ways to explore and analyse situations that are numerically based, by:
- Estimating efficiently.
- Working with formulae by hand and with a calculator.
- Showing awareness of the significance of digits.
- Checking statements and results by doing relevant calculations.

(The range of problem types includes percentage, ratio, rate and proportion.)
AS 11.1.2
Relate calculated answers correctly and appropriately to the problem situation by:
- Interpreting answers in terms of the context.
- Reworking a problem if the first answer is not sensible or if the initial conditions change.
- Interpreting calculated answers logically in relation to the problem and communicating processes and results.

AS 11.1.3
Apply mathematical knowledge and skills to plan personal finances and investigate opportunities for entrepreneurship inclusive of:
- Specifying and calculating the value of income and expenditure items.
- Estimating and checking profit.

Overview

In this lesson we continue our study of finance and look at calculating the cost price of items in a small business.

Lesson

Two words that are important when looking at a business are:

- Profit – occurs when income is greater than expenses; and
- Loss – occurs when income is less than expenses.

In the previous lesson we looked at three equations, which should govern the choices we make in both our personal lives and in the lives of businesses:

- income > expenses → money left to save – in a business we talk about profit
- income = expenses → "break even"
- income < expenses → have to use savings or take out a loan – in a business we talk about loss

The picture on the next page shows a typical informal trader. He buys a number of large packets of chips and a packet of small packets into which he will repackage the chips.

The trader hopes to make a profit from his business to pay for his personal living expenses. This means the income he gets from selling his smaller packets of chips must be greater than the money he invested in buying the large packets of chips and the smaller packets. To do this he must ensure he sells each smaller packet of chips for more than it cost him to put it together.

Methods and worked examples

The first step to ensure the chip trader makes a profit is to calculate what a small packet of chips actually costs him, i.e. the cost price of the small packet of chips.

If the chip trader bought four large bags of chips @ R15,50; the small packets @ R3,75 for 100 packets; and he needed a return taxi trip to fetch his goods at R5 a trip then we can summarise his expenses as follows:

Expenses	
Raw materials • chips • packets	 R15,50 × 4 = R62,00 R3,75 × 1 = R3,75
Rental	
Storage	
Transport	R5,00 × 2 = R10,00
TOTAL :	R75,75

The trader can fill 24 small packets from one large packet of chips, which means that the cost price of each small packet of chips is:

Cost price of small packet of chips $= \frac{R75,75}{4} \times 24$

$\approx$ R0,79 or 79c

The trader must sell his small packets of chips for more than 79c if he wants to make a profit. All the chip traders sell their chips at R1 a packet so, if we assume that our trader manages to sell all his packets, his income will be:

Income from sales = 96 × R1

= R96

His income and expenditure budget will then look as follows:

Expenses		Income	
Raw materials • chips • packets	 R15,50 × 4 = R62,00 R3,75 × 1 = R3,75	 Sales	 R1,00 × 96 = R96,00
Rental			
Storage			
Transport	R5,00 × 2 = R10,00		
TOTAL :	R75,75	TOTAL :	R96,00

The difference between his income and expenses will give us his profit:

Profit = R96 – R75,75 = R20,25.

One way the trader could increase his profit is to decrease the amount of chips he puts into each small packet. However, he must be careful because he might lose customers if he puts too few chips into each packet.

Activity

INDIVIDUAL

SUMMATIVE ASSESSMENT

Practice examples

1.1 Our chip trader tries to increase his profit by doing the following:

- Buys six packets of large chips per trip; and
- Fills 25 small packets per one large packet.

All other expenses stay the same and he still sells all his small packets at R1 per packet. Draw up a new expense and income statement for the chip trader and calculate his profit.

1.2 What does the chip trader have to be careful of?

2.1 Use the information provided and draw up an expense and income statement to determine the profit for a small informal trader who makes vetkoek at home and sells them at the local market. One recipe can make 36 vetkoek. She makes two batches daily and she manages to sell all her vetkoek at R1,30 each.

Recipe needs	**She can buy**	**Cost**
840g flour	1kg	R11,49
2 teaspoons salt	500g	R1,85
2 teaspoons sugar	1kg	R5,95
1×10g instant yeast	1 10g sachet	R1,29
75g butter	250 g	R11,49
625mℓ water		negligible
750ml cooking oil	750mℓ	R7,99

Lesson 16

FINANCE

Constant and linear functions

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues.
Assessment Standard
AS 11.1.1
In a variety of contexts find ways to explore and analyse situations that are numerically based, by:
- Estimating efficiently.
- Working with formulae by hand and with a calculator.
- Showing awareness of the significance of digits.
- Checking statements and results by doing relevant calculations.

(The range of problem types includes percentage, ratio, rate and proportion.)
AS 11.1.2
Relate calculated answers correctly and appropriately to the problem situation by:
- Interpreting answers in terms of the context.
- Reworking a problem if the first answer is not sensible or if the initial conditions change.
- Interpreting calculated answers logically in relation to the problem and communicating processes and results.

AS 11.1.3
Apply mathematical knowledge and skills to plan personal finances and investigate opportunities for entrepreneurship inclusive of:
- Specifying and calculating the value of income and expenditure items.
- Estimating and checking profit.

Learning Outcome 2
Functional relationships
The learner is able to recognise, interpret, describe and represent various functional relationships to solve problems in real and simulated contexts.
Assessment Standard
AS 11.2.1
Work with numerical data and formulae in a variety of real-life situations, including:
- Finding break-even points involving linear functions by solving simultaneous equations.

(Types of relationships to be dealt with include linear and inverse proportion relationships.)
AS 11.2.2
Draw graphs (by hand and/or by means of technology, where available) as required by the situations and problems being investigated.
AS 11.2.3
Critically interpret tables and graphs in a variety of real-life and simulated situations by:
- Estimating input and output values.

Overview

In this lesson we continue our study of finance and we look at some formulae or relationships, which we might use to calculate costs. In particular we will focus on two kinds:

- Constant functions
- Linear functions

Lesson

It would be helpful to know the following definitions:

- Function or relationship – a rule that relates two variables or things, e.g. cost of a journey and the number of kilometres travelled by the taxi.
- Constant function – a function, the values of which values do not vary.
- Linear function – a function that has a constant rate of change and can be represented by a straight line on a graph.

An example of a constant function is the cost of a trip on a minibus taxi for a given route. The cost to use the minibus taxi from one region (zone) to another is determined by the regions (zones) through which the taxis travels and is not by the actual number of kilometres travelled.

Metered taxis, by contrast, determine the cost of a trip by multiplying the number of kilometres travelled by a constant rate per kilometre. In this case the cost of the journey is not constant, but is determined by the number of kilometres travelled. This would be an example of a linear function because it has a constant rate of change.

Methods and worked examples

We will look at a minibus taxi, as an example of a constant function, and at the metered taxis, as an example of linear functions.

Minibus taxi – example of a constant function:

The following table shows the taxi tariffs to various regions.

From \ To	Alexandra	Baragwanath	Crown Mines	Diepkloof	Dobsonville	Dube	Faraday	Fourways	Halway House
Alexandra	R3,00							R8,00	R6,00
Diepkloof		R3,50	R4,00	R3,50	R4,50	R4,00			
Dobsonville		R4,50							
Dube		R4,00	R5,00	R4,00	R4,50	R3,50			
Halfway House	R6,00								R5,00

To travel from Diepkloof to anywhere in Dube it will cost R4. Notice that to travel from Diepkloof to Baragwanath costs R3,50 or, if you travel within Diepkloof, it costs the same price of R3,50.

Note: the cost to use the minibus taxi from one region to another is fixed or constant. It does not depend on the number of kilometres you travel.

Metered taxis – an example of a linear function:

The above picture is of a taxi and the tariffs he/she charges per trip. Usually these tariffs are found on the side door of the taxi. Notice that the rate is R8 per kilometre, in addition to a service charge (or Flag Fall amount) of R2.

We would calculate the cost of a 15 km trip as follows:

$$\text{Cost} = \text{R}2 + \text{R}8/\text{km} \times 15 \text{ km} = \text{R}122$$

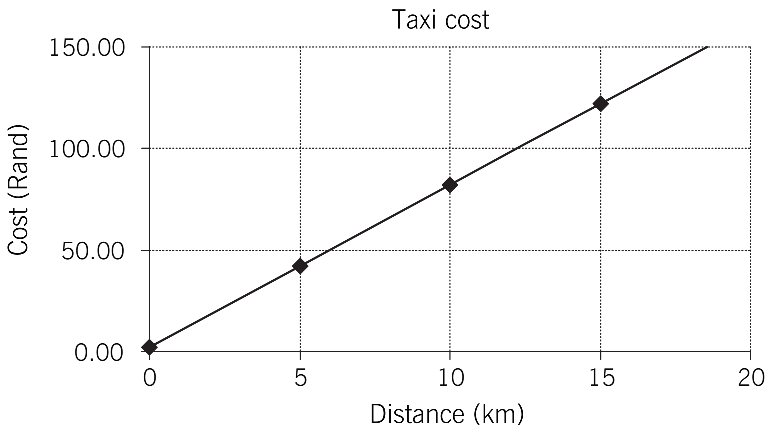
INDIVIDUAL

FORMATIVE ASSESSMENT

Every trip would be worked out using the above formula. Another way we could show the cost is to draw up a table, showing the kilometres travelled and the cost involved. Notice that the cost increases at a constant rate.

Distance (km)	0	5	10	15	20	25	30
Cost (rand)	R2	R42	R82	R122	R162	R202	R242

Now plot these values on a graph to enable us to read off any cost we want to know without having to use the fomula each time to calculate the cost. The points of the graph lie in a straight line, which is why this type of relationship is known as a linear function. The graph is shown below:



Activity

INDIVIDUAL

SUMMATIVE ASSESSMENT

Practice examples

1

Covered parking	Hours	Open-shade parking
N/A	0–15 min	R0
R12	0–1*15 min – 1	R7
R16	1 – 2	R9
R26	2 – 4	R13
R44	4 – 12	R20
R80	12 – 24	R28
Thereafter R40 for every additional 12 hours or part thereof		Thereafter R15 for every additional 12 hours or part thereof

* Lost ticket penalty of R150 + applicable tariff. **www.airports.co.za**

Use the table of parking tariffs to answer the following questions:

1.1 How much will you pay for parking if you parked for:

(a) $1\frac{1}{2}$ hours in the open-shade parking?

(b) $1\frac{1}{2}$ hours in the covered parking?

(c) 10 minutes in the open-shade parking?

(d) 10 minutes in the covered parking?

(e) Two hours and 10 minutes in the covered parking?

(f) Three hours and 55 minutes in the covered parking?

1.2 How much will you pay for parking if you parked for:

(a) Two days and three hours in the covered parking?

(b) How much would you have saved if you parked in the open-shade parking?

2 The tariffs for a taxi are as follows:

R4,60 – service charge

R7,20 per kilometre

2.1 Set up a formula to calculate the cost of a journey of 20 km.

2.2 Draw up a table to show how the cost changes with the number of kilometres travelled. Use the same distances as in the lesson.

2.3 Draw a graph to represent this linear function.

Lesson 17

FINANCE

Constant step functions

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues.
Assessment Standard
AS 11.1.1
In a variety of contexts find ways to explore and analyse situations that are numerically based, by:
- Estimating efficiently.
- Working with formulae by hand and with a calculator.
- Showing awareness of the significance of digits.
- Checking statements and results by doing relevant calculations.

(The range of problem types includes percentage, ratio, rate and proportion.)
AS 11.1.2
Relate calculated answers correctly and appropriately to the problem situation by:
- Interpreting answers in terms of the context.
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AS 11.1.3
Apply mathematical knowledge and skills to plan personal finances and investigate opportunities for entrepreneurship inclusive of:
- Specifying and calculating the value of income and expenditure items.
- Estimating and checking profit.

Learning Outcome 2
Functional relationships
The learner is able to recognise, interpret, describe and represent various functional relationships to solve problems in real and simulated contexts.
Assessment Standard
AS 11.2.1
Work with numerical data and formulae in a variety of real-life situations, including:
- Finding break-even points involving linear functions by solving simultaneous equations

(Types of relationships to be dealt with include linear and inverse proportion relationships.)
AS 11.2.2
Draw graphs (by hand and/or by means of technology, where available) as required by the situations and problems being investigated.
AS 11.2.3
Critically interpret tables and graphs in a variety of real-life and simulated situations by:
- Estimating input and output values.

Overview

In the previous lesson we looked at two functions:

- Constant functions – functions, the values of which do not change.
- Linear functions – functions that have a constant rate of change and can be represented by a straight line on a graph.

In this lesson we take a closer look at constant functions and particularly constant step functions.

Lesson

DVD

Some examples of constant step functions include:

- Minibus taxi costs;
- Tariffs for public telephones; and
- Tariffs for parking garages.

Methods and worked examples

We will look at the tariffs for a public telephone as an example of a constant step function.

Below is the Telkom tariff table for public phones:

Telephone call charges from public phones		Rand (excl VAT)	Rand (incl VAT)
	Unit charge per metering period	0,439	0,50

Calls to conventional landline telephones	Metering period in seconds	
	Standard time: Monday to Friday 07:00 to 19:00	Callmore time: Monday to Friday 19:00 to 07:00 and Friday 19:00 to Monday 07:00
Local (0-50 km)	60,0	110,0
Long distance (> 50km)	23,3	46,0

	Metering period in seconds	
Calls to mobile cellular telephones	Rate 1: Weekdays from 07:00 to 20:00	Rate 2: Monday to Friday 20:00 to 07:00 and Friday 20:00 to Monday 07:00
	11,2	17,6

source: www.telkom.co.za

The table consists of three different tables:

- Top table – tells us the cost of one metering period.
- Middle table – tells us the length of the metering period for calls to conventional landlines. There are four different lengths depending on the time of day you make your call and the distance of the call. Calls are either:
    - o Local – distances of less than 50 km; or
    - o Long distance – distances of more than 50 km.
- Bottom table – tells us the length of the metering period for calls to cellular telephones. There are two different lengths depending on the time of day that you make your call.

Worked example

INDIVIDUAL

FORMATIVE ASSESSMENT

How much would you pay for the following telephone call to a landline?

Time of call: Monday 15:30

Distance: 75 km

Duration: 7,5 minutes

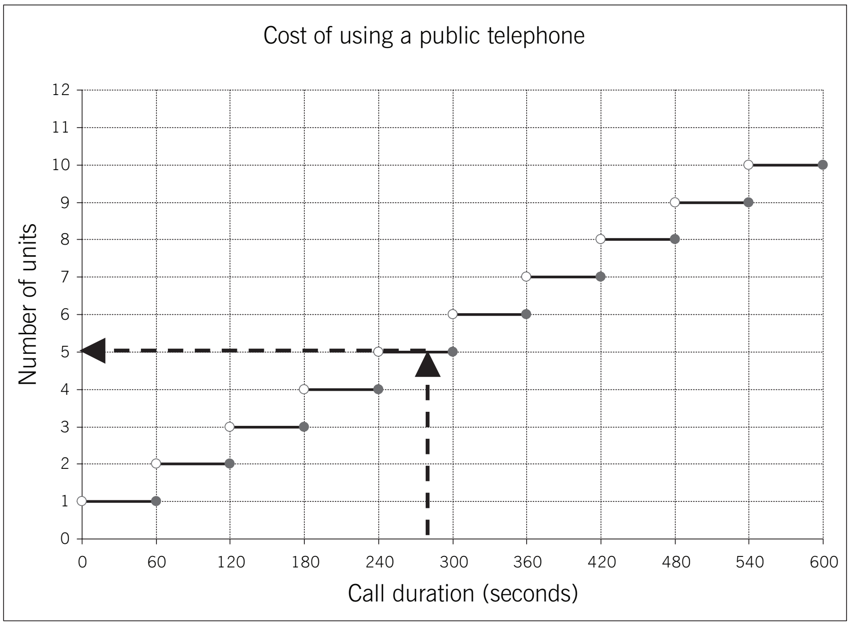
Solution: This call has been made in standard time and since the distance is more than 50 km, it is a long-distance call.

This means that the caller is credited with 23,3 seconds for each 50c coin he/she puts in the telephone. The cost of the call is worked out as follows:

$$\text{Number of seconds} = 7{,}5 \text{ min} \times 60 \text{ sec/min}$$
$$= 450 \text{ sec}$$
$$\text{Units} = \frac{450 \text{ sec}}{23.3 \text{ sec/unit}}$$
$$= 19{,}31 \text{units}$$
$$\approx 20 \text{ units (need to round up: Telkom charges per whole unit)}$$
$$\text{Cost} = 20 \text{ units} \times 50 \text{ c/unit}$$
$$= \text{R}10$$

Note: with a local call to a landline number during standard time you get 60 seconds of credit for each coin you put into the telephone. In other words, for one coin you can speak for any length of time between 0 and 60 seconds. This is why you always need to round up when you are calculating the number of units that you speak. Whether you speak for 1 sec or 59 *sec*, you still use 1 unit. This can be seen easily on a graph of the function.

The costs of a phone call also can be determined by reading off a graph. A graph of the Telkom tariff table for a standard time local call to a distance is shown below:



Use the graph to determine the cost of a local call of 4 min 24 sec during standard time.

Solution

$$\text{Number of seconds} = 4 \text{ min} \times 60 \text{ sec/min} + 24 \text{ sec}$$
$$= 264 \text{ sec}$$

Reading off the graph we see that this requires five units

$$\text{Cost} = 5 \text{ units} \times 50 \text{ c/unit}$$
$$= 250 \text{ c}$$
$$= \text{R}2{,}50$$

Activity

INDIVIDUAL

SUMMATIVE ASSESSMENT

Practice examples

Using the Telkom tariff table above, calculate the cost of the following calls:

1.1 All the these calls are to landlines:

(a) A local call at standard time for $1\frac{1}{2}$ minutes.

(b) A long-distance call at standard time for 3 min 36 sec.

(c) A long-distance call at Callmore time for 3 min 36 sec.

(d) A local call made on Thursday evening at 19:30 for 2 min 21 sec.

1.2 If the above calls were made to a cellular phone, what would each call cost now? Callmore time in question (c) can be replaced by Rate 2 for cellular phones.

1.3 What lessons can you learn from the calculations you did in 1.1 and 1.2?

2 Use the set of axes in the worked example above to do the following activity.

2.1 Draw a constant step function graph for local Callmore time tariffs.

2.2 Which of the two graphs has the steeper gradient?

2.3 Why do you think this graph has the steeper gradient?

2.4 Show on your graph where you would read off the number of units needed for a local call during Callmore time for 5 min 15 sec. Calculate the cost of this call.

3 Below is the tariff table for the parking garage we looked at in Lesson 16;

Covered parking	**Hours**	**Open-shade parking**
N/A	0–15 min	R0
R12	0–1*15 min – 1	R7
R16	1 – 2	R9
R26	2 – 4	R13
R44	4 – 12	R20
R80	12 – 24	R28
Thereafter R40 for every additional 12 hours or part thereof		Thereafter R15 for every additional 12 hours or part thereof

3.1 Draw a constant step function graph for the covered parking for 24 hours.

3.2 On the same system of axes, draw a graph for the open-shade parking.

3.3 Why do you think it is useful to draw both graphs on the same axes?

3.4 Determine how much you will save by parking in the open-shade parking, rather than the covered parking if you parked for 4 hr 45 min.

3.5 Calculate how much it will cost you to leave your car at the airport for two days and 16 hours if you parked it in:

(a) Open-shade parking; and

(b) Covered parking.

Lesson 18

FINANCE

Linear piecewise functions

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues.
Assessment Standard
AS 11.1.1
In a variety of contexts find ways to explore and analyse situations that are numerically based, by:
- Estimating efficiently.
- Working with formulae by hand and with a calculator.
- Showing awareness of the significance of digits.
- Checking statements and results by doing relevant calculations.

(The range of problem types includes percentage, ratio, rate and proportion.)
AS 11.1.2
Relate calculated answers correctly and appropriately to the problem situation by:
- Interpreting answers in terms of the context.
- Reworking a problem if the first answer is not sensible or if the initial conditions change.
- Interpreting calculated answers logically in relation to the problem and communicating processes and results.

AS 11.1.3
Apply mathematical knowledge and skills to plan personal finances and investigate opportunities for entrepreneurship inclusive of:
- Specifying and calculating the value of income and expenditure items.
- Estimating and checking profit.

Learning Outcome 2
Functional relationships
The learner is able to recognise, interpret, describe and represent various functional relationships to solve problems in real and simulated contexts.
Assessment Standard
AS 11.2.1
Work with numerical data and formulae in a variety of real-life situations, including:
- Finding break-even points involving linear functions by solving simultaneous equations

(Types of relationships to be dealt with include linear and inverse proportion relationships.)
AS 11.2.2
Draw graphs (by hand and/or by means of technology, where available) as required by the situations and problems being investigated.
AS 11.2.3
Critically interpret tables and graphs in a variety of real-life and simulated situations by:
- Estimating input and output values.

Overview

In Lesson 16 we looked at two functions:

- Constant functions – functions, the values of which do not change.
- Linear functions – functions that have a constant rate of change and can be represented by a straight line on a graph.

In this lesson we take a closer look at linear functions and in particular at what are known as: linear piecewise functions.

Lesson

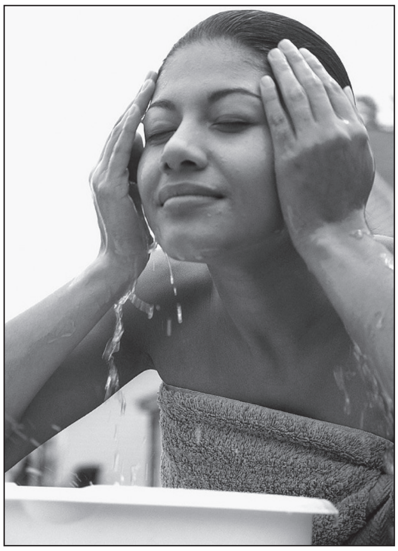
Some examples of linear piecewise functions include:

- Water;
- Sewerage; and
- Electricity tariffs.

You have come across linear functions before when looking at metered taxis. Piecewise linear functions are linear functions where the gradient of the line changes for certain intervals (in other words, where the constant rate changes over different intervals).

DVD

Methods and worked examples



We will look at the tariffs for water consumption as our example of a linear piecewise function.

Your water consumption is measured in kilolitres and you are charged using the rates shown in the table below. Notice that the more water you use, the more expensive it is per kilolitre.

Tariffs per kilolitre (kl) of water (excl VAT)	
0 to 6 kl	R0,00
+6 to 12 kl	R2,15
+12 to 20 kl	R4,30
+20 to 40 kl	R5,48
+40 to 60 kl	R6,67
+60 kl	R8,60

The table shows that the first 6 kl of water is free, but that the next 6 kl is charged at a rate of R2.15 per kilolitre. This looks similar to the constant step function we looked at in the previous lesson. The difference, however, is that in the case of water consumption you are not charged a constant fee for this interval, but you are charged at a rate per kilolitre. The example below will help to clarify this.

Worked example

INDIVIDUAL

Use the tariff table provided to determine the cost of using 18,9 kl of water.

FORMATIVE ASSESSMENT

Solution

Step 1: 6 kl @ R0/kl = R0
Step 2: 6 kl @ R2,15/kl = R12,90
Step 3: 6,9 kl @ R4,30/kl = R29,67
Total = R42,57

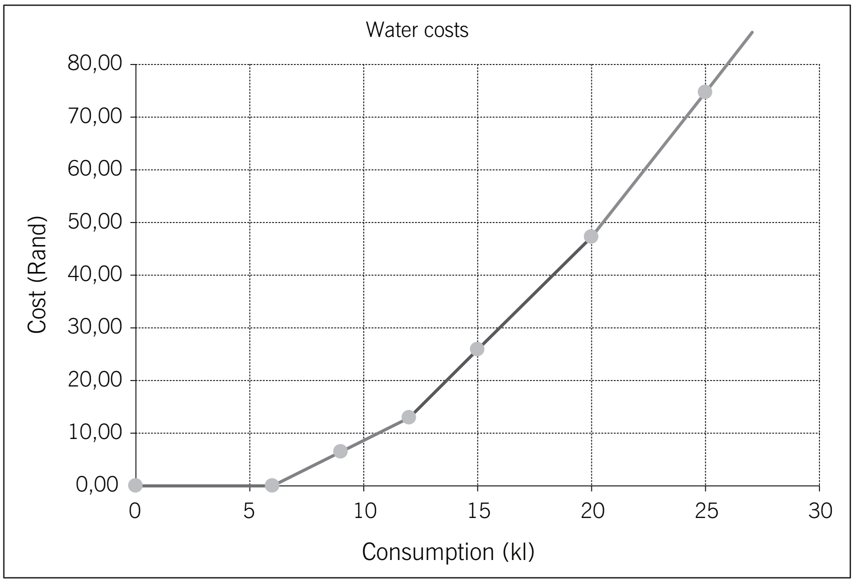
Note: the tariff intervals do not go up evenly, i.e. the first two intervals are for 6 kl of water; the next is for 8 kl and the following interval is for 20 kl of water.

You can draw a graph for the water tariffs by first working out a table of values. The table is shown below:

Consumption (kl)	0	6	9	12	15	20	25
Cost (rand)	0	0	6,45	12,90	25,80	47,30	74,70

Note: It is important to work out the end points of each interval, i.e. at 6 kl; 12 kl; 20 kl and so on.

The graph of the water tariffs is show below:



Note: the steeper the gradient or line, the greater the rate.

Consider the following examples:

Use the tariff table provided to determine the cost of using 18,9 kl of water.

Solution

Step 1:	6kl @ R0/kl	= R0
Step 2:	6kl @ R2.15/kl	= R12,90
Step 3:	8kl @ R4,30/kl	= R34,40
Step 4:	17,25kl @ R5,48/kl	= R94,53
	Total	= R141,83

Activity

INDIVIDUAL

SUMMATIVE ASSESSMENT

Practice examples

1. The water tariffs for Johannesburg and eThekwini (Durban) are give below:

Johannesburg (domestic consumption)	
Tariffs per kilolitre (kl) of water (excl VAT)	
First 6kl	Free
7 to 10kl	R3,60
11 to 15kl	R4,80
16 to 20kl	R6,00
21 to 40kl	R7,19
> 40kl	R8,50

eThekwini (domestic consumption)	
Tariffs per kilolitre (kl) of water (excl VAT)	
0kl to 6kl	nil
From 6kl to 30kl	R5,42
More than 30kl	R10,48
Fixed charge per month	
0kl to 6kl	nil
Greater than 6kl, less than 12kl	R29,58
Equal to or greater than 12kl	R42,32

1.1 Calculate the cost of 7,5 kl of water in:

Johannesburg; and

eThekwini (Durban).

1.2 Calculate the cost of 28 kl of water in:

Johannesburg; and

eThekwini (Durban).

1.3 Calculate the cost of 43 kl of water in:

Johannesburg; and

eThekwini (Durban).

1.4 Why do you think municipalities have piecewise tariffs?

Lesson 19

FINANCE

Calculating costs – breaking even I

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts which include financial aspects of personal, business and national issues.
Assessment Standard
AS 11.1.1
In a variety of contexts, find ways to explore and analyse situations that are numerically based, by:
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AS 11.1.3
Apply mathematical knowledge and skills to plan personal finances and investigate opportunities for entrepreneurship inclusive of:
- Specifying and calculating the value of income and expenditure items.
- Estimating and checking profit.

Learning Outcome 2
Functional relationships
The learner is able to recognise, interpret, describe and represent various functional relationships to solve problems in real and simulated contexts.
Assessment Standard
AS 11.2.1
Work with numerical data and formulae in a variety of real-life situations, including:
- Finding break-even points involving linear functions by solving simultaneous equations.

(Types of relationships to be dealt with include linear and inverse proportion relationships.)
Assessment Standard 11.2.2
Draw graphs (by hand and/or by means of technology where available) as required by the situations and problems being investigated.
Assessment Standard 11.2.3
Critically interpret tables and graphs in a variety of real-life and simulated situations by:
- Estimating input and output values.

Overview

In this lesson we continue with the mathematics of finance and the relationships that we may have to use to be able to calculate costs. In particular, we will consider a situation in which we need to find what is called a break-even point.

Lesson

LC DVD

A break-even point is the value at which the two options we are considering have the same value.

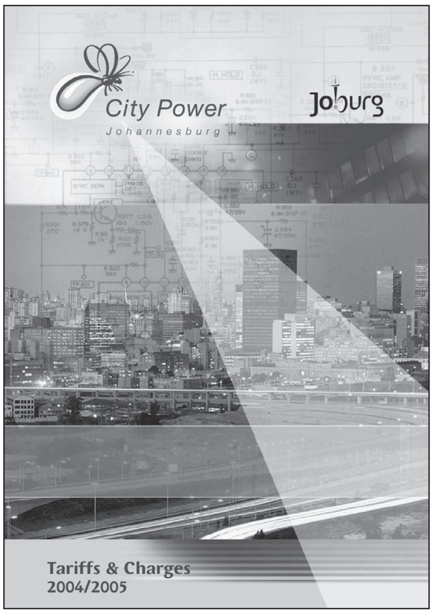
Some examples of situations where you may want to determine break even points include:

- deciding which is the better cell phone contract for your purposes;
- deciding which is the better tariff if two tariff options are given – e.g. *City Power*

in Johannesburg offers its clients different tariff options to choose from; and

- deciding which is the better option for your purposes when hiring a car.

Methods and worked examples



The Johannesburg Municipality offers three different tariff options for domestic users:

- the Life Line option;
- the Two Part Flat rate option; and
- the Two Part Seasonal rate option.

Below is the tariff table for Johannesburg:

PHASE	SERVICE CHARGE	ENERGY CHARGE	UNIT
Life Line	R 0.00	38,05	c/kWh

50kWh free per month

TWO PART FLAT

PHASE	SERVICE CHARGE	ENERGY CHARGE	UNIT
Single	R 67.76	27,23	c/kWh
Three	R 86.52	27,23	c/kWh

TWO PART SEASONAL

CHARGE	SEASONAL	SINGLE PHASE	THREE PHASE
Service		R67.76	R86.52
Energy	Summer	23,25	23,25
c/kWh	Winter	34,99	34,99

We will only look at the first two options.

- With the *Life Line* option you get your first 50 kWh per month free after which you pay 38,05c per kWh of electricity that you use.
- With the *Two Part Flat* option you pay a fixed monthly fee of R67,76 –called a service charge – in addition to which you pay 27,23c per kWh of electricity that you use.

Worked example

INDIVIDUAL

FORMATIVE ASSESSMENT

The problem we are faced with is deciding which the best option is. It should be clear that the *Life Line* option is the better choice for low consumptions. It should also be clear that the *Two Part Flat* option will be a better option for very high consumptions. Our challenge is to determine for what consumption the two would cost the same amount. This consumption is called the break-even point.

There are at least three different ways to determine the break even point for these options. You can use:

- a table of values;
- a graph; or
- solve equations.

In the example of the Johannesburg electricity costs we are going to look at a table of values and a graph.

Solution

The table below shows the cost of electricity for a few different consumptions. The method of working out these costs is shown in the table:

Consumption (kWh)	**Life line**	**Two Part Flat**
0	R0,00	R67,76 + 0 kWh × 27,23c/ kWh = R67,76
50	0,00	R67,76 + 50 kWh × 27,23c/ kWh = R81,38
75	50 kWh × 0,00c/ kWh + (75 – 50) kWh × 38,05c/ kWh = 951,25c = R9,51	R67,76 + 75 kWh × 27,23c/ kWh = R88,18
150	(150 – 50) kWh × 38,05c/ kWh = R38,05	R67,76 + 150 kWh × 27,23c/ kWh = R108,61
300	R95,13	R149,45
600	R209,28	R231,14
750	R266,35	R271,99
850	R304,40	R299,22
1000	R361,48	R340,06

From the table we can see that:

- As expected, for lower consumptions, the *Life Line* option is the better option;
- Similarly, for higher consumptions, the *Two Part Flat* option is better; and
- At some point between 750 *kWh* and 850 *kWh* there is a value where both options will cost the same.

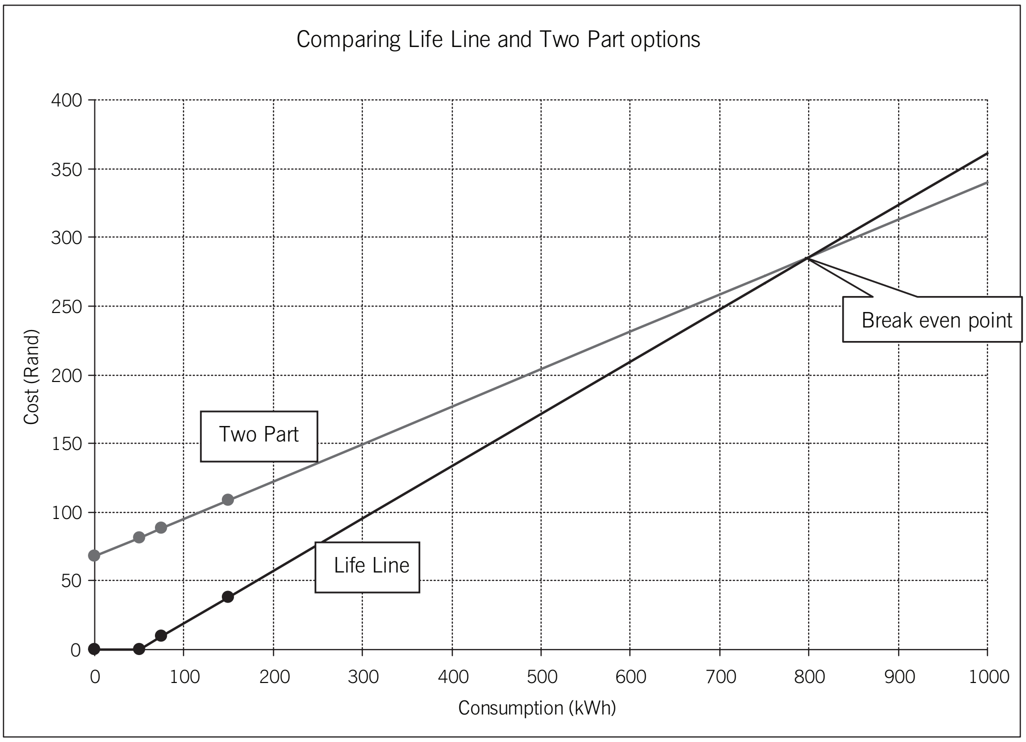
Since monthly household consumption is likely to vary and since the difference between the options at 750 kWh and 850 kWh is small – in the order of R5,00 – we probably do not need to be more accurate than this. In other words for values less than 750 kWh the *Life Line* seems to be the better option and for values greater than 850 kWh the *Two Part Flat* option is better. For values between 750 kWh and 850 kWh, it really does not make a great difference.

If we plotted a few of the values form our table and drew lines through them we would get the graph on the next page.

Looking at the graph it would appear as if the break-even point occurs at 800 kWh – be careful although it's not far away is it not at exactly 800 kWh. Because of the scale of the graph we cannot see that the breakeven point is actually at 803,56 kWh.

However, it is not important that the breakeven point is exactly at 803,56 kWh, what is important is that both the graph and the table have shown us that up to 700 or 750 kWh the *Life Line* option is better and that beyond 850 or even 900 kWh the *Two Part Flat* option is better. Both the graph and the table clearly show that between these values there isn't really a great difference between the options.

The advantage of the table and graph is that the both show the relationship between the two options very well and yet neither is able to show us exactly for what value the two are equal – a value that in this context doesn't matter that much anyway.



You should have noticed that the rate per kWh is greater with the *Life Line* option and that this is reflected in the steeper graph. You should also have noticed that the rate per kWh is less for the *Twp Part Flat* option and that this is reflected in the graph being less steep.

Activity

INDIVIDUAL

SUMMATIVE ASSESSMENT

1. A bank offers their clients a choice of three different accounts:
   - Mzansi (low cost accounts to encourage people to open an account).
   - Bronze
   - Silver

   Each account has a different fee structure and you would need to compare all of the different fees for all of the transactions before being able to decide which account is better suited to you. In this problem we will compare the fees for two different transactions.

   The fee structure for depositing cash into your account at the bank's own ATM is as follows:

Type of transaction	Mzansi	Bronze	Silver
Cash deposit at own ATM	R4,00	R4,00; 0,40%	R2,90; 0,90%

Note: R4,00; 0,04% means there is a basic transaction fee of R2,90 and an additional fee of 0,04% of the transaction value. R2,90; 0,90% should be interpreted in the same way.

2. Fill in the table by calculating the cost of depositing the following amounts into all three accounts:

Amount deposited	**Mzansi**	**Bronze**	**Silver**
R50,00			
R100,00			
R150,00			
R200,00			
R250,00			
R500,00			
R1000,00			

(a) Using the values in your table, draw graphs to represent the three different accounts.

(b) What can you deduce from the table and the graphs?

1.1 The deposit fee to deposit cash over the counter is given below:

Type of transaction	Mzansi	Bronze	Silver
Cash deposit at counter	R8,00	R4,00; 0,53%	R2,90; 0,90%

(a) Is it cheaper for Mzansi account holders to deposit money over the counter or at an ATM? Why do you think the bank has structured its' fee like this?

(b) Is it cheaper for bronze account holders to deposit money over the counter or at an ATM? How can you tell this without actually calculating any costs

(c) Is it cheaper for silver account holders to deposit money over the counter or at an ATM? Why do you think the bank has structured its' fee like this?

FINANCE

Calculating costs – breaking even II

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues.
Assessment Standard
AS 11.1.1
In a variety of contexts find ways to explore and analyse situations that are numerically based, by:
- Estimating efficiently.
- Working with formulae by hand and with a calculator.
- Showing awareness of the significance of digits.
- Checking statements and results by doing relevant calculations.

(The range of problem types includes percentage, ratio, rate and proportion.)
AS 11.1.2
Relate calculated answers correctly and appropriately to the problem situation by:
- Interpreting answers in terms of the context.
- Reworking a problem if the first answer is not sensible or if the initial conditions change.
- Interpreting calculated answers logically in relation to the problem and communicating processes and results.

AS 11.1.3
Apply mathematical knowledge and skills to plan personal finances and investigate opportunities for entrepreneurship inclusive of:
- Specifying and calculating the value of income and expenditure items.
- Estimating and checking profit.

Learning Outcome 2
Functional relationships:
The learner is able to recognise, interpret, describe and represent various functional relationships to solve problems in real and simulated contexts.
Assessment Standard
AS 11.2.1
Work with numerical data and formulae in a variety of real-life situations, including:
- Finding break-even points involving linear functions by solving simultaneous equations.

(Types of relationships to be dealt with include linear and inverse proportion relationships.)
AS 11.2.2
Draw graphs (by hand and/or by means of technology, where available) as required by the situations and problems being investigated.
AS 11.2.3
Critically interpret tables and graphs in a variety of real-life and simulated situations by
- Estimating input and output values.

Overview

In our previous lesson we determined the break-even point for two electricity tariff options, the Life Line and Two Part Flat. We explored how the problem could be solved by means of tables of values and graphs.

Lesson

DVD

In this lesson we look at a similar problem. It also happens to be related to electricity, but this time we will use graphs and equations to determine the break-even point.

A break-even point is the value at which the two options we are considering have the same value? Value-for-money problems sometimes reduce to break-even type problems.

Methods and worked examples

Consider the choices you have when buying an electric light bulb. You can either buy:

- The traditional incandescent light bulb (below left); or the "energy saving" compact fluorescent lamp or CFL (below right).

A consideration when buying a light bulb is to determine the running cost – this is done by considering the power rating. In our example we will use a traditional light bulb with a 60 Watt rating and an energy-saving bulb with an 11 Watt rating. The Watt is the scientific unit of electrical power that gives the rate of electricity consumption of an electrical appliance.

The unit of electricity is called a kilo-Watt-hour (kWh) and corresponds to the work performed by one kilowatt of electric power in one hour. The municipality charges for electricity in terms of kilo-Watt-hours – for this example we will assume the municipality charges 38,05 c/kWh for electricity. It follows that:

A 60 W bulb uses $\frac{60\text{ W}}{1\,000\text{ W}} = 0{,}06$ kW in one hour and costs

$0{,}06\text{ kWh} \times 38{,}05\text{ c/kWh} = 2{,}283c$ per hour.

An 11 W bulb uses $\frac{11\text{ W}}{1\,000\text{ W}} = 0{,}011$ kW in one hour and costs

$0{,}011\text{ kWh} \times 38{,}05\text{ c/kWh} = 0{,}41855c$ per hour.

Worked example

Given that:

- A regular 60 W bulb costs R3,99; and
- An energy-saving 11 W bulb costs R13,99.

It is quite obvious that although the 11 W lamp is more economical to use, it costs a lot more than the 60 W bulb, which is less economical to use. How many hours of use will it take for the two bulbs to cost the same amount?

Solution

We need to determine the break-even point. We will use two different methods:

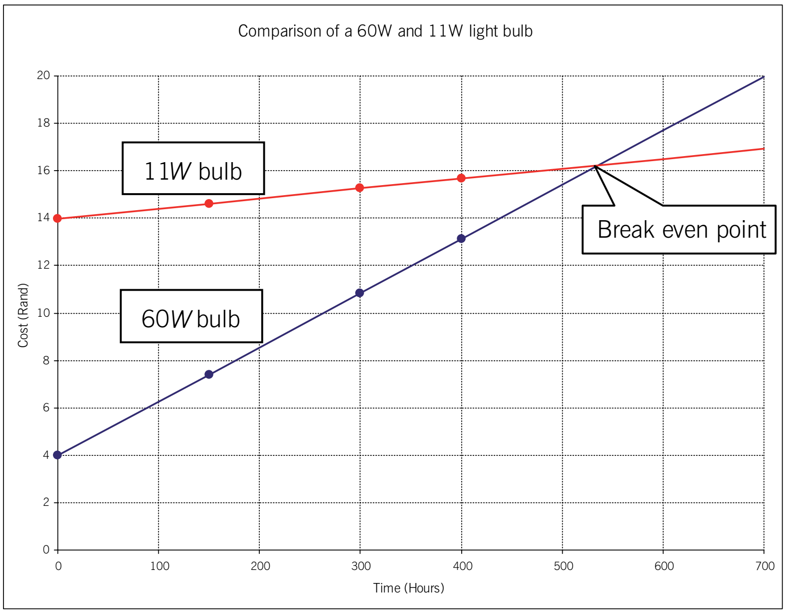
- By drawing graphs and reading off the value; and
- By solving equations.

Method 1 – Drawing a graph:

To draw the graphs we first need to determine a few values:

Time (hours)	Cost (rand) 60 W	Cost (rand) 11 W
0	R3,99	R13,99
150	R3,99 + 150 h × 2,283 c = R7,41	R13,99 + 150 h × 0,41855 c = R14,62
300	R10,84	R15,25
400	R13,12	R15,66

Using these values we can draw a graph.



From the graph it is clear that, as expected, the 60 W option starts out being less expensive, but that with time the 11 W option becomes the less-expensive option.

From the graph it is apparent that for some time interval between 400 h and 500 h there is a value where both options will cost the same. This point is called the break-even point and is the point where the two graphs of the options intersect.

Observe that the graph for the 60 W option is steeper than that for the 11 W option. This is because the 60 W option costs more per hour than the 11 W option does – the rate is greater. It is difficult to read off the exact values for the break-even point from this graph, although the graph does help us to see how the two relationships compare.

Although we can determine the exact values of the break-even point by solving equations (as shown below), it is sometimes enough to have a sense of the order of magnitude of the problem. In the case of the light bulbs the graphs have given us this. The point is the exact value is not that critical since, for example, different light bulbs will have different life-spans.

Method 2 – Solving equations:

At the break-even point:

$$\begin{aligned}
\text{Cost 60 W bulb} &= \text{Cost 11 W bulb} \\
399\text{ c} + 2{,}283\text{ c/h} \times \text{time} &= 1{,}399\text{ c} + 0{,}41855\text{ c/h} \times \text{time} \\
2{,}283\text{ c/h} \times \text{time} - 0{,}41855\text{ c/h} \times \text{time} &= 1{,}399\text{ c} - 399\text{ c} \\
1{,}86445\text{ c/h} \times \text{time} &= 10c \\
\text{time} &= \frac{10\text{ c}}{1.86445\text{ c/h}} \\
\text{time} &\approx 536{,}35\text{ h}
\end{aligned}$$

This means that the 11 W bulb becomes the more cost-effective option after 536 hours.

Our solution has not taken into account that it is claimed that the 11 W bulb lasts up to six times as long as the 60 W bulb. Incorporating this into our solution would be an interesting challenge.

There are, of course, other reasons besides cost that one might take into consideration when buying bulbs. Since the 11 W bulb is an energy-saving bulb and uses less electricity than the 60 W bulb, less fossil fuels are used and less greenhouse gases (which contribute to global warming) are produced in the manufacture of electricity to power it. This makes the 11 W bulb the more environmentally responsible choice.

INDIVIDUAL

SUMMATIVE ASSESSMENT

Activity

Practice examples

A bank offers its clients a choice of three different accounts:

- Mzansi (low-cost accounts to encourage people to open an account)
- Bronze
- Silver

Each account has a different fee structure and you would need to compare all the different fees for all the transactions before upi decide which account is better suited to you. In this problem we will compare the fees for different transactions.

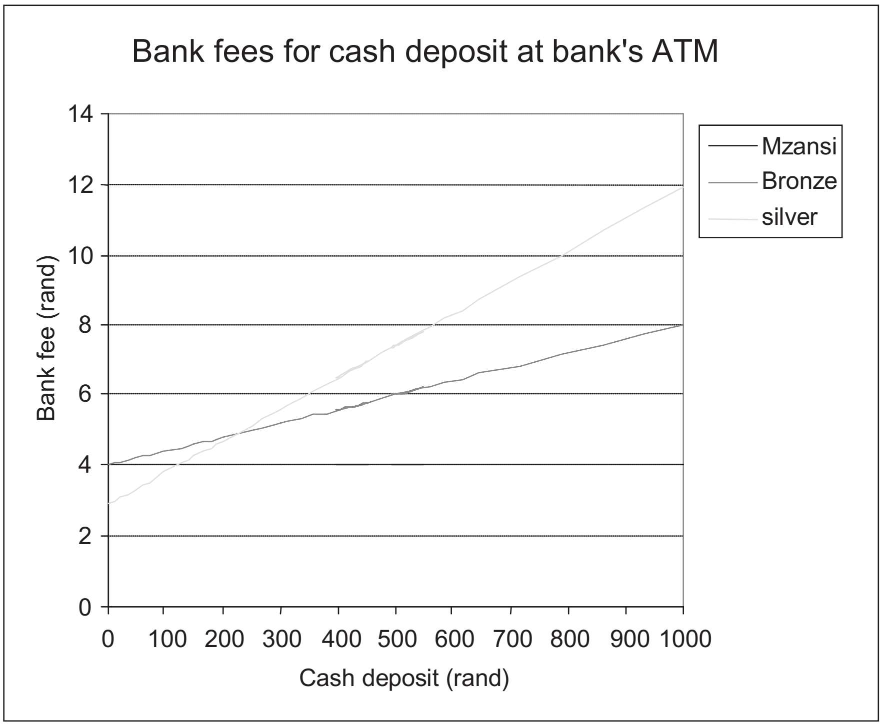
1. This activity on bank transaction fees is a continuation of the one you were working with in the previous lesson. The information you require is found below:

Type of transaction	Mzansi	Bronze	Silver
Cash deposit at own ATM	R4	R4; 0,40%	R2,90; 0,90%

Type of transaction	Mzansi	Bronze	Silver
Cash deposit at counter	R8	R4; 0,53%	R2,90; 0,90%

Note: R4; 0,04% means there is a basic transaction fee of R4 and an additional fee of 0,04% of the transaction value – the other fees should be interpreted in the same way.

a. You have already drawn the graph for depositing cash at the bank's own ATM as follows:



The equations used to calculate the costs are:

- Mzansi: cost = R4
- Bronze cost = R4 + $\frac{0.4}{100}$ × Amount
- Silver cost = R2,90 + $\frac{0,9}{100}$ × Amount

Set up and solve equations to calculate the break-even point for the transaction fees for the:

i. The Mzansi and Silver accounts; and

ii. The Bronze and Silver accounts.

b. Complete the table below using the formulae for cash deposits at the bank counter.

Amount deposited	Mzansi	Bronze	Silver
R50			
R100			
R150			
R200			

c. Using the values in your table, draw graphs to represent the counter cash deposit fee for the three different accounts.

d. Develop and solve equations to calculate the break-even point for the transaction fees for the:

i. The Mzansi and Bronze accounts;

ii. The Mzansi and Silver accounts; and

iii. The Bronze and Silver accounts.

Lesson 21

LOOKING BACK

Basic concepts

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues.
Assessment Standard
AS 11.1.1
In a variety of contexts find ways to explore and analyse situations that are numerically based, by:

- Estimating efficiently.
- Working with formulae by hand and with a calculator.
- Showing awareness of the significance of digits.
- Checking statements and results by doing relevant calculations.

(The range of problem types includes percentage, ratio, rate and proportion.)
AS 11.1.2
Relate calculated answers correctly and appropriately to the problem situation by:

- Interpreting answers in terms of the context.
- Reworking a problem if the first answer is not sensible or if the initial conditions change.
- Interpreting calculated answers logically in relation to the problem and communicating processes and results.

AS 11.1.3
Apply mathematical knowledge and skills to plan personal finances and investigate opportunities for entrepreneurship inclusive of:

- Specifying and calculating the value of income and expenditure items.
- Estimating and checking profit.

Learning Outcome 2
Functional relationships
The learner is able to recognise, interpret, describe and represent various functional relationships to solve problems in real and simulated contexts.
Assessment Standard
AS 11.2.1
Work with numerical data and formulae in a variety of real-life situations, including:

- Finding break-even points involving linear functions by solving simultaneous equations

(Types of relationships to be dealt with include linear and inverse proportion relationships.)
AS 11.2.2
Draw graphs (by hand and/or by means of technology, where available) as required by the situations and problems being investigated.
AS 11.2.3
Critically interpret tables and graphs in a variety of real-life and simulated situations by:

- Estimating input and output values.

Overview

The next three lessons are a revision of all the key concepts, knowledge and skills we have covered so far.

Lesson

L C DVD

We can divide the work into two main groups:

- Basic concepts; and
- Calculations dealing with matters of finance.

The basic concepts, knowledge and skills dealt with during the lessons are summarised on the next page:

Basic concepts

Percentage

- Definition
- Determining a percentage
- Adding a % of an amount to an amount
- Subtracting a % of an amount from an amount
- Calculating a % change
- Determining the original amount when the amount was changed by some %

Ratio/proportion

- Definition
- Calculations involving mixing in a ratio
- Calculations involving sharing in a ratio
- Direct proportion
- Inverse proportion

Rate

- Definition
- Unit rate
- Constant rate
- Varying rate
- Average rate

Methods and worked examples

The definitions for each of the three main concepts are:

- A percentage is a fraction – it is the part of the whole expressed in hundredths. It allows us to express fractions; ratios and proportions as whole numbers: 35% instead of $\frac{21}{60}$ or 0,35.
- A ratio is a comparison of two (or more) numbers, called the terms in the ratio. A ratio has no units since the quantities being compared are of the same kind. A ratio can be expressed in different ways: in words – *a* to *b*; with a colon – a : *b*; and as a fraction – $\frac{a}{b}$.
- A rate is a ratio where the two quantities being compared have different units. The unit of the rate is given by the ratio of the unit of the first quantity to the unit of the second quantity. Unit rates compare the first quantity to one unit of the second quantity.

Worked examples

INDIVIDUAL

FORMATIVE ASSESSMENT

Percentage:

Use the data from the table below to answer the following questions:

Dam	Capacity (Ml)	Year	% full
Voelvlei	164,122	2003	57,9%
		2004	33,9%
Wemmershoek	58,644	2003	39,7%
		2004	42,5%

(1) Determine the percentage change in volume for each dam from 2003 to 2004.

(2) Which dam had the greatest amount of water in 2004?

Solutions

(1) % change $= \frac{\text{final amount} - \text{original amount}}{\text{original amount}}$

Voelvlei $= \frac{33{,}9\% - 57{,}9\%}{57{,}9\%}$

$= -41{,}45\%$

Wemmershoek $= \frac{42{,}5\% - 39{,}7\%}{39{,}7\%}$

$= 7{,}05\%$

Note: a positive percentage change indicates that the volume of water in the dam has increased, whereas the negative percentage change indicates that the volume of water has decreased.

(2) Voelvlei = 33,9% of 164 122 ml
= 55 637 ml

Wemmershoek = 42,5% of 58 644 ml
= 24 923 ml

Note: this example shows that although Wemmershoek was 42,5% full, which is greater than 33,9% for Voelvlei, it had far less water in it than Vloelvlei did. This highlights two important issues when working with percentage:

- It is important to know not only the percentages involved in a problem, but also the actual values; and
- It is probably better to get a small percentage of a large amount, rather than a larger percentage of a small amount.

Ratio/proportion:

(1) A hairdresser needs to make up 60 ml of a mixture of tint and peroxide. The ratio of tint : peroxide = 1 : 2. Calculate how many ml of each substance she needs.

(2) A rectangle has dimensions 8 cm by 5 cm. What will the breadth be if the length increases to 10 cm and the area remains the same?

(3) The scale on a building plan is 1:50. If the dimensions of a room on the plan are 9 cm by 6 cm. What will the dimensions of the room be on the ground?

Solution

1. The 60 ml mixture needs to broken into three equal parts.
Each part = 60 ml ÷ 3 = 20 ml.

Tint needed = 1 × 20 ml
= 20 ml

Peroxide needed = 2 × 20 ml
= 40 ml

2. Area = 8 cm × 5 cm = 40 cm^2

The breadth will be = $\frac{40\ cm^2}{10cm}$
= 4 cm

3. 9 cm : × = 1 : 50 ∴ × = 9 cm × 50 = 450 cm = 4,5 m

6 cm : × = 1 : 50 ∴ × = 6 cm × 50 = 300 cm = 3,0 m

Rate:

1. Refer to the picture below: the cost of 2,5kg of flour is R10,98. The unit (per kg) cost is R5,44/kg.

   What is the unit (per kg) cost of a 750 g packet of flour that costs R4,32?

2. If the average petrol consumption of a car in the past year has been 8,4 litres/100 km, how much petrol do you expect to use on a 1 650 km journey?

Solution

(1) Unit cost of 750 g packet $= \frac{R4,32}{0,750\ kg}$

$= R5,76/kg$

(2) Average consumption $= \frac{8,4l}{100\ km}$

For of 1 650 km $= 1\ 650\ km \times \approx 140$ litres

Activity

See Lessons 1 to 13 for activities.

Lesson 22

LOOKING BACK

Finance

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues.
Assessment Standard
AS 11.1.1
In a variety of contexts find ways to explore and analyse situations that are numerically based, by:
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AS 11.1.3
Apply mathematical knowledge and skills to plan personal finances and investigate opportunities for entrepreneurship inclusive of:
- Specifying and calculating the value of income and expenditure items.
- Estimating and checking profit.

Learning Outcome 2
Functional relationships
The learner is able to recognise, interpret, describe and represent various functional relationships to solve problems in real and simulated contexts.
Assessment Standard
AS 11.2.1
Work with numerical data and formulae in a variety of real-life situations, including:
- Finding break-even points involving linear functions by solving simultaneous equations.

(Types of relationships to be dealt with include linear and inverse proportion relationships.)
AS 11.2.2
Draw graphs (by hand and/or by means of technology, where available) as required by the situations and problems being investigated.
AS 11.2.3
Critically interpret tables and graphs in a variety of real-life and simulated situations by:
- Estimating input and output values.

Overview

Our work on finance can be divided into two key ideas:

- The vocabulary of finance; and
- Calculations.

Lesson

LC DVD

The vocabulary and calculations dealt with during the lessons on finance are summarised on the next page:

Finance

Vocabulary

- Income and expenses
  - income > expenses
  - income = expenses
  - income < expenses
- Planning your finances
  - income and expenditure statements
  - budgets
  - fixed and variable income and expenses
  - high and low priority expenses

Calculations

Unit price

Functions

- Linear
  - direct proportion
  - linear
  - piecewise linear
- Constant functions
  - constant
  - constant step

Break-even problems

- Table
- Graphs
- Equations

Methods and worked examples

We looked at the following vocabulary in our lessons on finance:

- Income – the money received by an individual or a company; and
- Expenses – the money spent by an individual or a company.

These variables give rise to three important equations:

- Income > expenses: when income exceeds expenses there will be money left over that can be saved; in the case of a company we speak about profit;
- Income = expenses: when income equals expenses we say that we have a break-even situation; and
- Income < expenses: when income is less than expenses we need to borrow money to cover the costs; in the case of a company we speak about loss.

An **income and expenditure statement** provides you with a record of your actual income and expenses. This enables you to draw up a **budget** to plan for the future. Since planning involves making choices it is important to classify expenses into high and low-priority expenses.

Worked examples

The following examples and calculations were dealt with the lessons on finance:

Unit cost: (see Lesson 15)

The example we looked at was the trader who bought a number of large bags of chips, which he repackaged into smaller packets. He had to calculate the unit price of the smaller bag of chips to determine the amount of profit he made on each sale.

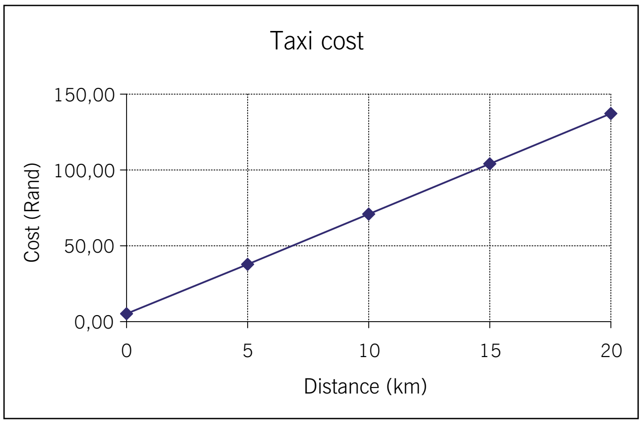
Linear functions:

Definition: a linear function is a function that has a constant rate of change and can be represented by a straight line.

We looked at linear functions and at a variation called piecewise linear functions.

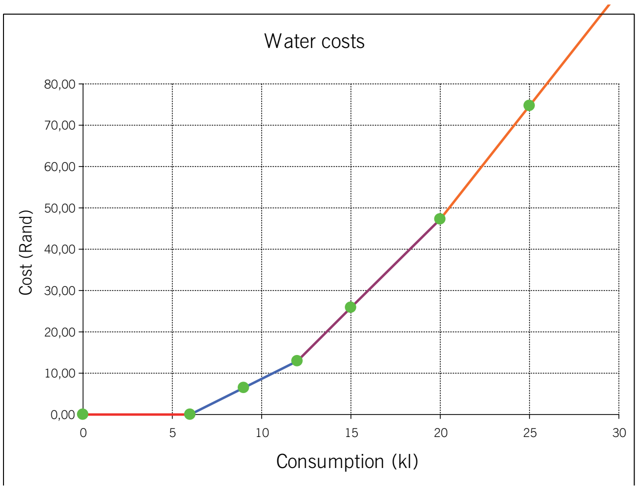
An example of a linear function (see Lesson 16) is the cost of using a metered taxi. The taxi fee is made up of a basic charge of R5 and a charge for the distance determined using the rate R6,60/km. This is illustrated in the table and graph below:

Distance	0	5	10	15	20	25	30
Cost (Rand)	5	38	71	104	137	170	203



We used water tariffs (see Lesson 18) as an example of a piecewise linear function. The rate at which water is charged is not constant, but changes from one interval to the next. The graph, therefore, is made up of series of lines – linear functions. This is illustrated in the table and graph below:

Tariffs per kilolitre (kl) of water (excl VAT)	
0 to 6 kl	R0,00
+6 to 12 kl	R2,15
+12 to 20 kl	R4,30
+20 to 40 kl	R5,48
+40 to 60 kl	R6,67
+60 kl	R8,60



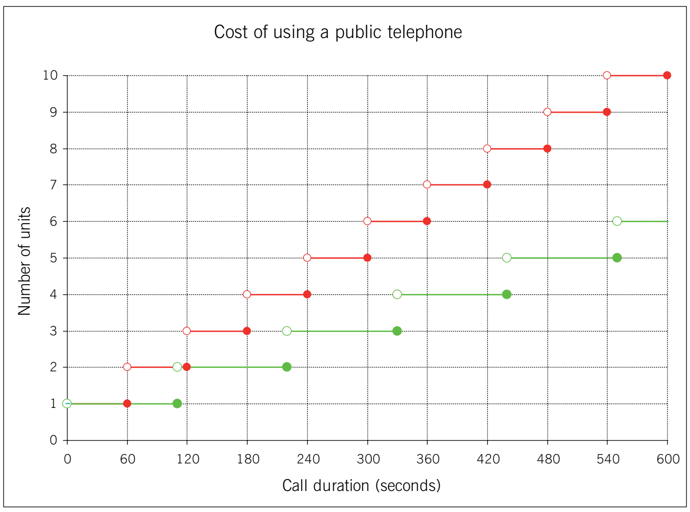
Constant functions:

Definition: a constant function is a function, the values of which values do not vary.

An example of a constant function is a minibus taxi tariff (see Lesson 16). The price is determined by the zones through which you travel and not by the actual distance you travel. This is illustrated in the table below:

From \ To	Alexandra	Baragwanath	Crown Mines	Diepkloof	Dobsonville	Dube	Faraday	Fourways	Halway House
Alexandra	R3,00							R8,00	R6,00
Diepkloof		R3,50	R4,00	R3,50	R4,50	R4,00			
Dobsonville		R4,50							
Dube		R4,00	R5,00	R4,00	R4,50	R3,50			
Halfway House	R6,00								R5,00

The public telephone tariff (see Lesson 17) is an example of a constant step function. This is illustrated in the graph below:



Break-even problems:

Finding the “break-even” point helps people to choose between two given options. The example we used was the Johannesburg electricity tariffs (see Lesson 19). We explored why (which values) you should choose the Life Line option and why (which values) you should chose the Two Part Flat option. This is illustrated in the table and graph below:

PHASE	SERVICE CHARGE	ENERGY CHARGE	UNIT
Life Line	R 0.00	38,05	c/kWh

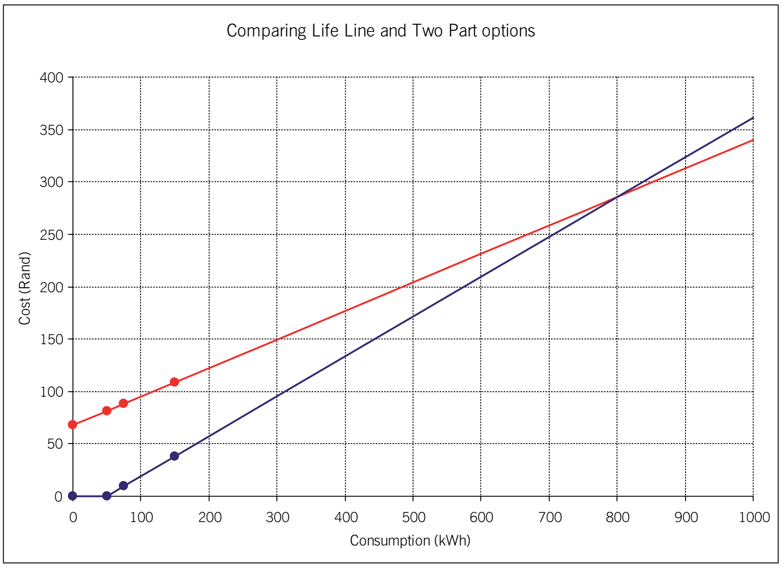
50kWh free per month

TWO PART FLAT

PHASE	SERVICE CHARGE	ENERGY CHARGE	UNIT
Single	R 67.76	27,23	c/kWh
Three	R 86.52	27,23	c/kWh

TWO PART SEASONAL

CHARGE	SEASONAL	SINGLE PHASE	THREE PHASE
Service		R67.76	R86.52
Energy	Summer	23,25	23,25
c/kWh	Winter	34,99	34,99



Activity

See Lessons 14 to 20 for activities.

LOOKING BACK

Comparing situations

Lesson 23

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues.
Assessment Standard
AS 11.1.1
In a variety of contexts find ways to explore and analyse situations that are numerically based, by:
- Estimating efficiently.
- Working with formulae by hand and with a calculator.
- Showing awareness of the significance of digits.
- Checking statements and results by doing relevant calculations.

(The range of problem types includes percentage, ratio, rate and proportion.)
AS 11.1.2
Relate calculated answers correctly and appropriately to the problem situation by:
- Interpreting answers in terms of the context.
- Reworking a problem if the first answer is not sensible or if the initial conditions change.
- Interpreting calculated answers logically in relation to the problem and communicating processes and results.

AS 11.1.3
Apply mathematical knowledge and skills to plan personal finances and investigate opportunities for entrepreneurship inclusive of:
- Specifying and calculating the value of income and expenditure items.
- Estimating and checking profit.

Learning Outcome 2
Functional relationships
The learner is able to recognise, interpret, describe and represent various functional relationships to solve problems in real and simulated contexts.
Assessment Standard
AS 11.2.1
Work with numerical data and formulae in a variety of real-life situations, including:
- Finding break-even points involving linear functions by solving simultaneous equations.

(Types of relationships to be dealt with include linear and inverse proportion relationships.)
AS 11.2.2
Draw graphs (by hand and/or by means of technology, where available) as required by the situations and problems being investigated.
AS 11.2.3
Critically interpret tables and graphs in a variety of real-life and simulated situations by:
- Estimating input and output values.

Overview

In the previous two lessons we looked at the topics we have covered already in this Mathematical Literacy series.

In this lesson we are going to work through a problem that brings together a great number of these topics.

Lesson

L C DVD

In the problem we are going to work through we will compare two options: in particular we will compare the transaction fees charged by two different banks for the same transaction. Not only will we be comparing the fees, but also the different ways in which these are calculated.

Methods and worked examples

INDIVIDUAL

FORMATIVE ASSESSMENT

Worked example

Bank A advertises its transaction fee for a cheque as follows: R2,90 / 1,10% / 28

Bank A explains that this expression means that it charges a basic fee of R2,90 per cheque plus 1,10% of the transaction amount with the maximum fee for a cheque being R28.

Bank B advertises its transaction fee for a cheque as follows: R4 / R3,25 / 26

Bank B explains that this expression means that it charges a basic fee of R4 for the first R250 or part thereof and it charges another R3,25 for each subsequent R250 or part thereof. The maximum fee for a cheque being R26.

Note:

We will see that:

- Bank A's method of calculating the fee leads to a linear piecewise function (see Lesson 18); and
- Bank B's method of calculating the fee leads to a constant step function (see Lesson 17).

Determine the transaction fees for each bank for the following cheques:

(3) R50

(4) R350

(5) R2,500

Solutions

(1) Bank A: Cost = basic charge + transaction amount × 1,10%
= R2,90 + R50 × 1,10%
= R3,45

Bank B: Cost = R4

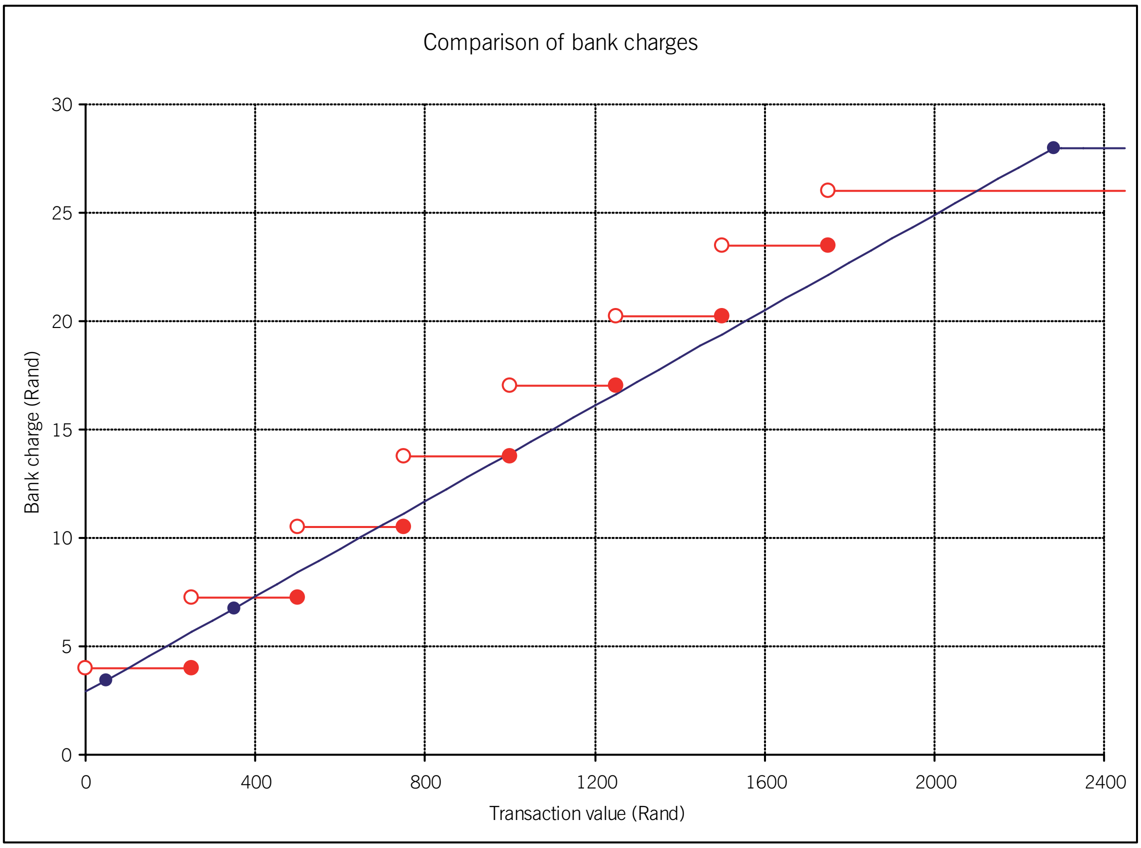
(2) Bank A: Cost = R2,90 + R350 × 1,10%
= R6,75

Bank B: Cost = R4 + 3,25
= R7,25

(3) Bank A: Cost = R2,90 + R2,500 × 1,10%
= R30,40 (this is more than the maximum value)
∴ transaction fee = R28 (maximum fee)

Bank B: Cost = R4 + 9 × R3,25
= R33,25 (this is more than the maximum value)
∴ transaction fee = R26 (maximum fee)

In this case a graph provides the easiest way of comparing the fee structure of the two banks.



What is clear is that there are a range of values for which the one bank is more expensive than the other. We certainly cannot say that up to some point the one is more than the other, although Bank B does seem to charge less for all cheques with values greater than about R2 100.

This is an excellent example of a situation in which only a graph can really help us to make sense of these charges and to compare them. Neither a table of values nor equations would have helped us here.

Activity

See Lessons 14 to 20

Lesson 24

INTEREST

An introduction

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context:
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues.
Assessment Standard
AS 11.1.1
In a variety of contexts, find ways to explore and analyse situations that are numerically based, by:
- Estimating efficiently.
- Working with formulae by hand and with a calculator.
- Showing awareness of the significance of digits.
- Checking statements and results by doing relevant calculations.

(The range of problem types includes percentage, ratio, rate and proportion.)
AS 11.1.2
Relate calculated answers correctly and appropriately to the problem situation by:
- Interpreting answers in terms of the context.
- Reworking a problem if the first answer is not sensible or if the initial conditions change.
- Interpreting calculated answers logically in relation to the problem and communicating processes and results.

AS 11.1.3
Apply mathematical knowledge and skills to plan personal finances and investigate opportunities for entrepreneurship inclusive of:
- Specifying and calculating the value of income and expenditure items.
- Estimating and checking profit.

Learning Outcome 2
Functional relationships
The learner is able to recognise, interpret, describe and represent various functional relationships to solve problems in real and simulated contexts.
Assessment Standard
AS 11.2.1
Work with numerical data and formulae in a variety of real-life situations, including:
- Finding break-even points involving linear functions by solving simultaneous equations.

(Types of relationships to be dealt with include linear and inverse proportion relationships.)
AS 11.2.2
Draw graphs (by hand and/or by means of technology, where available) as required by the situations and problems being investigated.
AS 11.2.3
Critically interpret tables and graphs in a variety of real-life and simulated situations by:
- Estimating input and output values.

Overview

In the next few lessons we continue to look at the mathematics of finance and focus more particularly on the concept of interest.

Lesson

LC DVD

Interest is:

- The fee paid by a borrower to a lender for the use of the borrowed money; and
- Is usually expressed as an annual percentage of the amount used. This percentage is known as the interest rate.

There are two parties involved when you are considering interest:

- The person who is borrowing the money – the borrower; and

- The person who has the money to loan – the lender.

Borrower ← Lender

After an agreed time the borrower repays the loan to the lender and pays a fee for borrowing the money. This fee is called interest. Usually the interest is determined using a rate expressed as a percentage.

Borrower → Lender

It is important to realise that we can all be borrowers and lenders. When we take a loan from the bank we are borrowing the money and the bank is lending us the money – in this case we would like the interest rate to be as low as possible. However, when we deposit money in a savings account, the bank is effectively borrowing the money from us and must pay us interest – in this case we would like the interest rate to be as high as possible. Of course, life isn't like that and the banks usually use a higher interest rate to determine the interest that we must pay them when we make a loan and pay us a lower interest rate when we leave (save) money with the bank.

Borrower ← Lender

Me Bank loan Bank

Bank Savings/Investment Me

Some examples of where you might want to borrow money and have to pay interest include:

- Emergencies, such as funerals; and
- Making a large purchase, such as buying a car or a house.

It is important to understand the effect of interest – both as a lender and as a borrower. To do so we will focus on the following aspects of interest in the next series of lessons:

- Fixed and varying interest rates;
- Simple interest;
- Compound interest;
- Hire-purchase and loan repayments; and
- The effect of changes in:
    - Interest rate; and
    - Monthly repayments

  on repaying a loan.

INDIVIDUAL

FORMATIVE ASSESSMENT

Methods and worked examples

When you borrow money from a bank, i.e. take out a loan, you have to pay a fee (interest) to the bank when you repay the loan. The interest would reflect as an expense in your financial statement.

When you save money with a bank, you are the lender and the bank is the borrower. The bank pays you a fee (interest) for borrowing the money. This fee or interest reflects as income in your financial statements.

INDIVIDUAL

SUMMATIVE ASSESSMENT

Activity

1. Buhle borrows R500 from her uncle. After six months she pays him R525.
    - 1.2 Who is the borrower in this situation?
    - 1.3 Who is the lender?
    - 1.4 How much interest did the borrower pay?
2. Delani deposits R1 000 into his savings account. A year later he withdraws R1 026,54.
    - 2.1 Who is the borrower in this situation?
    - 2.2 Who is the lender?
    - 2.3 How much interest did the borrower pay?

INTEREST

Simple interest

Lesson 25

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues.
Assessment Standard
AS 11.1.1
In a variety of contexts, find ways to explore and analyse situations that are numerically based, by:
- Estimating efficiently.
- Working with formulae by hand and with a calculator.
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- Checking statements and results by doing relevant calculations.

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- Reworking a problem if the first answer is not sensible or if the initial conditions change.
- Interpreting calculated answers logically in relation to the problem and communicating processes and results.

AS 11.1.3
Apply mathematical knowledge and skills to plan personal finances and investigate opportunities for entrepreneurship inclusive of:
- Specifying and calculating the value of income and expenditure items.
- Estimating and checking profit.

Learning Outcome 2
Functional relationships
The learner is able to recognise, interpret, describe and represent various functional relationships to solve problems in real and simulated contexts.
Assessment Standard
AS 11.2.1
Work with numerical data and formulae in a variety of real-life situations, including:
- Finding break-even points involving linear functions by solving simultaneous equations.

(Types of relationships to be dealt with include linear and inverse proportion relationships.)
AS 11.2.2
Draw graphs (by hand and/or by means of technology, where available) as required by the situations and problems being investigated.
AS 11.2.3
Critically interpret tables and graphs in a variety of real-life and simulated situations by:
- Estimating input and output values.

Overview

In this lesson we continue to look at the mathematics of finance and interest. We focus on a particular kind of interest called simple interest.

Lesson

DVD

Simple interest is when interest is calculated only on the loan amount. The loan amount is called the principal.

In subjects, such as accounting and business studies, calculating simple interest is summarised using the formula: $I = P \times r \times t$

Where: I = interest; P = principal or loan amount; r = the interest rate per year; and t = time in years or fractions of years.

Methods and worked examples

Although we will use of the formula above, it is more important to understand the mechanism by which simple interest is calculated – this is illustrated in the examples that follow.

Worked example

A micro-lender offers short-term loans and charges a fee (interest) of 15% of the loan amount. How much interest will you pay if you take out a loan of R2 500?

Solution: interest = R2 500 × 15%

= R375

(calculator keys: enter **2500×15%**)

When the person repays the loan, he/she must pay R2 500 + R375 = R2 875 – this amount is called the repayment.

Consider the following additional examples:

INDIVIDUAL

FORMATIVE ASSESSMENT

Additional example 1

1. A micro-lender who offers short-term loans, charges a client R400 interest on a R2 500 loan. What interest rate did the micro-lender use to calculate the interest?

Additional example 2

2. A donor wants to create an annual prize of R500. If the R500 is the interest on an investment and the annual rate is 11%, how much should the donor invest?

Additional example 3

3. A person loans R500 from a bank. How much interest must he/she pay after four months if the annual interest rate is 15%?

Solutions

1. Since the interest is a certain percentage of the loan amount, the rate is a fraction of the loan amount expressed as a percentage.

$$\text{rate} = \frac{400}{2\,500} \text{ expressed as a percentage}$$
$$= 16\%$$

(calculator keys: enter **400/2500** and then **%**)

Note: the interest rate used to determine the repayment amount was 16%. Clearly the higher the interest rate, the larger the amount of interest that needs to be paid (compare with the worked example where the interest rate was 15%).

2. interest = loan amount (investment) × interest rate (as a percentage)

$$\therefore \text{investment} = \frac{\text{interest}}{\text{interest rate}}$$
$$= \frac{\text{R500}}{11\%}$$
$$= \text{R4 545}$$

This means that if the donor invests R4 545 at an interest rate of 11% then there will be an interest payment of R450 at the end of each loan period.

3. In this problem the interest rate is not quoted for the period of four months, but for a year. If the interest rate for the whole year is 15% then the interest rate for part of the year is calculated proportionally.

Four months represents 412th of the year, so the rate for the period would be $\frac{4}{12}$th of 15%.

interest = R500 × $\frac{4}{12}$ × 15%

= R25

We could also use the formula:

I = P × r × t = R500 × 15% per year × $\frac{4}{12}$ years = R25

Activity

INDIVIDUAL

SUMMATIVE ASSESSMENT

1. How much interest will you pay if you borrow R3 000 from a micro-lender who charges 12,5% interest for short-term loans?
2. If you paid the micro-lender R1 130 back for a loan of R1 000, what interest rate did the micro-lender use to determine the interest?
3. A donor wants to donate an annual R250 prize to your school – this prize is to be the interest on an investment. How much must he invest, if he earns 4,5% interest on his money?
4. How much do you need to repay the micro-lender if you borrowed R1 500 and he charges 13,5% interest on short-term loans?
5. Viwe borrows R1 000 from the bank at an interest rate of 12,5% pa. How much must he pay the bank if he pays back the loan after three months?

Lesson 26

INTEREST

Compound interest

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues.
Assessment Standard
As 11.1.1
In a variety of contexts find ways to explore and analyse situations that are numerically based, by:

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AS 11.1.3
Apply mathematical knowledge and skills to plan personal finances and investigate opportunities for entrepreneurship inclusive of:

- Specifying and calculating the value of income and expenditure items.
- Estimating and checking profit.

Learning Outcome 2
Functional relationships
The learner is able to recognise, interpret, describe and represent various functional relationships to solve problems in real and simulated contexts.
Assessment Standard
AS 11.2.1
Work with numerical data and formulae in a variety of real-life situations, including:

- Finding break-even points involving linear functions by solving simultaneous equations.

(Types of relationships to be dealt with include linear and inverse proportion relationships.)
AS 11.2.2
Draw graphs (by hand and/or by means of technology, where available) as required by the situations and problems being investigated.
AS 11.2.3
Critically interpret tables and graphs in a variety of real-life and simulated situations by:

- Estimating input and output values.

Overview

In lesson 24 we introduced the concept of interest and defined interest as:

The fee paid by a borrower to a lender for the use of the borrowed money, which is usually expressed as an annual percentage of the amount borrowed (known as the interest rate).

In the previous lesson we considered one type of interest – simple interest, which we said referred to situations where interest is calculated only on the loan amount.

In this lesson we consider another type of interest – compound interest.

Lesson

Compound interest refers to situations where interest is calculated not only on the loan/investment amount (principal), but also the accumulated interest of previous periods.

Methods and worked examples

When you calculate compound interest you need to determine the interest paid in the first time period and add it to the principal amount. This new amount becomes the principal for the next time period. This process is repeated for however many time periods you need. The interest is incorporated into the amount each time, which means that your original amount grows.

Worked example

R1 000 is invested at an interest rate of 12% per investment period. What will the value of the investment be after seven periods?

Solution

Loan/investment amount	R1 000
Interest rate (per period)	12%

Description	**Interest**	
Opening balance		[illegible]
end of period 1	12% × R1 000 = R120	R1 120
end of period 2	R134,40	R1 254 40
end of period 3	R150,53	R1 404 93
end of period 4	R168,59	R1 573 52
end of period 5	R188,82	R1 762 34
end of period 6	R211,48	R1 973 82
end of period 7	R236,86	R2 210 68

$\frac{R134.40}{R1000}$ as a percentage = 13,4% of the original investment

Note:

- The interest you receive increases for each time period as the balance increases.
- By earning 12% on the accumulated balance of the investment and interest, you earn more interest, i.e. greater than 12%, than you would if the interest was calculated on the investment alone. This is shown at the end of the 2nd period where the interest is R134,40, which is 12% of R1 120, but 13,4% of R1 000 – the initial investment.
- After the 6th period the investment had nearly doubled.

For compound interest situations there is a Rule of Thumb that can be used to estimate how long it will take for an investment to double in value. It is called the rule of 72 and states that:

Rule of 72: *in compound interest situations the value of the investment will double after a number of investment periods equal to 72 divided by the interest rate per period.*

In our case 72 ÷ 12 = 6.

Consider the following additional examples:

INDIVIDUAL

FORMATIVE ASSESSMENT

Additional example 1

1. R1 000 is invested at an interest rate of 18% per investment period. What will the value of the investment be after seven periods?

Additional example 2

2. R1 000 is invested at an interest rate of 18% per annum. The bank compounds the interest every month.
   i. What will the value of the investment be after 12 months?
   ii. Compare the value of your investment after compounding monthly for 12 months with the value of your investment after compounding it yearly (question 1). What do you notice?
   iii. Calculate the percentage interest that this investment actually earned after a year when compounded monthly.

Solutions

1. According to the rule of thumb the investment should double after $72 \div 18 = 4$ investment periods.

Loan/investment amount	R1 000
Interest rate (per period)	18,0%

Description	Interest	Balance
Opening balance		R1 000
end of period 1	R180	R1 180
end of period 2	R212,40	R1 392,40
end of period 3	R250,63	R1 643,03
end of period 4	R295,75	R1 938,78
end of period 5	R348,98	R2 287,76
end of period 6	R411,80	R2 699,55
end of period 7	R485,92	R3 185,47

Note: according to the rule of thumb the investment should double after $72 \div 18 = 4$ investment periods. The values in the table confirm this. If you compare this with the worked example where the interest rate was much lower (12%), you see that the higher interest rate causes your investment to double in a much shorter period of time.

2. The interest rate per month is calculated as follows:

   $18\% \div 12\text{months} = 1{,}5\%$ per month

   (a)

Loan/investment amount	R1 000
Interest rate (per period)	1,5%

Description	Interest	Balance
Opening balance		R1 000
end of period (month) 1	R15	R1 015
end of period (month) 2	R15,23	R1 030,23
end of period (month) 3	R15,45	R1 045,68
end of period (month) 4	R15,69	R1 061,36
end of period (month) 5	R15,92	R1 077,28
end of period (month) 6	R16,16	R1 093,44
end of period (month) 7	R16,40	R1 109,84
end of period (month) 8	R16,65	R1 126,49
end of period (month) 9	R16,90	R1 143,39

end of period (month) 10	R17,15	R1 160,54
end of period (month) 11	R17,41	R1 177,95
end of period (month) 12	R17,67	R1 195,62

(b) After one year the value of the investment is R1 195,62 when compounded monthly and R1 180 the value when compounded yearly. The more times the interest is compounded the greater the "effective interest rate" and hence the return on your investment.

(c) Percentage interest $= \frac{R195,62}{R1\ 000}$ expressed as a percentage

$\approx 19{,}6\%$

Note: This interest rate is higher than the interest rate specified for the investment (18%) and is known as the effective interest rate.

Activity

INDIVIDUAL

SUMMATIVE ASSESSMENT

1. The table below gives the current interest rates per annum for a particular type of savings account.

Balance	Rate
R0 - R500:	1.75%
R501 - R1 000:	2.25%
R1,001 - R2,000:	2.75%
R2,001 - R5,000:	2.75%
R5,000 and above:	3.25%

1.1 You have R450 in your savings account.

1.1.1 What interest will you get per annum?

1.1.2 If the bank compounds your interest monthly, what interest will you get per month?

1.1.3 Calculate the value of your investment after six months.

2.1 After working in the holidays you deposit more money into your account and now have a balance of R1 325.

2.2 What interest will you get per annum now?

2.3 If the bank compounds your interest monthly, what interest will you get per month?

2.4 Calculate the value of your investment after 12 months.

2.5 Calculate the effective interest rate that your savings earned.

(1) Choose any bank to find out the following:

1. What savings schemes does it offer.
2. Do all the savings accounts offer a tiered interest rate like the one in question 1?
3. Which scheme gives the best interest rate per annum?
4. What is the prime lending rate for your particular bank?
5. What are the interest rates (for money owing and for money saved) for a credit card?

Lesson 27

INTEREST

Repaying a loan

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues.
Assessment Standard
AS 11.1.1
In a variety of contexts find ways to explore and analyse situations that are numerically based, by:
- Estimating efficiently.
- Working with formulae by hand and with a calculator.
- Showing awareness of the significance of digits.
- Checking statements and results by doing relevant calculations.

(The range of problem types includes percentage, ratio, rate and proportion.)
AS 11.1.2
Relate calculated answers correctly and appropriately to the problem situation by:
- Interpreting answers in terms of the context.
- Reworking a problem if the first answer is not sensible or if the initial conditions change.
- Interpreting calculated answers logically in relation to the problem and communicating processes and results.

AS 11.1.3
Apply mathematical knowledge and skills to plan personal finances and investigate opportunities for entrepreneurship inclusive of:
- Specifying and calculating the value of income and expenditure items.
- Estimating and checking profit.

Learning Outcome 2
Functional relationships
The learner is able to recognise, interpret, describe and represent various functional relationships to solve problems in real and simulated contexts.
Assessment Standard
AS 11.2.1
Work with numerical data and formulae in a variety of real-life situations, including:
- Finding break-even points involving linear functions by solving simultaneous equations.

(Types of relationships to be dealt with include linear and inverse proportion relationships.)
AS 11.2.2
Draw graphs (by hand and/or by means of technology, where available) as required by the situations and problems being investigated.
AS 11.2.3
Critically interpret tables and graphs in a variety of real-life and simulated situations by:
- Estimating input and output values.

Overview

In the previous three lessons we defined interest and looked at the concepts of simple and compound interest. Now we will focus on a problem that involves interest – repaying of a loan.

Lesson

LC DVD

The variables that affect repaying a loan are:

- The interest rate;
- The number of periods over which the loan will be repaid; and
- The monthly instalments.

Methods and worked examples

The key difference in the loans that we look at in this lesson and the simple and compound interest problems we looked at previously is that you must pay back part of the loan each month.

Worked example

INDIVIDUAL

FORMATIVE ASSESSMENT

R1 000 is borrowed at an interest of 18% per annum.

(a) How long will it take to repay the loan if the borrower repays R125 per month?

(b) How much interest will the borrower have paid?

Solution

In the solution the balance has a negative sign because the money is owed to the lender. The interest also has a negative sign because this is the fee that is paid for borrowing the money.

Loan amount	R1 000
Interest rate (per annum)	18,0%
Interest rate (per month)	1,5%
Monthly instalment	R125

Description	**Interest**	**Instalment**	**Balance**
Opening balance			**R1000,00**
End of month 1	-R15	R125	-R890
End of month 2	-R13,35	R125	-R778,35
End of month 3	-R11,68	R125	-R665,03
End of month 4	-R9,98	R125	-R550,01
End of month 5	-R8,25	R125	-R433,25
End of month 6	-R6,50	R125	-R314,75
End of month 7	-R4,72	R125	-R194,47
End of month 8	-R2,92	R125	-R72,39
End of month 9	-R1,09	R73,48	R0,00

Note: a fee of R15 was paid for borrowing the money, so, although the instalment was R125, the balance has decreased by only R110.

(a) It takes nine months to repay the loan.

Note:

- The interest decreases each month because the balance on which it is calculated decreases.
- The instalment value stays the same each month, meaning that as the amount of interest decreases so more of the repayment goes towards reducing the balance, which will decrease by a larger amount each month. In other words, each instalment has a greater impact on the outstanding balance than the instalment of the previous month.
- After eight months there is only R73,47 owing and hence the last repayment is R73,47 and not R125.

(b) Total repayments $= 8 \times R125 + R73{,}48$

$= R1\ 073{,}48$

Interest paid $= R73{,}48$

Percentage interest $= \frac{R73{,}48}{R1\ 000}$ expressed as a percentage

$\approx 7{,}53\%$

Note:

- This example illustrates that although repaying a loan involves interest, the amount that must be repaid cannot be calculated using either the simple or compound interest approaches. This is because the amount of the loan is effectively being changed every month in two ways: the interest and the repayment.

Activity

INDIVIDUAL

SUMMATIVE ASSESSMENT

1. Calculate how many months it will take to repay a loan of R2 500 borrowed at an interest rate of 13% per annum. Monthly loan repayments are R250.
2. Calculate the value of your investment after six months if you save R250 every month at an interest rate of 7,5% per annum compounded monthly.

INTEREST

Repaying a loan the impact of interest

Lesson 28

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts, which include financial aspects of personal, business and national issues.
Assessment Standard
AS 1.1.1
In a variety of contexts find ways to explore and analyse situations that are numerically based, by:

- Estimating efficiently.
- Working with formulae by hand and with a calculator.
- Showing awareness of the significance of digits.
- Checking statements and results by doing relevant calculations.

(The range of problem types includes percentage, ratio, rate and proportion.)
AS 11.1.2
Relate calculated answers correctly and appropriately to the problem situation by:

- Interpreting answers in terms of the context.
- Reworking a problem if the first answer is not sensible or if the initial conditions change.
- Interpreting calculated answers logically in relation to the problem and communicating processes and results.

AS 11.1.3
Apply mathematical knowledge and skills to plan personal finances and investigate opportunities for entrepreneurship inclusive of:

- Specifying and calculating the value of income and expenditure items.
- Estimating and checking profit.

Learning Outcome 2
Functional relationships
The learner is able to recognise, interpret, describe and represent various functional relationships to solve problems in real and simulated contexts.
Assessment Standard
AS 11.2.1
Work with numerical data and formulae in a variety of real-life situations, including:

- Finding break-even points involving linear functions by solving simultaneous equations.

(Types of relationships to be dealt with include linear and inverse proportion relationships.)
AS 11.2.2
Draw graphs (by hand and/or by means of technology, where available) as required by the situations and problems being investigated.
AS 11.2.3
Critically interpret tables and graphs in a variety of real-life and simulated situations by:

- Estimating input and output values.

Overview

In this lesson we continue to explore interest and will focus on one of the variables that affect the repayment of a loan; namely the interest rate. A change in the interest rate will affect peoples' lives in different ways. For example, if there is a decrease in the interest rate:

- Old-age pensioners who rely on the interest earned on their savings will experience a drop in income; whereas
- People repaying loans will experience a decrease in their monthly repayment, which leaves them with money to spend on other things.

Lesson

There are many loans for which the interest is fixed and a change in the interest rate will have no effect at all. Irrespective of whether a loan/investment has a fixed or variable interest rate it is important to remember that:

- If you are borrowing money, you should aim to get as low an interest rate as possible; and
- If you are investing money, you should aim to get as high an interest rate as possible.

Loans, such as home loans, and investments and savings accounts are affected by changing interest rates. The interest rates that banks charge and pay are linked to an interest rate called the repo rate.

Definition: the repo rate is the interest rate at which the Reserve Bank lends money to private banks. The governor of the Reserve Bank varies this rate to influence, among other things, inflation.

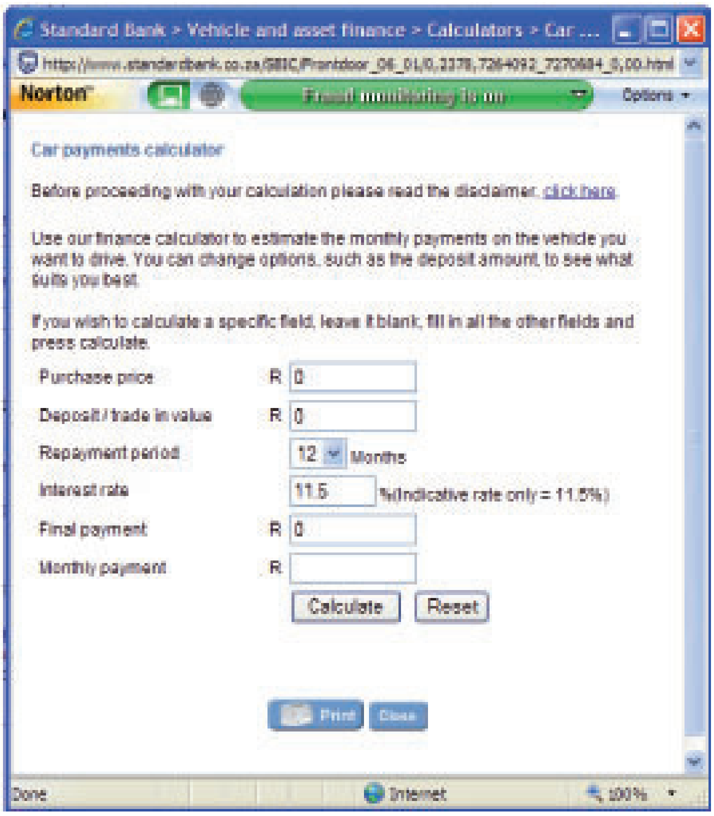
The interest rate used by micro-lenders and others is determined in part by the Usury Act. This Act was established by Parliament to protect the consumer and, in particular, the poor. Although the interest rate can vary from one lender to another, there is a range of values within which the interest rates will fall.

Methods and worked examples

There are complicated equations that can be used to determine the effect of a change in interest rate on loan repayments. However, there are loan calculators available via the internet that can calculate any of the variables involved in a typical loan. Some examples are shown below:

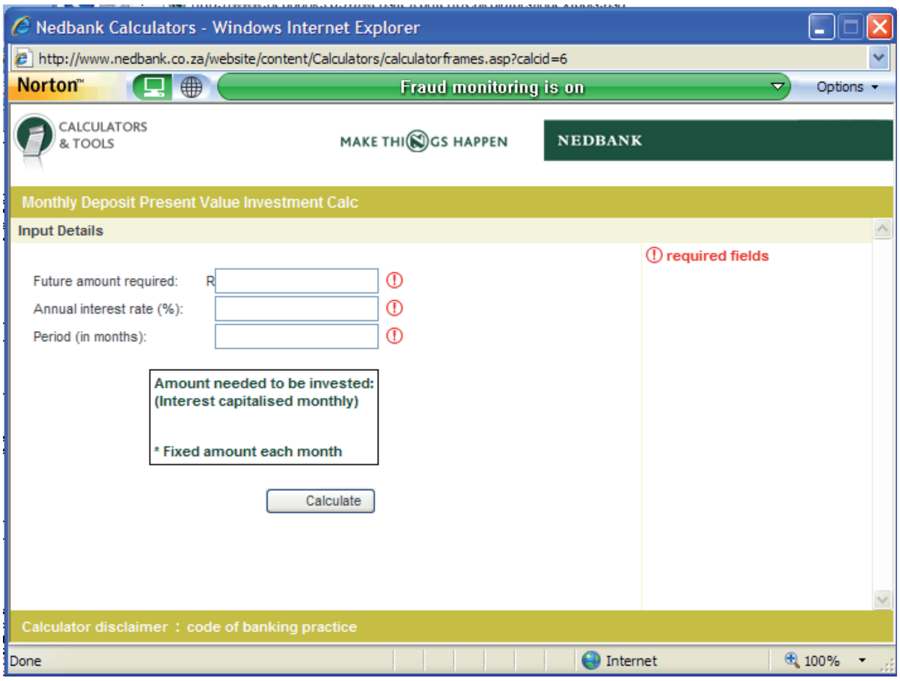
The first calculator (which can be found at: www.standardbank.co.za) can be used to calculate the monthly instalment you would make if you wanted to buy a car with a particular value. All you do is enter the value of the car in the purchase price box, the amount of the deposit that you plan to pay, the number of months you expect to take to repay the loan and the interest rate that you expect to be charged. Once you have entered the information you can press “calculate” and the calculator will tell you how much you have to pay each month.

You can use the calculator to calculate the price of the car you can afford by filling in the monthly payment that you can pay; the interest rate you expect to be charged; the number of months you expect to repay the loan and the deposit you plan to pay. When you press “calculate”, the calculator will tell you the value of the car you can afford.



The second calculator (which can be found at: www.absa.co.za) is a personal loan calculator. You enter the loan amount and the number of months you decide to take to repay the loan and the calculator will calculate the monthly repayment. This calculator automatically works with the bank's current interest rate.

The third calculator (found at www.nedbank.co.za) tells you how much you need to invest each month if you want the investment plus interest to equal a certain value after a given time. This information is useful if you want to save for a big event or even retirement.



What each of these calculators have in common is that the formula is hidden. They give you an answer without showing you how it was calculated. The challenge for the mathematically literate person is to be able to use these calculators to his/her advantage.

INDIVIDUAL

FORMATIVE ASSESSMENT

Worked example

Use a bank loan calculator to investigate the effect of different interest rates on the following loan: loan amount of R45 000, to be repaid over four years, with one payment every month.

Solution

Loan amount: R45 000

Repayment period: Four years

Interest rate	Monthly payment	Total amount
8%	R1 098,58	R52 731
9%	R1 119,83	R53 751
10%	R1 141,32	R54 783
11%	R1 163,05	R55 826
12%	R1 185,02	R56 881

To investigate the effect of interest we will look at the percentage change of the monthly payment.

The formula for % change is as follows:

$$\%\text{ change} = \frac{\text{final value} - \text{original value}}{\text{original value}} \text{ (expressed as a \%)}$$

For a 1% change in interest rate, i.e. from 8% to 9%, the percentage change in the monthly payments will be:

$$\%\text{ change} = \frac{\text{R1 119,83} - \text{R1 098,58}}{\text{R1 098,58}} \text{ (expressed as a \%)}$$

% change ≈ 1,93%

% change ≈ 2%

For a 4% change in interest rate, i.e. from 8% to 12%, the percentage change in the monthly payments will be:

% change $= \frac{\text{R1 185,02} - \text{R1 098,58}}{\text{R1 098,58}}$ (expressed as a %)

% change ≈ 7,87%

% change ≈ 8%

The above calculations illustrate that although the interest rate has changed by 1% or 4%, it seems as if the effect on monthly payments is almost twice that. The actual amounts in this example do not seem so large, but we should remember that the loan is small compared with, say, a home loan.

Note: a small change in interest rate does make a significant difference so it is worth your while to try to negotiate a lower interest rate for a loan or a higher interest rate for an investment.

Activity

INDIVIDUAL

SUMMATIVE ASSESSMENT

1. Use the values given in the worked example to answer the questions that follow.
   - 1.1 Calculate the impact on the monthly payments of an increase in the interest rate from:
     - a) 8% to 10%, i.e. a 2% increase; and
     - b) 8% to 11%, i.e. a 3% increase.
   - 1.2 Comment on the % change in the interest rate compared with the percentage change in the monthly payments.

Lesson 29

INTEREST

Repaying a loan the impact of the repayment amount

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts which include financial aspects of personal, business and national issues.
Assessment Standard
AS 11.1.1
In a variety of contexts, find ways to explore and analyse situations that are numerically based, by:
- Estimating efficiently.
- Working with formulae by hand and with a calculator.
- Showing awareness of the significance of digits.
- Checking statements and results by doing relevant calculation.

(The range of problem types includes percentage, ratio, rate and proportion.)
AS 11.1.2
Relate calculated answers correctly and appropriately to the problem situation by:
- Interpreting answers in terms of the context.
- Reworking a problem if the first answer is not sensible or if the initial conditions change.
- Interpreting calculated answers logically in relation to the problem, and communicating processes and results.

AS 11.1.3
Apply mathematical knowledge and skills to plan personal finances and investigate opportunities for entrepreneurship inclusive of:
- Specifying and calculating the value of income and expenditure items.
- Estimating and checking profit.

Learning Outcome 2
Functional relationships
The learner is able to recognise, interpret, describe and represent various functional relationships to solve problems in real and simulated contexts.
Assessment Standard
AS 11.2.1
Work with numerical data and formulae in a variety of real-life situations, including:
- Finding break-even points involving linear functions by solving simultaneous equation.s

(Types of relationships to be dealt with include linear and inverse proportion relationships.)
AS 11.2.2
Draw graphs (by hand and/or by means of technology where available) as required by the situations and problems being investigated.
AS 11.2.3
Critically interpret tables and graphs in a variety of real-life and simulated situations by:
- estimating input and output values.

Overview

Mathematically literate people want to exercise power over their lives and to do so they need to understand how the variables of a problem relate to each other and impact on the situation that the problem describes.

In the previous lesson we looked at the impact of a change in the interest rate on the repayment of a loan. In this lesson we will look at the impact of a change in the monthly instalment or repayment amount on the repayment of a loan.

Lesson

If you decrease the amount you pay monthly, then you will need to make more payments in order to repay your loan. So remember: Do not let a money lender try to "help" you by decreasing the amount you pay – you will pay for much longer and in the process pay a lot more in total.

Methods and worked examples

In order to repay a loan, you need to pay in more money per month than the interest for that month. This can be summarised as follows:

- If the monthly repayment amount < monthly interest amount – the loan amount owed will increase and your debt will increase;
- If the monthly repayment amount = monthly interest amount – the loan amount will remain unchanged and you will never pay off your loan; and
- If the monthly repayment amount > monthly interest amount – the loan amount will decrease and you will eventually pay off your loan. The greater the monthly repayment the sooner you will pay off the loan.

Worked example

Investigate the impact of different monthly instalments on the following loan:

Loan amount of R30 000,00 with a fixed interest rate of 12,5% per annum on which you make monthly instalments.

Solution

We will start off by working out the critical amount i.e. the interest you need to pay on the R30 000,00

The interest rate per month is calculated as follows:

$$12{,}5\% \div 12\text{months} = 1{,}042\% \text{ per month}$$

$$\text{Therefore interest} = 1{,}042\% \times R30\,000{,}00$$

$$= R312{,}60$$

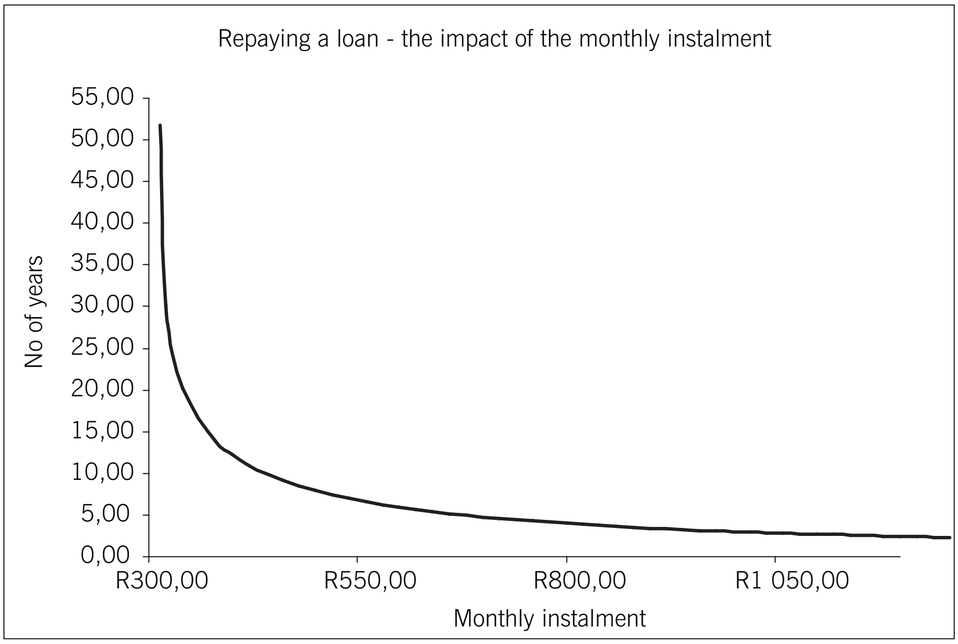
In order to be able to repay this loan, the repayment amounts must greater than R312,60. This is illustrated in the three scenarios below:

Description	Interest	Instalment	Balance
Scenario 1: Paying back an amount less than the interest amount			
Opening balance			-R30 000,00
End of month 1	-R312,60	R300,00	-R30 012,60
End of month 2	-R312,73	R300,00	-R30 025,33
Note: The loan amount (balance) is increasing			
Scenario 2: Paying back an amount equal to the interest amount			
Opening balance			-R30 000,00
End of month 1	-R312,60	R312,60	-R30 000,00
End of month 2	-R312,60	R312,60	-R30 000,00
Note: The loan amount remains the same			
Scenario 3: Paying back an amount more than the interest amount			
Opening balance			-R30 000,00

End of month 1	-R312,60	R340,00	-R29 972,60
End of month 2	-R312,31	R340,00	-R29 944,92

Note: The loan amount is decreasing

Below is a graph that shows the impact of increasing the monthly repayments for the R30 000,00 loan on the number of years it will take to pay off the loan:



There are two important lessons to be learnt from the graph:

- a small increase in the repayment value near the minimum repayment amount makes a huge difference to the number of years that you will take to repay the loan; and
- by contrast, a small increase in the repayment value when the repayment is large does not make as significant a difference to the number of years it will take to repay the loan.

INDIVIDUAL

SUMMATIVE ASSESSMENT

Activity

1. Investigate the effect of the monthly instalments on the repayment of a loan of R10 000 with a fixed interest rate of 10,8% by answering the following questions:

   1.1 Calculate the amount of interest you will pay in the first month of repayment.

   Develop and complete a table such as the one below for each of the following scenarios – that is, complete the table until the loan has been repaid (you will nee no more than 20 lines).

   Scenario 1: Interest (calculated in 1.1) + R500,00

   Scenario 2: Interest (calculated in 1.1) + R700,00

Description	Interest	Instalment	Balance
Opening balance			-R10 000,00
End of month 1			
End of month 2			
End of month 3			
End of month 4			
End of month 5			
End of month 6			

1.2 Based on the tables you have completed, determine how many months it will take you to repay the loan if your monthly instalment in:

(A) Scenario 1

(B) Scenario 2

1.3 Based on the tables you have completed, determine the total value of all the payments for:

(B) Scenario

(C) Scenario 2

1.4 Calculate the actual percentage interest paid on the loan for:

(A) Scenario 1

(B) Scenario 2

Lesson 30

FINANCE

Buying in bulk

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts which include financial aspects of personal, business and national issues.
Assessment Standard
AS 11.1.1
In a variety of contexts, find ways to explore and analyse situations that are numerically based, by:
- Estimating efficiently.
- Working with formulae by hand and with a calculator.
- Showing awareness of the significance of digits.
- Checking statements and results by doing relevant calculations.

(The range of problem types includes percentage, ratio, rate and proportion.)
AS 11.1.2
Relate calculated answers correctly and appropriately to the problem situation by:
- interpreting answers in terms of the context.
- reworking a problem if the first answer is not sensible or if the initial conditions change.
- interpreting calculated answers logically in relation to the problem, and communicating processes and results.

AS 11.1.3
Apply mathematical knowledge and skills to plan personal finances and investigate opportunities for entrepreneurship inclusive of:
- specifying and calculating the value of income and expenditure items.
- estimating and checking profit.

Learning Outcome 2
Functional relationships
The learner is able to recognise, interpret, describe and represent various functional relationships to solve problems in real and simulated contexts.
Assessment Standard
AS 11.2.1
Work with numerical data and formulae in a variety of real-life situations, including:
- Finding break-even points involving linear functions by solving simultaneous equations.

(Types of relationships to be dealt with include linear and inverse proportion relationships.)
AS 11.2.2
Draw graphs (by hand and/or by means of technology where available) as required by the situations and problems being investigated.
AS 11.2.3
Critically interpret tables and graphs in a variety of real-life and simulated situations by:
- estimating input and output values.

Overview

In this lesson we will bring together many of the ideas that we have addressed in the previous lessons. Dealing with finance nearly always requires you to make a decision and in today's lesson we will look at the options of whether to buy in bulk or not. We will deal with the perception that people have that buying in bulk is always economical.

Lesson

DVD

Methods and worked examples

The question is whether buying a 2 ℓ bottle of cold drink will always give better value for money than buying a 1ℓ bottle of cold drink.

If you assume that the unit cost of the 2 litre bottle is less than the unit cost of the 1 litre bottle then you will get better value for your money if you buy the 2 litre bottle.

However, this is only true if:

(a) you actually want to buy 2 ℓ of cold drink; and

(b) you have enough money to buy the 2 ℓ of cold drink .

Worked example

Which would give you the better buy:

- a two litre bottle of cold drink which costs R10,79; or
- a one litre bottle of cold drink which costs R7,39?

Solution

The unit cost of the two litre bottle of cold drink is R5,40/ℓ and the unit cost of the one litre bottle of cold drink is R7,39/ℓ.

Buying the two litre bottle of cold drink is better value for money, it will, however, cost you more money. So unless you need and/or want 2 ℓ of cold drink you are better off buying the 1ℓ bottle.

Activity

INDIVIDUAL

FORMATIVE ASSESSMENT

Additional examples:

In this example we will investigate the cost of the ingredients of cookies which are baked in order to sell for a profit. The cookies are sold in small packets.

The recipe for the cookies is as follows:

Basic Butter Cookies

Ingredients (enough for 4 packets)

1 cup (250 g) butter

2 cups (400 g) sugar

3 cups (360 g) flour

1 Tsp (15 g) baking powder

1 pinch salt

2 eggs

1 tsp (5ml) vanilla essence

Note: Different substances have different densities, which is why, butter weighs 250 g/cup; sugar weighs 200 g/cup and flour weighs 120 g/cup.

The unit cost of the product is shown on the shelving.

A list of products with their size options and unit price follows:

Butter (250g)	Baking powder (15 g)
250 g : R9,95 (R39,80/kg)	50 g : R2,89 (R57,80/kg)
500 g : R16,45 (R32,90/kg)	100 g : R7,59 (R75,90/kg)
Sugar (400 g)	200 g : R11,49 (R57,45/kg)
500 g : R3,49 (R6,98/kg)	Salt (a pinch)
1 kg : R6,75 (R6,75/kg)	500 g : R2,59 (R5,18/kg)
2,5 kg : R14,99 (R6,00/kg)	Eggs (2 eggs)
5 kg : R30,95 (R6,19/kg)	6 : R7,29 (R1,22/egg)
Flour (360 g)	18 : R18,99 (R1,06/egg)
1 kg : R6,59 (R6,59/kg)	24 : R23,50 (R0,98/egg)
2,5 kg : R10,95 (R4,38/kg)	Vanilla essence (5ml)
5 kg : R22,99 (R4,60/kg)	30 ml : R4,15 (R138,33/l)
10 kg : R49,99 (R5,00/kg)	100 ml : R8,79 (R87,90/l)

Note: The cost of a large, 5 kg bag of sugar is less economical than a smaller, 2,5 kg bag of sugar. It is easy to compare the unit cost of different sized packets or different brands of the same product as the unit cost of the products are found on the shelving at supermarkets. (see picture above).

Solution:

We will start with calculating the cost of making one recipe of cookies i.e. four packets of cookies. We will then explore what happens to the cost of the packets of cookies as we buy ingredients for a larger number of packets.

Cost for 1 recipe (4 packets)

Ingredients	Quantities	Cost
Butter (250 g)	1 × 250 g @ R9,95	R9,95
Sugar (400 g)	1 × 500 g @ R3,49	R3,49
Flour (360 g)	1 × 1 kg @ R6,59	R6,59
Baking powder (15 g)	1 × 50 g @ R2,89	R2,89
Salt (a pinch)	1 × 500 g @R2,59	R2,59
Eggs (2 eggs)	1 × 6 @ R7,29	R7,29
Vanilla essence (5 ml)	1 × 30 ml @ R4,15	R4,15
	TOTAL:	R36,95

The cost of each packet = $\frac{\text{R36,95}}{4}$ = R9,24/pkt.

There are other costs e.g. electricity that need to be considered when setting the selling price. To make a profit , you would need to sell each packet for a amount greater than at least R9,24. Let us say that we are going to sell the packets for R12,00.

Cost for 3 recipes (12 packets)

Ingredients	Quantities	Cost
Butter (250 g) × 3 = 750 g	1 × 250 g @ R9,95 1 × 500 g @ R16,45	R26,40
Sugar (400 g) × 3 = 1,2 kg	1 × 500 g @ R3,49 1 × 1 kg @ R6,75	R10,24
Flour (360 g) × 3 = 1,080 kg	1 × 2,5 kg @ R10,95	R10,95
Baking powder (15 g) × 3 = 45 g	1 × 50 g @ R2,89	R2,89
Salt (a pinch)	1 × 500 g @R2,59	R2,59
Eggs (2 eggs) × 3 = 6 eggs	1 × 6 @ R7,29	R7,29
Vanilla essence (5 ml) × 3 = 15 ml	1 × 30 ml @ R4,15	R4,15
	TOTAL:	R64,51

The cost of each packet = $\frac{\text{R64,51}}{12}$ = R5,38/pkt.

Making more cookies decreases the cost of each packet of cookies. This means that if you still sell the packets at R12,00 you will be making a much bigger profit —nearly R4,00 more per packet.

Cost for 10 recipes (40 packets)

Ingredients	Quantities	Cost
Butter (250 g) × 10 = 2,5 kg	5 × 500 g @ R16,45	R82,25
Sugar (400 g) × 10 = 4 kg	1 × 500 g @ R3,49 1 × 1 kg @ R6,75 1 × 2,5 kg @ R14,99	R25,23
Flour (360 g) × 10 = 3,6 kg	2 × 2,5 kg @ R10,95	R21,90
Baking powder (15 g) × 10 = 150 g	3 × 50 g @ R2,89	R8,67
Salt (a pinch)	1 × 500 g @R2,59	R2,59
Eggs (2 eggs) × 10 = 20 eggs	1 × 24 @ R23,50	R23,50
Vanilla essence (5 ml) × 10 = 50 ml	1 × 100 ml @ R8,79	R8,79
	TOTAL:	R172,93

The cost of each packet = $\frac{\text{R172,93}}{40}$ = R4,32/pkt.

This is certainly less than the cost of making one recipe and three recipes but the difference between three and ten recipes is not as marked as the difference between one recipe and three recipes.

Note: We need to do each calculation from scratch as the relationship between the number of recipes and the cost is not a direct proportion situation. We cannot simply say that if one recipe costs R10 then two recipes cost R20. This is because we have different options to chose from when we are selecting the most economical combination of ingredients to buy.

In conclusion we can say:

- as the number of recipes increases so it is possible to reduce the unit cost of each packet;
- you can only buy in bulk if you have the money to do so; and
- you must make sure that you have a market to sell all the packets of cookies to.

INDIVIDUAL

SUMMATIVE ASSESSMENT

Activity

1 Use the recipe and the table of ingredients given in the worked example to answer the following:

1.1 Calculate the cost of :

i. 5 recipes and hence each packet of cookies;

ii. 20 recipes and hence each packet of cookies.

1.2 Fill in the following table. Assume that you sell each packet of cookies for R12,00. Ignore extra costs like electricity etc when calculating the last column

Number of recipes	Number of packets of cookies	Unit cost per packet	Profit per packet
1			
3			
5			
10			
20			

1.3 Discuss any observations you make from the table.

FINANCE

Lesson 31

Breaking even

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts which include financial aspects of personal, business and national issues.
AS 11.1.1
In a variety of contexts, find ways to explore and analyse situations that are numerically based, by:
- Estimating efficiently.
- Working with formulae by hand and with a calculator.
- Showing awareness of the significance of digits.
- Checking statements and results by doing relevant calculations.

(The range of problem types includes percentage, ratio, rate and proportion.)
AS 11.1.2
Relate calculated answers correctly and appropriately to the problem situation by:
- interpreting answers in terms of the context.
- reworking a problem if the first answer is not sensible or if the initial conditions change.
- interpreting calculated answers logically in relation to the problem, and communicating processes and results.

AS 11.1.3
Apply mathematical knowledge and skills to plan personal finances and investigate opportunities for entrepreneurship inclusive of:
- specifying and calculating the value of income and expenditure items.
- estimating and checking profit.

Learning Outcome 2
Functional relationships
The learner is able to recognise, interpret, describe and represent various functional relationships to solve problems in real and simulated contexts.
Assessment Standard
AS 11.2.1
Work with numerical data and formulae in a variety of real-life situations, including:
- Finding break-even points involving linear functions by solving simultaneous equations.

(Types of relationships to be dealt with include linear and inverse proportion relationships.)
AS 11.2.2
Draw graphs (by hand and/or by means of technology where available) as required by the situations and problems being investigated.
AS 11.2.3
Critically interpret tables and graphs in a variety of real-life and simulated situations by:
- estimating input and output values.

Overview

This lesson is the last lesson on finance and we will be working through a problem that brings together many of the ideas that we have addressed in previous lessons in order to make a decision. We will look at a decision that involves the concept of breaking even.

Lesson

LC DVD

When we are dealing with income and expenses, we say that we have broken even when:

- income = expenses

Methods and worked examples

The problem that we are going to consider is that of a youngster who can earn money by delivering take-away food but he has no transport. He sees an advertisement for a motorcycle and needs to determine whether he can earn enough money to purchase the motorcycle. We will consider the problem in two parts (income and expenses) and then combine them to help the youngster to make a decision. The two parts are:

- calculating the cost and the running costs of the motorcycle; and
- calculating the amount of money he can earn.

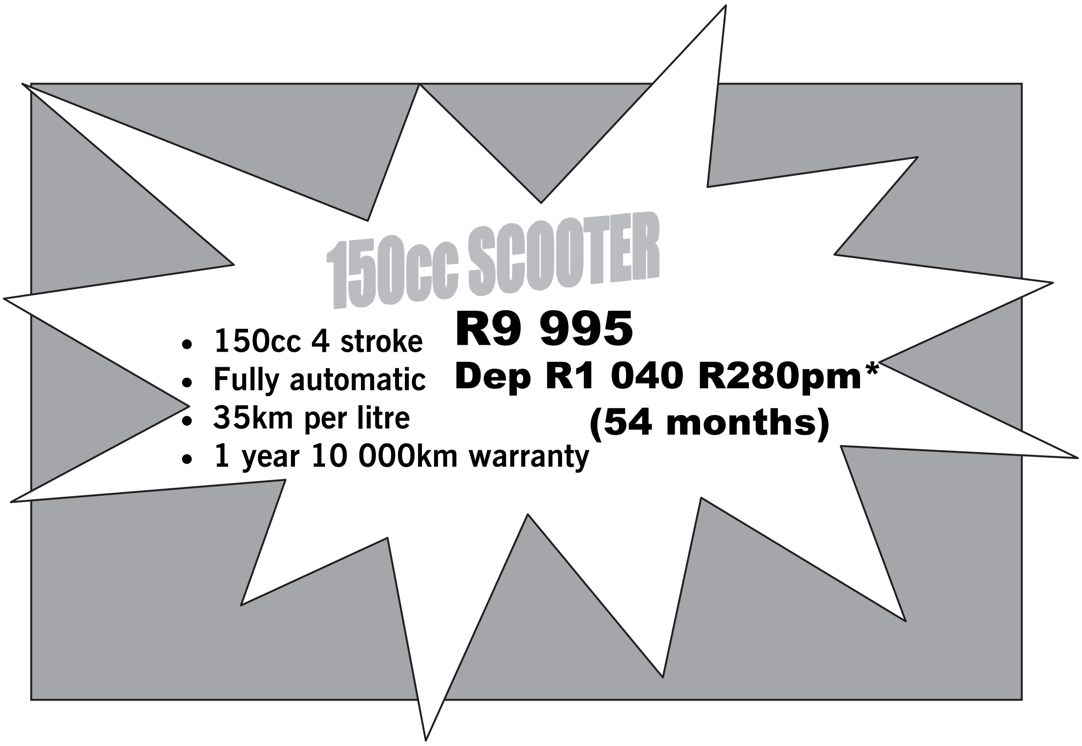
INDIVIDUAL

FORMATIVE ASSESSMENT

Worked example

Part 1: Expenses – motorcycle costs:

This is the advertisement that the youngster saw:



Determine a formula with which to calculate the monthly cost of the buying and running of this scooter.

Solution to Part 1:

Ananlysis of the advertisement gives us the following information:

- the scooter costs R9 995 cash;
- you can buy it by putting down a R1 040 deposit and paying a monthly installment of R280 for 54 months;
- the * indicates that there is some information missing – in this case that there is a R100,00 per month compulsory insurance; and
- the scooter can travel an average of 35 km on 1 litre of petrol, i.e. the average rate of petrol consumption.

For our purposes we will assume that the youngster can pay the R1 040 deposit, so we will only look at his monthly expenses.

There are two expenses each month:

- a fixed expense (the insurance and the repayment); and
- a variable expense (petrol)

His fixed expense = R280 + R100 = R380

His variable expense is calculated as follows:

Consumption rate = 35 km/ℓ

Cost of petrol = R5,75/ℓ

Therefore, the cost per km $= \frac{\text{R5,75 } \ell}{\text{35 km/ } \ell}$

≈ R0,16/km

Therefore his monthly cost = fixed cost + variable cost

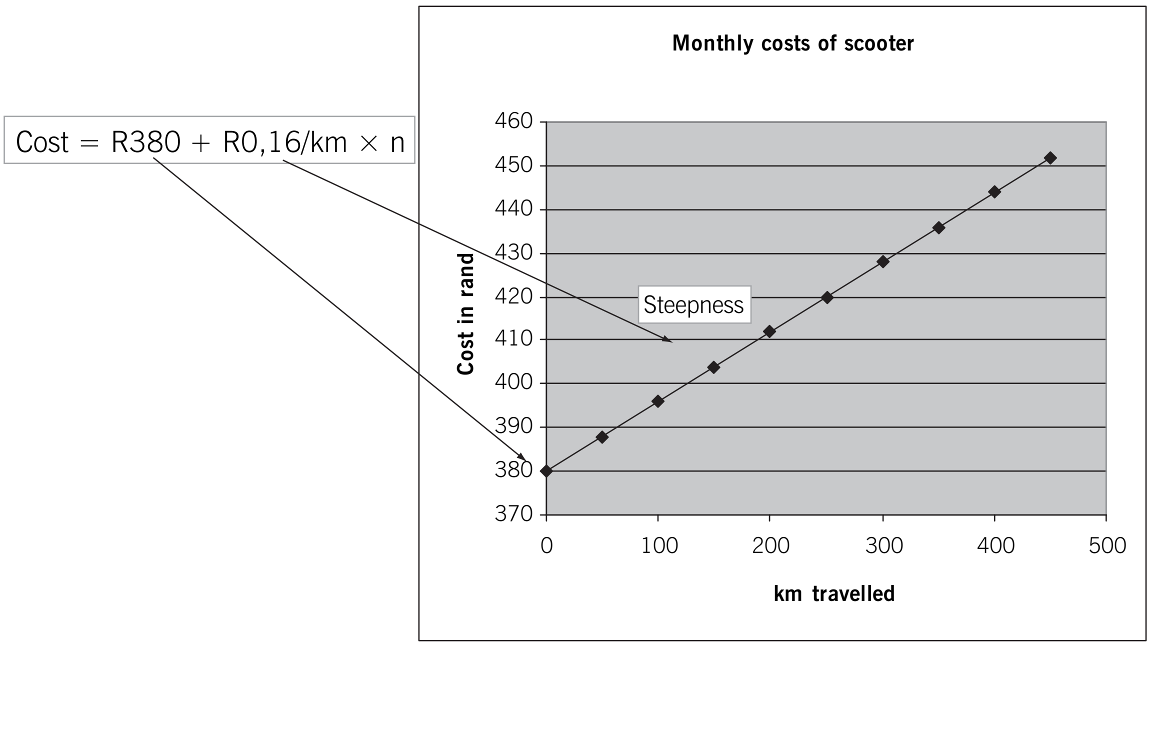
= R380 + R0,16/km × number of kilometers travelled (n)

= R380 + R0,16/km × n

Note: This formula is similar to the formulae we used for the electricity and the taxi tariffs. It represents a linear relationship or function. This is a function with a constant rate of change and can be represented by a straight line.

A table of values and a graph representing his monthly costs per km travelled is shown below:

Number of km travelled	Cost
0	R380 + R0,16/km × 0 km = R380
50	R380 + R0,16/km × 50 km = R388
100	R380 + R0,16/km × 100 km = R396
150	R404
200	R412
250	R420
300	R428
350	R436
400	R444
450	R452



Part 2: Income – delivering take-away food

The youngster saw the advertisement below in the newspaper:

316

Part time employment

DRIVERS WANTED

Do you have your own transport (car or m/cycle) and a drivers license?

Deliver meals and earn R1,50 per km

Enquire 073-245-6489

Determine a formula with which to calculate his income.

Solution to Part 1:

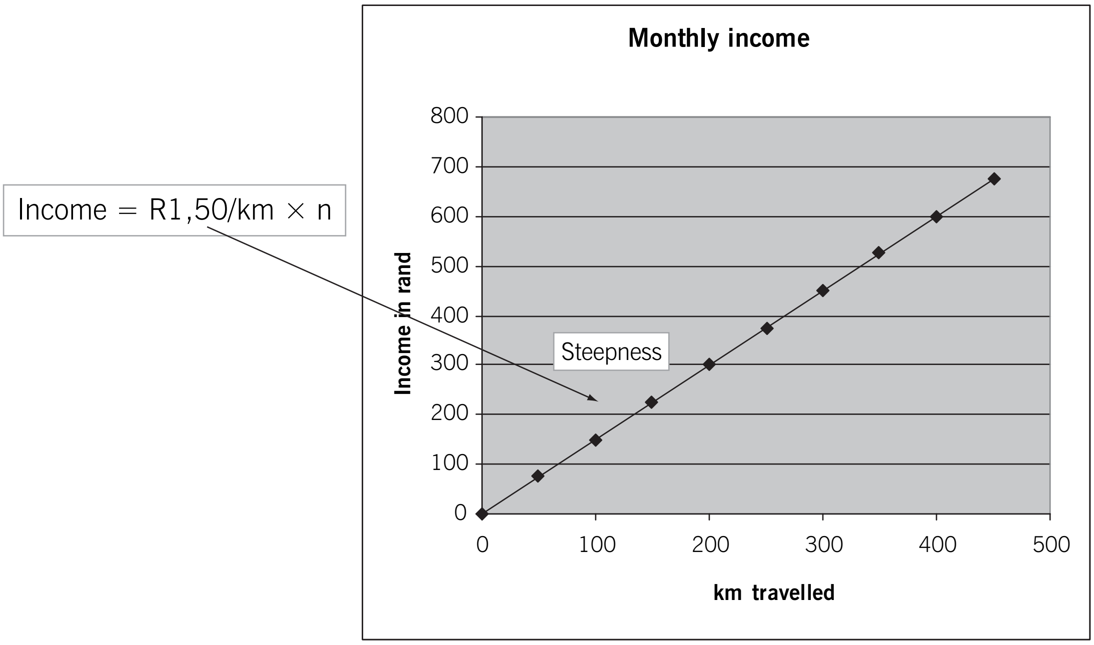
The rate at which he would get paid is R1,50/km.

$$\text{Therefore his income } = \text{number of km driven} \times \text{rate per km}$$
$$= n \times \text{R}1{,}50$$

Note: This is also a linear function and can be represented by a straight line. It is also an example of a direct proportional relationship as both quantities increase in the same ratio.

A table of values and a graph representing his monthly income per *km* travelled is shown below:

Number of km travelled	Income
0	R1,50/km × 0 km = R0,00
50	R1,50/km × 50 km = R75
100	R1,50/km × 100 km = R150
150	R225
200	R 300
250	R 375
300	R 450
350	R 525
400	R 600
450	R 675



We are now in a position to help the youngster make a decision – whether or not to buy the scooter and take the job to pay for it.

We need to determine the number of km at which his income will equal his expenses. We called this point the "break even" point. We can determine this point in three different ways:

- by looking at tables of values;
- by reading off values from graphs; and
- by solving simultaneous equations.

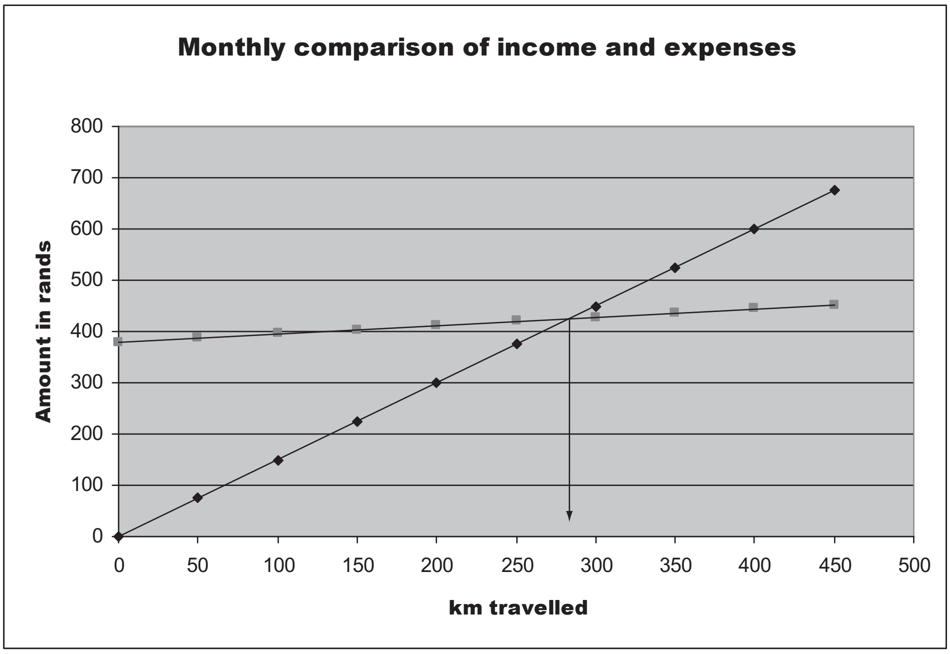
Each method has its advantages. We will consider each method.

Tables:

When you compare the values in the two tables given above, you can see that the break even point lies between 250 km and 300 km. This means that if he travels less than 250 km, he will definitely not earn enough money to cover his costs and if he travels more than 300 km, he will definitely cover his costs and have money over for other things.

Graph:

In order to compare the graphs, we need to draw both graphs on the same set of axes. See below.



The break even point is the point where the two graphs intersect. We cannot read off the value accurately but we can see from the graph that the break even point lies between 250 km and 300 km but closer to 300 km.

Equations:

You can calculate the exact value of the break even point by solving the two equations, which we developed earlier, simultaneously as follows:

$$\begin{aligned} \text{Income} &= \text{Expense} \\ \text{R1,50/km} \times n &= \text{R380} + \text{R0,16/km} \times n \\ \text{R1,50n} - \text{R0,16}n &= \text{R380} \\ \text{R1,34}n &= \text{R380} \\ n &= \text{R380/R1.34} \\ &\approx 283{,}58 \text{ km} \\ &\approx 285 \text{ km} \end{aligned}$$

The youngster can now make a decision as to whether to buy the scooter or not, as he knows approximately how many km he must travel per month in order to break even. He will need to check with the Take-away company whether he can expect to travel at least this number of *km* per month.

LC INDIVIDUAL

LC SUMMATIVE ASSESSMENT

Activity

1. The youngster in the problem above manages to find another delivery company who is prepared to pay him R1,80 per km. His father also agrees to pay the compulsory insurance of R100 each month.

   1.1 Determine a formula to represent his new income?

1.2 Fill in the table below:

Number of km travelled	Income
0	
50	
100	
150	
200	
250	
300	
350	
400	

1.3 Draw a graph to represent his income.

1.4 Determine a formula to represent his new expenses.

1.5 Fill in the table below:

Number of km travelled	Cost
0	
50	
100	
150	
200	
250	
300	
350	
400	

1.6 On the same system of axes, draw a graph to represent his expenses.

1.7 Determine the break even point by means of:

(A) The table of values.

(B) The graphs. Show how the break even value can be read off on your graph.

(C) By solving the two equations simultaneously.

Lesson 32

DATA HANDLING

An introduction

Learning Outcomes and Assessment Standards

Learning Outcome 4
Data Handling
The learner is able to collect, summarise, display and analyse data and to apply knowledge of statistics and probability to communicate, justify, predict and critically interrogate findings and draw conclusions.
NOTE: Assessment Standard 11.4.4. is the focal Assessment Standard of LO4. The other LO4 Assessment Standards serve to develop the skills that will enable learners to achieve this one.
Assessment Standard
AS 11.4.4
Critically interpret data and representations thereof (with awareness of sources of error and bias), in order to draw conclusions on problems investigated and make predictions and to critique other interpretations.
AS 11.4.1
Investigate a problem on issues such as those related to:
- Social, environmental and political factors.
- People's opinions.
- Human rights and inclusivity by:
- Collecting or finding data by appropriate methods (e.g. interviews, questionnaires, the use of data bases) suited to the purpose of drawing conclusions to the questions.
- Using appropriate statistical methods.
- Selecting a representative sample from a population with due sensitivity to issues relating to bias.
- Comparing data from different sources and samples.

AS 11.4.2
Select, justify and use a variety of methods to summarise and display data in statistical charts and graphs inclusive of:
- Tallies.
- Tables.
- Pie charts.
- Single and compound bar graphs.
- Line and broken-line graphs.

AS 11.4.3
Understand that data can be summarised and compared in different ways by calculating, and using measures of central tendency and spread (distribution), for more than one set of data inclusive of the:
- Mean.
- Median.
- Mode.
- Range.

AS 11.4.5
Work with simple notions of likelihood/probability in order to make sense of statements involving these notions.
AS 11.4.6
Demonstrate an awareness of how it is possible to use data in different ways to justify opposing conclusions.

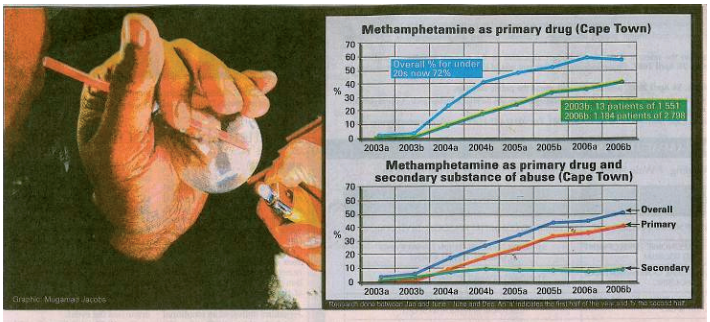
Overview

We are bombarded with "statistically based" arguments on a day-to-day basis. We receive information in the news and in advertisements via newspapers, magazines television, radio etc that is statistical in nature. An important attribute of a mathematically literate person is their ability to ask critical questions, so it is important that you understand how these statistics are developed so that you are in a position to critique those that you come across.

Methods and worked examples

The following examples from the media show the use of data and representations of data:

The data that was used to write this article has been illustrated by means of graphs.



Study finds tik demand is soaring

A'EYSHA KASSIEM
Education Writer

THE type of illegal substances in South Africa has changed over the past 10 years because drug addicts want to try new drugs.

This is part of a study by Charles Parry, director of the Alcohol and Drug Abuse Research Unit at the Medical Research Council (MRC).

He says that drug trafficking in South Africa has increased "considerably" since 1994.

The latest findings into substance abuse, for the period from June to December last year, indicate that children as young as 10 experiment with drugs. Some are addicted to tik.

Statistics from treatment centres in the province show that the age of addicts treated ranges from 10 to 73 – prompting organisations to consider targeting awareness campaigns at younger audiences.

Parry said the changing market meant the use of drugs such as the once-popular Mandrax and ecstasy was decreasing and that the demand for tik was increasing.

MRC statistics of addicts being treated also indicate a drop in the use of some drugs and an emergence of others.

Parry said the first six months of last year had seen a decrease in the demand for dagga in Cape Town and Durban, but there was a particularly high treatment demand by dagga-users under the age of 20 in the Eastern Cape.

The demand for Mandrax had also decreased across the country, except in Gauteng.

The use of "club drugs", such as ecstasy and LSD, also appeared to be tapering off.

"In the late 1990s there was an increased demand for 'club drugs' among rave club attendees, but treatment demand for ecstasy remained low over the years (across the country)."

A demand for treatment for addiction to tik, cocaine and heroin was increasing in the Western Cape.

Statistics also indicate the introduction of methcathinone, although this is still being used minimally.

"The past few years have seen a dramatic increase in local manufacturing and abuse of tik in the Western Cape.

"From the first part of 1999 to the end of 2006, the proportion of patients reporting tik as their primary substance of abuse rose from a mere 0.1% to a staggering 42% – with (users') average age 21 years.

"An astonishing 58% of patients under the age of 20 reported tik as their primary substance of abuse," he said.

The majority of addicts being treated were coloured men.

Since the first half of 1996, the demand for treatment relating to cocaine and heroine had increased drastically, he said.

Parry said South Africa was "desirable" for the trans-shipment of drugs because of its location and "lax border controls". "We have a weak criminal justice system, modern telecommunications and banking system as well as international trade links with South America, Northern America, Europe and Asia."

He said heroin from Asia and cocaine from South America were imported into South Africa and exported to Europe, North America and Australia.

"South Africa is the leading market for illicit drugs entering southern Africa," she said.

The data used to write the article below is represented in a table and a bar graph. The table speaks about the average personal income of the readers and shows that this newspaper is leading the other newspapers in terms of the income of their readers. The bar graph shows that the actual number of readers of this newspaper is less than that of one of their competitors. The headline emphasises the data in the table rather than the data in the bar graph.

Newspapers gain in popularity

Sunday Times still leading the pack in terms of readers' income, writes **Adele Shevel**

NEWSPAPERS have moved up from fifth to fourth position in the South African media-preference stakes, according to the latest AMPS (All Media and Product Survey).

The Internet has jumped two positions from last on the list and is now the seventh most popular news and entertainment medium in SA.

Most popular of all is radio, followed by TV.

There has been a significant increase in the number of people who read newspapers.

Daily newspapers increased their readership most, by 9.4%, largely because of the 22% increase in the readership of the Daily Sun. That newspaper now has the highest readership in SA: 3.7 million a day.

The Daily Sun took the lead from the Sunday Times, but the latter remains the most widely

TOP 10 BY READERS' INCOME

	Average household income	Total household income	Average personal income	Total personal income
Sunday Times	R10 110	R33.283-bn	R5 752	R18.936-bn
Sunday Times Lifestyle	R10 535	R26.031-bn	R6 116	R15.112-bn
Sunday Times Bus. Times	R11 221	R21.276-bn	R6 757	R12.811-bn
Daily Sun	R4 550	R16.740-bn	R2 671	R9.828-bn
Sunday Times Extra	R10 783	R16.714-bn	R6 324	R9.802-bn
Rapport	R10 470	R16.208-bn	R5 743	R8.890-bn
Sunday Times Magazine	R11 354	R14.783-bn	R6 207	R8.082-bn
City Press	R6 688	R12.727-bn	R4 127	R7.854-bn
Sunday Times Metro	R10 129	R13.299-bn	R5 925	R7.780-bn
Sunday Sun	R5 702	R11.478-bn	R3 380	R6.803-bn

READERSHIP

	Million readers
Daily Sun	3.679
Sunday Times	3.292
Sunday Times Lifestyle	2.471
Sunday Sun	2.013
City Press	1.903
Sunday Times Business Times	1.896
Sunday Times Extra	1.550
Rapport	1.548
Sunday Times Metro	1.313
Sunday Times Magazine	1.302

This article has no representation of any data collected. The headline tells us that bullying is getting worse. To be able to report this, however, would require some form of evidence – some baseline of numbers, i.e. what was happening some years ago and what is happening now. Although some people may think that there has been an increase in bullying, thinking does not make it so – there has to evidence.

Bullying in South African schools is getting worse

Staff Reporter

Bullying in South African schools by pupils and teachers has reached "epidemic proportions" in the past two years.

Bullying was verbal, psychological or physical, Pierre du Plessis and Lloyd Conley – doctors at the University of Johannesburg – told the Inter national Conference on Learner Discipline in Potchefstroom.

"Most learners are targeted by bullies in taxis on the way to school, or in toilets and bathrooms at schools," Dr Du Plessis said. sion."

Boys and girls used different methods to bully fellow pupils.

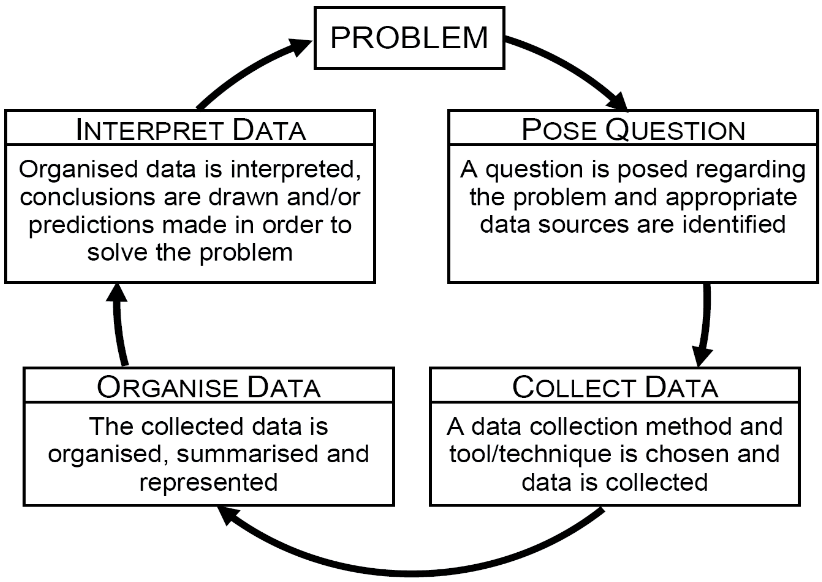
While boys were likely to engage in "direct bullying" in a physical manner, girls took to spreading rumours or enforcing "social isolation".

It was crucial for pupils to be supervised – particularly in the playground and in the hallways, Dr Du Plessis said.

"Whether the bullying is direct or indirect, the key component is that the physical or psychological intimidation occurs repeatedly over time to create a pattern of harassment and abuse."

Perpetrators and victims alike were

Data handling is a process which can be represented by a data handling cylce. A diagrammatic representation of the data handling cyle is shown below:



The reason for collecting data is to address a problem as there isn't any point in collecting data unless you want to do something with it. To ensure that you get the right answers to the problem, you have to ask the right questions. This is the first step in the data handling cycle – to pose the right question.

Having decided on the questions and who can best answer the questions, you need to collect data. This is the second step in the cycle. In a future lesson we will look at different techniques that can be used to collect data.

The third step in the cycle involves working with the data. This usually involves three processes:

- organising the data – taking from its rough form and putting it altogether;

- summarising the data –– reducing a large number of data to a more manageable amount; and
- representing the data – presenting it in such a way that you and other people can make sense of it.

The final step is to interpret the data. This involves drawing conclusion or making predictions based on the data – seeing how the data answers the question we originally posed and how that in turn helps us to resolve the problem we started out with. It is important to note that the same data can be interpreted differently by different people for their own and different purposes.

Activity

GROUPS

CLASS

1. Find articles in the newspaper that use statistics as a base for the article. Make a list of the questions they were answering with the data they collected.
2. Prepare a poster of different articles; highlighting the variety of subjects that use data as a way of communicating.

Lesson 33

DATA HANDLING

Posing questions

Learning Outcomes and Assessment Standards

Learning Outcome 4
Data Handling
The learner is able to collect, summarise, display and analyse data and to apply knowledge of statistics and probability to communicate, justify, predict and critically interrogate findings and draw conclusions.
Assessment Standard
AS 11.4.1
Investigate a problem on issues such as those related to:

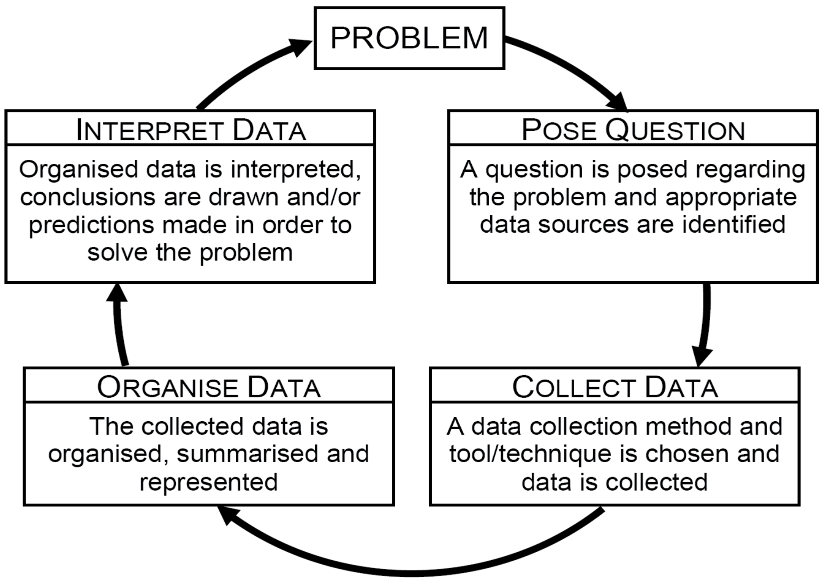
- Social, environmental and political factors.
- People's opinions.
- Human rights and inclusivity by:
- Collecting or finding data by appropriate methods (e.g. interviews, questionnaires, the use of data bases) suited to the purpose of drawing conclusions to the questions.
- Using appropriate statistical methods.
- Selecting a representative sample from a population with due sensitivity to issues relating to bias.
- Comparing data from different sources and samples.

Overview

In the previous lesson we introduced the data handling cycle (see below). In this lesson we will look at the first stage of the data handling cycle – posing questions.

Lesson

The data handling cycle begins with a problem. Some examples of problems include:



- Determining a description of the population of a country. In SA this is done by Stats South Africa by means of a census;
- Determining the characteristics of the people who buy a particular product. This is done by market researchers;
- Proving a theory/hypothesis by performing experiments in a laboratory. Scientists would be interested in this; and

- Trying to decide on a cellular telephone contract.

What each of these problems has in common, from the census to deciding on the cellular telephone contract, is that information or data is needed to solve the problem.

To make sure that we collect the right kind of data we need to formulate the right kind of question or list of questions. It is crucial to recognise that the way in which a question is asked will determine the way that the data collected can be interpreted.

If I ask you: “Do you enjoy school?” you may answer: “no” and yet you may still think that it is very important to go to school. If I want to know how important you think school is in your life, I need to ask that question.

Methods and worked examples

There are two main types of questions:

- open ended; and
- closed questions.

An open ended question does not provide a great deal of structure for the answer. An example of an open ended question would be something like: “What do you like most about our coffee shop?”

There are a whole range of answers that would be appropriate, e.g.:

- the atmosphere;
- the music played; and
- the coffee is served in mugs.

The advantage of an open ended question is that you may hear things that you would otherwise not hear, for example you may not have realised that a factor in bringing people to your shop is the music you play.

The disadvantage of an open ended question is that you have to deal with a large number of different answers. The answers could be long and it may be hard to determine a trend in the answers. They are time consuming to process.

A closed question provides a limit to the number of possible answers. Examples of closed questions include:

- date of birth;
- gender (male/female);
- true or false questions; and
- yes/no questions;

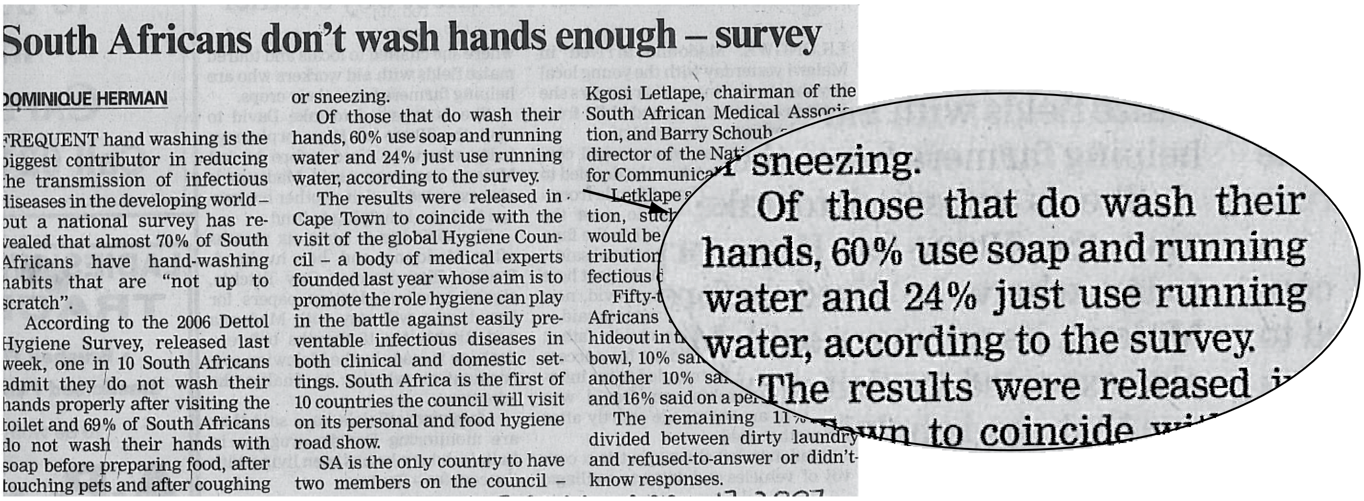
Questions like these only have one answer e.g. Did you watch Isidingo last night? (yes/no).

One question type that bridges the open ended and closed question spectrum is a question with a rating scale. There are different rating scales:

- always – occasionally – never;
- very good – good – average – poor – very poor;
- rate on a scale from 1 to 5, with 1 being the best and 5 the worst.

Worked example

What type of questions do you think the reporter would have used in collecting the data for this article? Give some examples of questions that could have been used.



South Africans don't wash hands enough – survey

DOMINIQUE HERMAN

FREQUENT hand washing is the biggest contributor in reducing the transmission of infectious diseases in the developing world – but a national survey has revealed that almost 70% of South Africans have hand-washing habits that are "not up to scratch".

According to the 2006 Dettol Hygiene Survey, released last week, one in 10 South Africans admit they do not wash their hands properly after visiting the toilet and 69% of South Africans do not wash their hands with soap before preparing food, after touching pets and after coughing or sneezing.

Of those that do wash their hands, 60% use soap and running water and 24% just use running water, according to the survey.

The results were released in Cape Town to coincide with the visit of the global Hygiene Council – a body of medical experts founded last year whose aim is to promote the role hygiene can play in the battle against easily preventable infectious diseases in both clinical and domestic settings. South Africa is the first of 10 countries the council will visit on its personal and food hygiene road show.

SA is the only country to have two members on the council – Kgosi Letlape, chairman of the South African Medical Association, and Barry Schoub ... director of the Nat... for Communica...

Letklape ... tion, such ... would be ... tribution ... fectious ...

Fifty-t... Africans ... hideout in t... bowl, 10% sa... another 10% sa... and 16% said on a pe...

The remaining 11% ... divided between dirty laundry and refused-to-answer or didn't-know responses.

Solution

The reporter would have used closed questions. Some examples are:

- Do you wash your hands? Please tick.
  - o After going to the toilet.
  - o Before preparing food.
  - o After touching pets.
  - o After coughing or sneezing.
- When you wash your hands, do you use soap? (yes/no)

Additional examples

INDIVIDUAL

FORMATIVE ASSESSMENT

What type of questions do you think the reporter would have used in collecting the data for the article alongside? Give an example of a question that could have been used.

Solution

The reporter could have used a rating question. An example could be:

How often have you been bullied at school? Please tick

- o Never
- o Occasionally
- o Regularly
- o Almost constantly

The reporter could also have used a closed question. An example could be:

- If you have been bullied, where did it happen? Please tick
  - o On public transport.
  - o In the classroom.
  - o In the playground.

- In the toilets/bathrooms.
- On the sports field.
- Any other places (please specify).

Activity

INDIVIDUAL

SELF ASSESSMENT

1. Below is a snippet from an article on South African men and their concerns over their image :

SA men go big on image, says poll

SOUTH African males have embraced the world of metrosexuality — with more men spending time and money on their appearance.

An international consumer survey by the Nielsen Company claims that South African men — along with the Portuguese and Greeks — are world leaders when it comes to caring for their appearance, with 94% of South African respondents claiming that men have become more interested in personal grooming.

The reasons given vary from hopes of attracting a new partner to looking good for a spouse and feeling good about themselves.

Nielsen product manager Meave Beckett-Leech said the survey of 25 000 Internet users across the world found that South African men spent more on beauty products and treatments than any other.

"We have found that South Africans are a lot more concerned about their appearance than other global respondents and South Africans are prepared to spend more on maintaining their appearance."

Thai men came second, with 93%, followed by ...ians (90%), Britons (89%) and Americans (78%) ... Kong men were found to be the ...

Read the articles and answer the questions below:

a. How many people took part in the reporter's survey?

b. How did the reporter go about getting the responses?

c. Give two examples of:

   i. closed questions;

   ii. rating questions;

   iii. open questions

that could have been asked in the survey.

Lesson 34

DATA HANDLING

Samples, populations and collection techniques

Learning Outcomes and Assessment Standards

Learning Outcome 4
Data Handling
The learner is able to collect, summarise, display and analyse data and to apply knowledge of statistics and probability to communicate, justify, predict and critically interrogate findings and draw conclusions.
NOTE: Assessment Standard 11.4.4. is the focal Assessment Standard of LO4. The other LO4 Assessment Standards serve to develop the skills that will enable learners to achieve this one.
Assessment Standard
AS 11.4.4
Critically interpret data and representations thereof (with awareness of sources of error and bias), in order to draw conclusions on problems investigated and make predictions and to critique other interpretations.
AS 11.4.1
Investigate a problem on issues such as those related to:

- Social, environmental and political factors.
- People's opinions.
- Human rights and inclusivity by:
- Collecting or finding data by appropriate methods (e.g. interviews, questionnaires, the use of data bases) suited to the purpose of drawing conclusions to the questions.
- Using appropriate statistical methods.
- Selecting a representative sample from a population with due sensitivity to issues relating to bias.
- comparing data from different sources and samples.

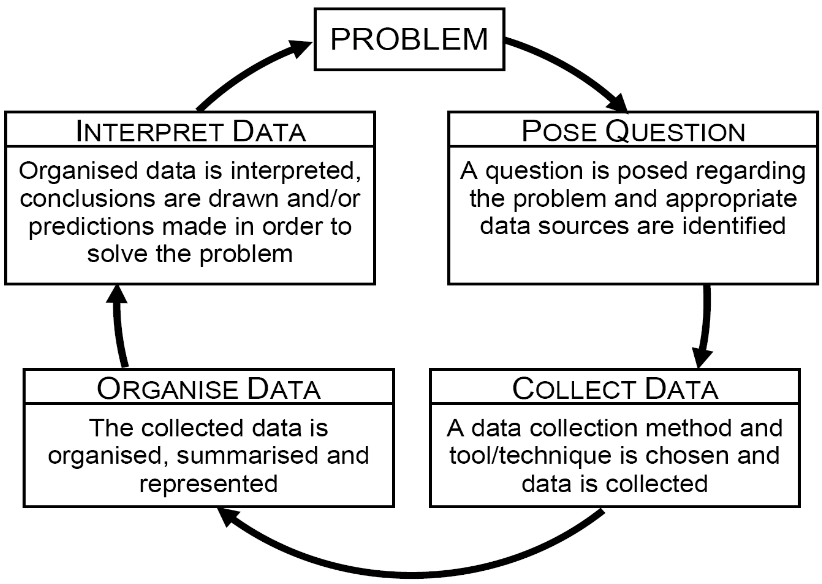
AS 11.4.6
Demonstrate an awareness of how it is possible to use data in different ways to justify opposing conclusions.

Overview

In this lesson we focus on the second stage of the data handling cycle – collecting the data (see below). We will look at the importance of selecting a data source that gives us the best possible information to be able to answer our problem.

Lesson

We will consider two aspects of data collection in this lesson:



- sampling; and
- techniques of data collection.

Methods and worked examples

Sampling:

It would be an enormous task to interview every person in a town e.g. Kimberley, if we wanted to get the opinion of the "people" of Kimberley on a particular issue. To get around this we interview a smaller group of Kimberley residents (called a sample) who represent the bigger group (called the population).

Data collection often involves sampling – the selection of a part of a population with the purpose of making conclusions about the whole population. The method used to determine the sample should ensure that the sample is as representative of the population as possible. In other words – the method should reduce bias.

There are many different ways of determining a sample (some of which will be discussed in a later lesson) two types of sampling that we will focus on now are:

- random sampling; and
- stratified sampling.

Random sampling involves selecting members of the sample in such a way that every member of the population has an equally likely chance of being selected.

Stratified sampling is used when there are identifiable groups within the population. This sampling technique ensures a random sample from each of the groups. Stratified sampling would be used if you wanted to get the opinion of a school. It would involve taking a random sample of the same number of people from each grade and maybe the same number of boys and girls from each grade.

The size of the sample is very important. The sample needs to be small enough so that the data is manageable, but it must also be big enough to ensure that we can reliably make predictions about the whole population e.g. if your best friend has agreed to wear a mini skirt to the party, it is unfair to say to your mother that "all" of the girls will be wearing mini skirts.

Worked example

INDIVIDUAL

FORMATIVE ASSESSMENT

The newspaper article on the next page is a report on an international internet survey of 25 000 users. It claims to have found that 94% of South African men have become more interested in personal grooming. Critique the sample size in the article and discuss whether you think that it is a good representation of the men in the world?

SA men go big on image, says poll

SOUTH African males have embraced the world of metrosexuality — with more men spending time and money on their appearance.

An international consumer survey by the Nielsen Company claims that South African men — along with the Portuguese and Greeks — are world leaders when it comes to caring for their appearance, with 94% of South African respondents claiming that men have become more interested in personal grooming.

The reasons given vary from hopes of attracting a new partner to looking good for a spouse and feeling good about themselves.

Nielsen product manager Meave Beckett-Leech said the survey of 25 000 Internet users across the world found that South African men spent more on beauty products and treatments than any other.

Solution

There are more than 2 billion men above 20 years of age in the world so 25 000 is only about 1% of the population. The survey is also biased in favour of internet users. The question that arises is: what proportion of South African men has access to the Internet? In other words how representative of South African men is the survey.

Data collection techniques:

In addition to choosing the right question and selecting an appropriate sample, where necessary, we must also decide on the appropriate collection technique. That is if we want our problem to be addressed properly.

We will look at four data collection techniques:

- observation;
- interviews;
- questionnaires and surveys; and
- using available information.

Observation:

This means actually watching something happen e.g. counting cars at an intersection.

- Advantages:
  - Actual observation of a phenomenon.
- Limitations:
  - Time consuming
- Application:
  - Recording the results of an experiment
  - Counting events.

Interviews:

- Advantages:
    - o Allows the interviewer to clarify responses to questions.
    - o High participation rate.
- Limitations:
    - o Time consuming
    - o Influence of interviewer.
    - o Reliability of responses.
- Application:
    - o Small scale detailed/in-depth investigations.

Questionnaires/surveys:

- Advantages:
    - o Permits anonymity which may contribute to more sincere (honest) responses.
    - o Time efficient.
- Limitations:
    - o Possible misinterpretation of questions by respondents.
    - o Unwillingness of respondents to complete the questionnaire.
- Application:
    - o Large scale investigations.

Using available information:

- Advantages:
    - o The information already exists.
- Limitations:
    - o The existing data may not be sufficiently detailed.
- Application:
    - o Looking for historical trends.

PAIRS

CLASS

Activity

1. Consider the articles below and on the next page, and:

   a. Discuss how you think the data for each article was collected.

   b. Discuss how you would choose a sample group to represent the population that the article is referring to.

CHILDREN AS YOUNG AS TEN ADDICTED

Scientists warn of youth tik crisis

A'EYSHA KASSIEM
Education Writer

CHILDREN as young as 10 are experimenting with drugs and are addicted to tik, as the province's drug epidemic spirals out of control.

And the number of tik patients seeking help at treatment centres has soared to 59%, from 5% at the end of 2003. More than a third of the patients were under 20.

According to the latest statistics from the Medical Research Council (MRC), for the period between June and December last year, substance abuse in the Western Cape over the past three years has skyrocketed, with the age of drug users ranging from 10 to 54.

... choice. Since the beginning of 2005, tik use for under-20s in treatment had increased from 11% to the current 59%.

While the face of tik is predominantly male, coloured, unemployed and single, the drug is also gaining popularity among females, with both sexes using the drug in almost equal numbers.

Alcohol is also starting to be seen as the possible "gateway" drug that leads more young people to experiment with other drugs.

In addition, researchers are starting to examine the link between tik use and risky sexual behaviour as the drug is seen to increase both confidence and sex drive.

The MRC's Alcohol and ...

... Cape Town over the past three years, with adolescents forming a "large proportion of tik users".

He said in the second half of 2002, 13 patients out of 1 551 had tik as their primary drug – but by the second half of last year, this had leapt to 1 184 patients out of 2 798.

"The majority (84%) were in treatment for the first time. The average age of patients who reported tik as their primary substance of abuse in the second half of 2006 was 22 years old and 72% were male.

"Most of the patients (90%) were coloured, 8% were white, 1% Asian/Indian and 1% were black. Notably, 37% of the patients were younger than 20 years of age," he said.

CHILDREN as young as 10 are experimenting with drugs and are addicted to tik, as the province's drug epidemic spirals out of control.

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Article 1

Bullying in South African schools is getting worse

Staff Reporter

Bullying in South African schools by pupils and teachers has reached "epidemic proportions" in the past two years.

Bullying was verbal, psychological or physical, Pierre du Plessis and Lloyd Conley – doctors at the University of Johannesburg – told the Inter national Conference on Learner Discipline in Potchefstroom.

"Most learners are targeted by bullies in taxis on the way to school, or in toilets and bathrooms at schools," Dr Du Plessis said.

... sion."

Boys and girls used different methods to bully fellow pupils.

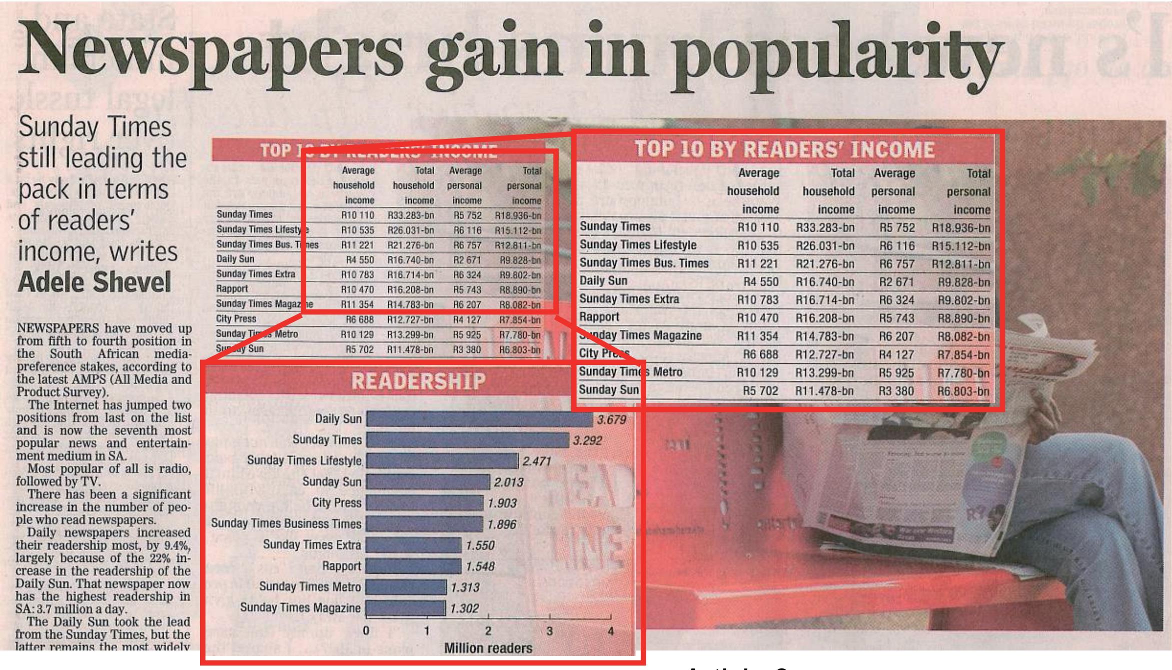
While boys were likely to engage in "direct bullying" in a physical manner, girls took to spreading rumours or enforcing "social isolation".

It was crucial for pupils to be supervised – particularly in the playground and in the hallways, Dr Du Plessis said.

"Whether the bullying is direct or indirect, the key component is that the physical or psychological intimidation occurs repeatedly over time to create a pattern of harassment and abuse."

Perpetrators and victims alike were

Article 2



Newspapers gain in popularity

Sunday Times still leading the pack in terms of readers' income, writes **Adele Shevel**

NEWSPAPERS have moved up from fifth to fourth position in the South African media-preference stakes, according to the latest AMPS (All Media and Product Survey).

The Internet has jumped two positions from last on the list and is now the seventh most popular news and entertainment medium in SA.

Most popular of all is radio, followed by TV.

There has been a significant increase in the number of people who read newspapers.

Daily newspapers increased their readership most, by 9.4%, largely because of the 22% increase in the readership of the Daily Sun. That newspaper now has the highest readership in SA: 3.7 million a day.

The Daily Sun took the lead from the Sunday Times, but the latter remains the most widely

TOP 10 BY READERS' INCOME

	Average household income	Total household income	Average personal income	Total personal income
Sunday Times	R10 110	R33.283-bn	R5 752	R18.936-bn
Sunday Times Lifestyle	R10 535	R26.031-bn	R6 116	R15.112-bn
Sunday Times Bus. Times	R11 221	R21.276-bn	R6 757	R12.811-bn
Daily Sun	R4 550	R16.740-bn	R2 671	R9.828-bn
Sunday Times Extra	R10 783	R16.714-bn	R6 324	R9.802-bn
Rapport	R10 470	R16.208-bn	R5 743	R8.890-bn
Sunday Times Magazine	R11 354	R14.783-bn	R6 207	R8.082-bn
City Press	R6 688	R12.727-bn	R4 127	R7.854-bn
Sunday Times Metro	R10 129	R13.299-bn	R5 925	R7.780-bn
Sunday Sun	R5 702	R11.478-bn	R3 380	R6.803-bn

READERSHIP

	Million readers
Daily Sun	3.679
Sunday Times	3.292
Sunday Times Lifestyle	2.471
Sunday Sun	2.013
City Press	1.903
Sunday Times Business Times	1.896
Sunday Times Extra	1.550
Rapport	1.548
Sunday Times Metro	1.313
Sunday Times Magazine	1.302

Article 3

Lesson 35

DATA HANDLING

Organising data

Learning Outcomes and Assessment Standards

Learning Outcome 4
Data Handling
The learner is able to collect, summarise, display and analyse data and to apply knowledge of statistics and probability to communicate, justify, predict and critically interrogate findings and draw conclusions.
Assessment Standard
AS 1.4.1
Investigate a problem on issues such as those related to:
- Social, environmental and political factors.
- People's opinions.
- Human rights and inclusivity by:
- Collecting or finding data by appropriate methods (e.g. interviews, questionnaires, the use of data bases) suited to the purpose of drawing conclusions to the questions.
- Using appropriate statistical methods.
- Selecting a representative sample from a population with due sensitivity to issues relating to bias.
- Comparing data from different sources and samples.

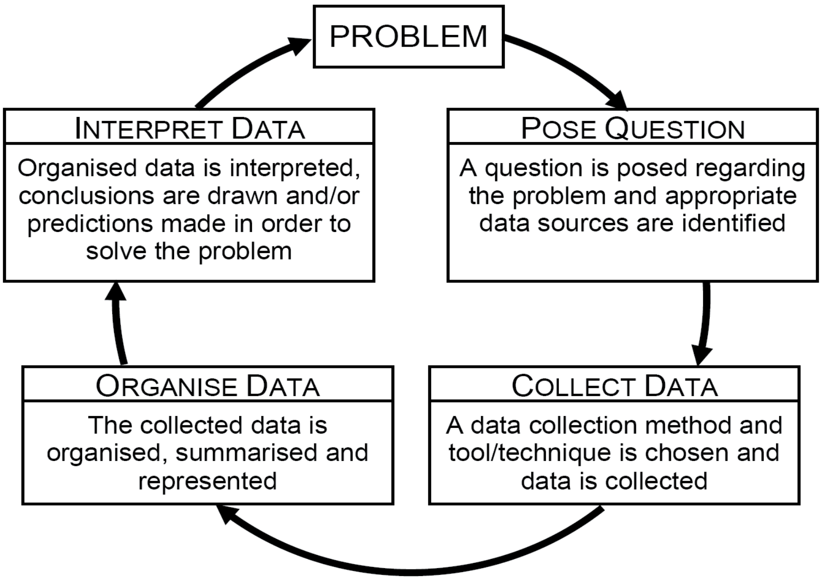
AS 11.4.2
Select, justify and use a variety of methods to summarise and display data in statistical charts and graphs inclusive of:
- Tallies.
- Tables.
- Pie charts.
- Single and compound bar graphs.
- Line and broken-line graphs.

AS 11.4.6
Demonstrate an awareness of how it is possible to use data in different ways to justify opposing conclusions.

Overview

We are now ready to look at the third stage of the data handling cycle – organising data. In this stage the data is organised in order to be able to summarise and represent it in a way that makes interpretation of the data easier. In this lesson we look at ways of organising the data. This step is made a great deal easier for large amounts of data by the use of computers.

Lesson



Methods and worked examples

There are two different types of data and these different types impact on the methods used when organising data:

- Categorical data – data with labels rather than numbers e.g. the club that a runner belongs to or the sex of a person; and
- Quantitative or numerical data. There are two types of numerical data:
    - discrete data – data that can only have a countable value e.g. the number of children in a family; and
    - continuous data – data that can have any value in a range of values e.g. the height of a person.

The advantage of organising data is that you reduce the amount of data you have to work with but the disadvantage is that you lose the detail.

Worked example

The example we will use to illustrate organising data is that of two runners – a girl and a boy running in the same race. The boy beats the girl in the race. The girl, however, claims that although the boy clocked a better time than she did, she is still the better runner. What she is really claiming is that she ran a better race when compared to the other women than the boy did when compared to the men.

In order to justify this claim the girl will need to work with the race results and show that she ran a better race. The results of the race follow. We will be using these results in this lesson and the next few lessons.

POS	NAME	CLUB	AGE	SEX	CAT	TIME
62	ABRA	TOP FORM	45	M	VET	0:46:40
97	ADAM	METROPOLITAN	42	M	VET	0:49:38
204	ADAM	TEMP	49	M	VET	0:58:14
285	ADAM	MP TITANS	55	M	MAS	1:05:32
137	ADON	NB GUGS	54	M	MAS	0:53:31

POS	NAME	CLUB	AGE	SEX	CAT	TIME
320	ALBE	TYGERBERG	53	M	MAS	1:13:19
368	ARCH	RUN WALK 4 LIFE	37	F	SEN	1:27:24
58	AREN	ATLANTIS	45	M	VET	0:46:24
308	AREN	HEWAT	37	F	SEN	1:08:50
312	AREN	HEWAT	37	M	SEN	1:10:33
256	ARUM	STRAGGLERS	50	M	MAS	1:02:20
93	AUGU	VOB	42	M	VET	0:49:26
255	BARN	STRAGGLERS	40	M	VET	1:02:19
55	BATH	TEMP	21	M	SEN	0:46:12
370	BENJ	TOP FORM	59	F	MAS	1:28:53
127	BESS	SOUTH STRIDERS	36	F	SEN	0:52:46
95	BEST	TEMP	38	F	SEN	0:49:32
362	BEUK	SANLAM	54	M	MAS	1:23:19
154	BEUS	TEMP	41	M	VET	0:54:44
1	BHIT	HARMONY WP	22	M	SEN	0:32:04
253	BINN	EDGEMEAD	56	M	MAS	1:02:11
294	BIRT	TEMP	54	M	MAS	1:06:12
269	BLAI	TEMP	24	F	SEN	1:03:56
218	BLAN	TEMP	28	M	SEN	0:59:20
119	BOTH	MAFIKENG	59	M	MAS	0:51:24
165	BOTH	TEMP	25	F	SEN	0:55:23
40	BOUW	SANLAM	22	M	SEN	0:44:01
87	BOUW	SANLAM	22	F	SEN	0:49:08
182	BOUW	SANLAM	50	F	MAS	0:56:40
283	BRAK	FISH HOEK	26	M	SEN	1:05:23
160	BRAN	FISH HOEK	37	M	SEN	0:55:09
366	BRAN	FISH HOEK	32	F	SEN	1:26:43
148	BRED	PAARL	43	F	VET	0:54:00
50	BREN	EDGEMEAD	24	M	SEN	0:45:25
326	BRET	OLD MUTUAL	38	F	SEN	1:15:54
339	BRET	TEMP	54	M	MAS	1:18:46
126	BRIN	SANLAM	54	M	MAS	0:52:38
163	BRIN	TEMP	23	M	SEN	0:55:16
268	BUCH	TEMP	19	M	JUN	1:03:51
19	BUNN	EDGEMEAD	42	M	VET	0:39:27
69	BUNN	HARMONY WP	46	M	VET	0:47:33
106	BUNN	EDGEMEAD	37	F	SEN	0:49:58
184	BURG	TYGERBERG	48	M	VET	0:56:49
226	BURG	WEST COAST	37	F	SEN	0:59:43
240	BURG	TYGERBERG	49	F	VET	1:01:17
309	BURG	HEWAT	50	M	MAS	1:08:51
345	BURG	HEWAT	53	F	MAS	1:20:15
286	BUYS	TEMP	30	F	SEN	1:05:38
149	CARO	SANDF WP	47	M	VET	0:54:08
156	CARS	BELLVILLE	41	M	VET	0:54:50
213	CHED	VOB	15	M	JUN	0:58:41
215	CHIM	ESKOM GIJIMAS	25	F	SEN	0:59:05
122	CLAA	HARMONY WP	58	M	MAS	0:51:50

POS	NAME	CLUB	AGE	SEX	CAT	TIME
103	COMB	TEMP	23	M	SEN	0:49:51
53	CONR	METROPOLITAN	37	M	SEN	0:45:48
229	CRON	VOB	52	M	MAS	1:00:30
68	CROW	TYGERBERG	44	M	VET	0:47:26
313	CROW	TELKOM	42	F	VET	1:11:30
56	CUPI	HARMONY WP	51	F	MAS	0:46:12
64	CURR	EDGEMEAD	38	M	SEN	0:46:57
76	CYST	FRANSCHHOEK	41	M	VET	0:48:27
52	DA S	TOP FORM	51	M	MAS	0:45:42
321	DA S	TYGERBERG	58	F	MAS	1:14:32
21	DANI	MAMRE	27	M	SEN	0:39:56
43	DARI	VOB	45	M	VET	0:44:21
42	DAVI	TOP FORM	37	M	SEN	0:44:11
45	DAVI	MP CELTIC	32	M	SEN	0:44:26
41	DE B	SANLAM	52	M	MAS	0:44:04
139	DE J	TEMP	34	M	SEN	0:53:39
188	DE J	HARMONY WP	39	F	SEN	0:56:55
115	DE K	WHALERS	36	M	SEN	0:51:18
237	DE K	DURBAC	49	F	VET	1:01:12
305	DE K	WHALERS	28	F	SEN	1:08:06
34	DE L	HARMONY WP	38	F	SEN	0:42:30
114	DE L	BRACKENFELL	41	M	VET	0:51:17
300	DE L	BRACKENFELL	40	F	VET	1:07:08
239	DE W	TEMP	33	M	SEN	1:01:15
272	DEAN	VOB	55	M	MAS	1:04:19
301	DEAN	VOB	55	F	MAS	1:07:33
73	DENN	EDGEMEAD	44	M	VET	0:47:57
293	DEVI	VOB	71	M	GGMAS	1:06:09
151	DHAN	VOB	46	M	VET	0:54:33
360	DOMI	NB GUGS	59	F	MAS	1:23:15
361	DOMI	NB GUGS	16	F	JUN	1:23:16
331	DU P	HELDERBERG	52	M	MAS	1:17:11
329	DU T	BELLVILLE	55	F	MAS	1:16:46
236	DUNN	EDGEMEAD	51	F	MAS	1:01:00
296	EBRA	MP CELTIC	47	M	VET	1:06:15
344	EDWA	BELLVILLE	57	F	MAS	1:20:07
351	ELKI	TYGERBERG	41	F	VET	1:21:51
172	ELS	BELLVILLE	37	F	SEN	0:55:54
162	ENGE	TEMP	46	M	VET	0:55:15
365	ESAU	MP TITANS	44	F	VET	1:26:23
33	ESTE	TYGERBERG	44	F	VET	0:41:55
332	ESTE	EDGEMEAD	39	F	SEN	1:18:01
107	FERR	EDGEMEAD	47	M	VET	0:49:58
273	FERR	EDGEMEAD	28	F	SEN	1:04:39
369	GEER	HOUT BAY	35	F	SEN	1:27:41
116	GELD	SANLAM	36	M	SEN	0:51:19
354	GELD	TEMP	27	F	SEN	1:22:14
219	GILB	MELKBOS	46	M	VET	0:59:21

POS	NAME	CLUB	AGE	SEX	CAT	TIME
144	GILL	TEMP	51	M	MAS	0:53:51
66	GODL	HARMONY WP	50	M	MAS	0:47:23
183	GOLI	TYGERBERG	31	F	SEN	0:56:43
2	GOQW	AAC	26	M	SEN	0:32:12
355	GOVE	TEMP	34	M	SEN	1:22:15
212	GRAH	BRACKENFELL	57	F	MAS	0:58:38
110	GREE	AAC	72	M	GGMAS	0:50:19
348	GRIF	EDGEMEAD	52	F	MAS	1:21:13
336	GROB	BELLVILLE	60	F	GMAS	1:18:29
98	GROE	HELDERBERG	37	F	SEN	0:49:44
24	HANE	HARMONY WP	37	M	SEN	0:40:53
185	HANS	TYGERBERG	37	M	SEN	0:56:51
359	HARL	UCT	49	M	VET	1:23:14
129	HARR	PINELANDS	42	M	VET	0:53:01
299	HARR	PINELANDS	40	F	VET	1:07:07
143	HART	PAARL	53	M	MAS	0:53:50
136	HASS	AAC	61	M	GMAS	0:53:25
179	HAYC	AAC	46	M	VET	0:56:30
176	HEIM	WEST COAST	61	M	GMAS	0:56:22
341	HEIM	WEST COAST	59	F	MAS	1:19:13
23	HEND	SANDF WP	34	M	SEN	0:40:50
63	HEND	SANLAM	45	M	VET	0:46:47
238	HEND	DURBAC	36	F	SEN	1:01:12
196	HENK	TEMP	43	M	VET	0:57:38
71	HERB	TEMP	27	M	SEN	0:47:49
99	HERB	BELLVILLE	62	F	GMAS	0:49:44
128	HIGG	PINELANDS	51	M	MAS	0:52:57
247	HILL	WEST COAST	35	F	SEN	1:01:57
197	HOPE	SANLAM	55	M	MAS	0:57:46
203	HOPE	SANLAM	53	F	MAS	0:58:12
120	HUGO	DURBAC	65	M	GMAS	0:51:25
189	IREL	WEST COAST	48	M	VET	0:57:00
278	JANS	RUN WALK 4 LIFE	25	F	SEN	1:05:11
83	JOHN	BELLVILLE	51	M	MAS	0:48:46
297	JOHN	TEMP	42	M	VET	1:06:35
310	JOLL	TEMP	42	M	VET	1:09:12
270	JOOS	HARMONY WP	53	F	MAS	1:04:08
177	JOSE	TYGERBERG	41	M	VET	0:56:23
174	JOUB	PAARL	64	M	GMAS	0:56:12
302	JULI	SANDF WP	49	F	VET	1:07:34
65	KALI	TEMP	29	M	SEN	0:47:10
74	KALI	TEMP	32	M	SEN	0:48:24
28	KELL	EDGEMEAD	42	M	VET	0:41:30
264	KETT	VOB	58	F	MAS	1:03:01
289	KHAN	TOP FORM	26	M	SEN	1:05:58
232	KLEM	SANDF WP	40	F	VET	1:00:48
202	KLOP	PAARL	47	M	VET	0:58:07
49	KOHL	EDGEMEAD	57	M	MAS	0:45:23

POS	NAME	CLUB	AGE	SEX	CAT	TIME
26	KONA	SALDANHA	29	M	SEN	0:41:20
130	KOOP	TEMP	29	M	SEN	0:53:01
322	LAVI	TEMP	26	F	SEN	1:15:03
12	LEBI	AFB MAKHADO	26	M	SEN	0:37:58
7	LEHM	OLD MUTUAL	38	M	SEN	0:35:40
17	LEHM	OLD MUTUAL	28	M	SEN	0:39:16
10	LIGA	AFB MAKHADO	24	M	SEN	0:36:52
254	LIND	TEMP	28	M	SEN	1:02:18
352	LODE	BELLVILLE	46	F	VET	1:21:58
210	LOUW	EDGEMEAD	51	M	MAS	0:58:34
334	LUBB	SANLAM	26	F	SEN	1:18:12
342	LUMB	SPARTAN	66	F	GMAS	1:19:33
169	MAAR	TYGERBERG	39	M	SEN	0:55:29
325	MAAR	TYGERBERG	37	F	SEN	1:15:50
231	MACC	VOB	29	F	SEN	1:00:46
318	MADI	RAVENSMEAD	39	M	SEN	1:12:02
216	MADJ	RAVENSMEAD	39	F	SEN	0:59:12
230	MADJ	RAVENSMEAD	42	M	VET	1:00:45
6	MAGA	VOB	19	M	JUN	0:34:37
75	MAGG	METROPOLITAN	46	M	VET	0:48:26
132	MAGG	EDGEMEAD	12	M	JUN	0:53:19
134	MAGG	EDGEMEAD	37	M	SEN	0:53:23
228	MAKO	TEMP	22	F	SEN	1:00:03
32	MARA	EDGEMEAD	36	M	SEN	0:41:54
217	MARA	TEMP	27	F	SEN	0:59:16
220	MARA	MP CELTIC	24	F	SEN	0:59:31
211	MARK	WEST COAST	50	M	MAS	0:58:36
60	MARR	STELLENBOSCH	31	M	SEN	0:46:26
315	MARR	STELLENBOSCH	27	F	SEN	1:11:42
317	MARR	NB GUGS	54	M	MAS	1:12:01
251	MART	METROPOLITAN	48	F	VET	1:02:08
353	MART	VOB	41	M	VET	1:22:08
102	MATT	UWC	48	M	VET	0:49:50
158	MAWD	WEST COAST	30	M	SEN	0:55:06
13	MBHE	EDGEMEAD	16	M	JUN	0:38:13
276	McDA	RUN WALK 4 LIFE	31	F	SEN	1:04:43
277	McDA	WEST COAST	31	M	SEN	1:04:44
11	MDE	VOB	19	M	JUN	0:36:54
123	MELD	MP CELTIC	25	M	SEN	0:51:58
167	MERT	PINELANDS	31	M	SEN	0:55:27
173	MEYE	TEMP	27	F	SEN	0:56:11
343	MEYE	TYGERBERG	60	M	GMAS	1:19:49
31	MHAM	AFB MAKHADO	24	M	SEN	0:41:51
159	MILL	PINELANDS	42	M	VET	0:55:08
364	MILN	SPARTAN	75	M	75	1:25:09
295	MISR	PINELANDS	39	F	SEN	1:06:13
5	MLUN	VOB	20	M	SEN	0:34:35
125	MOAT	HARMONY WP	31	M	SEN	0:52:37

POS	NAME	CLUB	AGE	SEX	CAT	TIME
22	MOGA	AFB MAKHADO	32	M	SEN	0:39:56
252	MOHA	TEMP	22	M	SEN	1:02:10
263	MOHA	TEMP	24	M	SEN	1:02:56
265	MOKA	TEMP	22	F	SEN	1:03:15
3	MOLE	MR PRICE WP	27	M	SEN	0:33:43
175	MONT	WEST COAST	36	F	SEN	0:56:16
323	MOOD	SANDF WP	27	F	SEN	1:15:35
44	MORG	VOB	35	M	SEN	0:44:21
146	MORK	RUN WALK 4 LIFE	46	F	VET	0:53:53
314	MORK	RUN WALK 4 LIFE	47	M	VET	1:11:41
14	MOTA	WHALERS	26	M	SEN	0:38:49
335	MOUT	SANLAM	29	M	SEN	1:18:13
18	MPUK	HARMONY WP	31	M	MAS	0:39:25
164	MULL	PAARL	30	F	SEN	0:55:22
166	MULL	CPUT	34	M	SEN	0:55:23
363	MULL	SANDF WP	56	M	MAS	1:24:26
356	MUNN	ESKOM GIJIMAS	51	F	MAS	1:22:18
92	MURP	SOUTH STRIDERS	42	M	VET	0:49:23
70	MURR	EDGEMEAD	44	F	VET	0:47:45
84	MURT	TELKOM	50	M	MAS	0:48:48
333	MVIN	NB GUGS	29	F	SEN	1:18:09
171	NAUD	VOB	50	M	MAS	0:55:46
37	NAUM	EDGEMEAD	35	M	SEN	0:43:36
180	NEL	HARMONY WP	41	M	VET	0:56:37
186	NEL	SANLAM	37	F	SEN	0:56:54
319	NELS	DURBAC	63	F	GMAS	1:12:10
349	NELS	BELLVILLE	41	M	VET	1:21:14
350	NELS	BELLVILLE	40	F	VET	1:21:16
36	NIEN	HARMONY WP	56	M	MAS	0:43:12
201	NIKA	NB GUGS	38	F	SEN	0:58:01
371	NO	TEMP	23	F	SEN	1:30:04
29	NO C	TEMP	24	M	SEN	0:41:32
30	NO C	TEMP	25	M	SEN	0:41:40
67	NO C	SANLAM	26	M	SEN	0:47:25
181	NO C		27	M	SEN	0:56:38
267	NO C	TEMP	28	F	SEN	1:03:51
337	NO C	TEMP	29	M	SEN	1:18:38
20	NO R	TEMP	30	M	SEN	0:39:48
138	NO R	TEMP	31	M	SEN	0:53:33
307	NO R	TEMP	32	M	SEN	1:08:41
8	NOKO	AFB MAKHADO	23	M	SEN	0:35:59
340	NORT	TEMP	50	M	MAS	1:18:53
242	NOSS	MP CELTIC	15	M	JUN	1:01:38
243	NOSS	MP CELTIC	47	M	VET	1:01:39
133	O`DO	EDGEMEAD	40	M	VET	0:53:23
59	OKRE	UCT	46	M	VET	0:46:25
150	OLIV	TEMP	27	F	SEN	0:54:24
221	OOST	SANLAM	51	F	MAS	0:59:32

POS	NAME	CLUB	AGE	SEX	CAT	TIME
224	OPPE	ATLANTIS	49	F	VET	0:59:40
257	OSBO	TEMP	40	F	VET	1:02:20
258	OSBO	TOP FORM	40	M	VET	1:02:21
246	PAIN	TEMP	35	F	SEN	1:01:52
194	PARI	TEMP	39	M	SEN	0:57:36
346	PARI	NB GUGS	38	F	SEN	1:21:10
347	PARI	NB GUGS	37	M	SEN	1:21:10
311	PAUL	SPARTAN	64	F	GMAS	1:09:41
291	PAYN	WHALERS	41	F	VET	1:06:06
113	PEDR	UWC	45	M	VET	0:51:02
15	PERS	ADIDAS	43	M	VET	0:38:55
104	PIET	TELKOM	35	M	SEN	0:49:51
191	POOL	CORR SERVICES	46	M	VET	0:57:13
208	POTG	SWARTLAND	33	F	SEN	0:58:19
82	PRET	SANLAM	40	M	VET	0:48:41
111	PRET	TOP FORM	36	F	SEN	0:50:26
200	PRET	NB GN	34	F	SEN	0:58:00
35	PROU	EDGEMEAD	48	M	VET	0:42:58
214	RAHI	TOP FORM	42	M	VET	0:58:56
155	RANE	METROPOLITAN	32	M	SEN	0:54:00
358	REED	TEMP	37	M	SEN	1:22:48
25	REID	EDGEMEAD	26	M	SEN	0:40:59
330	REND	EDGEMEAD	46	F	VET	1:16:52
88	RENE	WEST COAST	57	M	MAS	0:49:13
271	RENE	WEST COAST	55	F	MAS	1:04:13
140	RIEN	HARMONY WP	44	F	VET	0:53:40
157	RIJK	UAE	45	M	VET	0:55:02
357	RIPS	TEMP	39	M	SEN	1:22:48
205	RIST	PINELANDS	44	F	VET	0:58:15
153	RODG	MATIES	49	M	VET	0:54:41
72	ROEL	MP CELTIC	33	M	SEN	0:47:55
61	ROTH	BRACKENFELL	53	M	MAS	0:46:35
328	ROTH	BRACKENFELL	50	F	MAS	1:16:45
81	RYKL	VOB	46	M	VET	0:48:35
142	SAMU	PINELANDS	49	M	VET	0:53:44
316	SCHI	RUN WALK 4 LIFE	63	M	GMAS	1:11:44
327	SCHI	RUN WALK 4 LIFE	60	F	GMAS	1:16:19
101	SCHN	PINELANDS	53	M	MAS	0:49:48
298	SCHN	PINELANDS	25	F	SEN	1:07:07
275	SCHR	EDGEMEAD	46	F	VET	1:04:40
161	SEPT	EERSTERIVIER	51	M	MAS	0:55:10
118	SHAW	TYGERBERG	63	M	GMAS	0:51:23
4	SIBI	AAC	24	M	SEN	0:33:55
225	SIEB	SANLAM	49	F	VET	0:59:42
39	SMIT	WEST COAST	24	M	SEN	0:43:46
54	SMIT	PINELANDS	38	M	SEN	0:46:09
195	SMIT	TEMP	47	M	VET	0:57:37
57	SNYD	TYGERBERG	46	M	VET	0:46:18

POS	NAME	CLUB	AGE	SEX	CAT	TIME
100	SPAR	MP CELTIC	62	F	GMAS	0:49:47
241	SPIE	SOLIDARITY	38	M	SEN	1:01:22
135	STEC	EDGEMEAD	39	F	SEN	0:53:25
303	STEE	HARMONY WP	15	F	JUN	1:07:53
304	STEE	HARMONY WP	45	M	VET	1:07:53
261	STER	TEMP	44	M	VET	1:02:50
223	STEW	VOB	65	M	GMAS	0:59:38
281	STOF	SPARTAN	45	F	VET	1:05:17
90	STOL	TEMP	40	M	VET	0:49:18
290	STRA	VOB	62	M	GMAS	1:06:00
367	STRA	RUN WALK 4 LIFE	37	F	SEN	1:27:23
141	SULL	DURBAC	44	M	VET	0:53:43
89	SWAN	TEMP	47	M	VET	0:49:16
187	SWAN	HARMONY WP	35	F	SEN	0:56:55
266	TALI	ARD	50	M	MAS	1:03:26
145	TALJ	SAPS WP	48	M	VET	0:53:53
284	TALJ	SAPS WP	45	F	VET	1:05:28
91	TAYL	SPARTAN	37	M	SEN	0:49:21
198	TAYL	DURBAC	65	F	GMAS	0:57:49
234	TAYL	EDGEMEAD	38	F	SEN	1:00:59
279	TAYL	PINELANDS	70	M	GGMAS	1:05:16
48	THER	DURBAC	49	M	VET	0:45:11
244	THIR	RUN WALK 4 LIFE	35	F	SEN	1:01:49
193	THOR	PINELANDS	57	M	MAS	0:57:31
209	TITU	TEMP	39	M	SEN	0:58:20
16	TOM	ESKOM GIJIMAS	50	M	MAS	0:39:00
280	TRAU	SANDF WP	43	F	VET	1:05:17
152	TUNA	EDGEMEAD	45	M	VET	0:54:37
80	TWIG	VOB	34	M	SEN	0:48:33
9	VAN	TYGERBERG	29	M	SEN	0:36:18
51	VAN	EDGEMEAD	39	F	SEN	0:45:26
85	VAN	TELKOM	45	M	VET	0:48:49
96	VAN	BELLVILLE	38	M	SEN	0:49:33
105	VAN	DURBAC	46	M	VET	0:49:55
108	VAN	TYGERBERG	38	M	SEN	0:49:59
112	VAN	EDGEMEAD	42	F	VET	0:50:33
121	VAN	TEMP	29	M	SEN	0:51:45
124	VAN	MP CELTIC	32	F	SEN	0:52:12
147	VAN	TEMP	24	M	SEN	0:53:54
170	VAN	TYGERBERG	53	F	MAS	0:55:32
178	VAN	AAC	33	F	SEN	0:56:30
192	VAN	HELDERBERG	55	M	MAS	0:57:25
199	VAN	MP CELTIC	61	M	GMAS	0:57:57
207	VAN	SANLAM	42	M	VET	0:58:16
222	VAN	EDGEMEAD	71	M	GGMAS	0:59:34
233	VAN	BELLVILLE	42	M	VET	1:00:51
259	VAN	EDGEMEAD	38	F	SEN	1:02:22
260	VAN	TELKOM	41	F	VET	1:02:32

POS	NAME	CLUB	AGE	SEX	CAT	TIME
262	VAN	VOB	33	F	SEN	1:02:50
287	VAN	HARMONY WP	67	F	GMAS	1:05:45
292	VAN	BRACKENFELL	38	F	SEN	1:06:07
306	VAN	OLD MUTUAL	40	F	VET	1:08:30
324	VAN	TYGERBERG	45	F	VET	1:15:50
338	VAN	TEMP	53	M	MAS	1:18:45
168	VERC	TYGERBERG	40	M	VET	0:55:29
27	VIAN	TEMP	27	M	SEN	0:41:27
282	VINK	TEMP	24	F	SEN	1:05:23
79	VOLK	ESKOM GIJIMAS	51	M	MAS	0:48:31
117	VOLM	ESKOM GIJIMAS	54	M	MAS	0:51:22
227	VOS	BELLVILLE	55	F	MAS	0:59:53
235	WALK	EDGEMEAD	47	F	VET	1:01:00
248	WALK	WEST COAST	58	M	MAS	1:01:57
78	WALT	PAARL	42	M	VET	0:48:29
249	WATS	SOUTH STRIDERS	55	M	MAS	1:02:02
250	WEDD	WEST COAST	46	M	VET	1:02:02
288	WEID	TYGERBERG	39	M	SEN	1:05:52
47	WERT	HARMONY WP	37	F	SEN	0:45:04
109	WESS	SAPS WP	45	M	VET	0:50:14
131	WESS	SANDF WP	35	F	SEN	0:53:17
245	WESS	BELLVILLE	47	M	VET	1:01:49
46	WEST	WEST COAST	41	M	VET	0:44:48
38	WILL	UCT	20	M	SEN	0:43:37
86	WILL	TYGERBERG	54	M	MAS	0:49:00
94	WILL	PINELANDS	54	F	MAS	0:49:31
190	WILL	MP CELTIC	36	M	SEN	0:57:00
206	WILL	PINELANDS	57	M	MAS	0:58:16
274	WILS	EDGEMEAD	43	F	VET	1:04:40
77	WOOD	EDGEMEAD	33	M	SEN	0:48:27

Worked example 1

Task: Organise the above data in the following ways:

(1) The male and female runners according to their clubs.

(2) The male and female runners according to their running times.

GROUPS

CLASS

Solution

(1) The easiest way to organise this data is to draw up a tally table. Each runner is represented by a stripe or tally. Once there are four runners then the fifth runner is represented by a diagonal tally through the first four. See below for the:

Club	Female	Male
TOP FORM	|	|
METROPOLITAN		|
TEMP	|||	||||
MP TITANS		|
NB GUGS		|
TYGERBERG		|
RUN WALK 4 LIFE	|	
ATLANTIS		|
HEWAT	|	|
STRAGGLERS		||
VOB		|
SOUTHERN STRIDERS	|	
SANLAM		|
HARMONY WP		|
EDGEMEAD		
MAFIKENG		
FISH HOEK		
PAARL		
OLD MUTUAL		
WEST COAST		
BELVILLE		
SANDF WP		
ESKOM GIJIMAS		

The above tally table is not complete. You will complete the table as part of the activity at the end of the lesson. Some club names may not appear on the list and so you will need to add these.

Once you have made your tally table, you need to make a frequency table (see below). All the clubs that have fewer than five runners have been put into a category called other.

	Female	Male	Total
AAC	1	5	6
BELLVILLE	8	6	14
BRACKENFELL	4	2	6
DURBAC	4	4	8
EDGEMEAD	15	20	35
HARMONY WP	9	10	19
MP CELTIC	3	8	11
NB GUGS	5	3	8
PAARL	2	4	6
PINELANDS	5	10	15
RUN WALK 4 LIFE	7	2	9
SANDF WP	5	3	8
SANLAM	7	11	18
TEMP	16	44	60
TOP FORM	2	6	8
TYGERBERG	8	14	22
VOB	4	17	21

WEST COAST	5	10	15
Other	25	57	82
Total	135	236	371

The data is now organised and ready for the next stage – summarising the data.

GROUPS

CLASS

Worked example 2

(2) There are 371 runners and apart from a few who crossed the line at the same time almost every runner has a unique finishing time. This makes it difficult to record each individual's time. To overcome this difficulty, we group the data. To do this we need to find the difference between the fastest and the slowest times. For this race the fastest time was about 32min and the slowest time was just over 1 hr 30 min.

$$\text{Slowest time} - \text{fastest time} = 90 \text{ min} - 32 \text{ min}$$
$$\approx 60 \text{ min}$$

If we use an interval of 5 minutes, we will get about 13 intervals. If the first interval starts at 30 min then it will end at 35 min and the next interval will start at 35 min and end at 40 min. Notice that if the first interval includes 35 min then the second interval must exclude 35 min. An incomplete tally table is shown below.

Time (min)	Female	Male
$30 < t \leq 35$		𝍸 |
$35 < t \leq 40$		
$40 < t \leq 45$		𝍸 𝍸
$45 < t \leq 50$		
$50 < t \leq 55$		
$55 < t \leq 60$		
$60 < t \leq 65$		
$65 < t \leq 70$		
$70 < t \leq 75$		
$75 < t \leq 80$		
$80 < t \leq 85$		
$85 < t \leq 90$		
$90 < t \leq 95$		

GROUPS

CLASS

Activity

1 Complete the tally table for the club data. Confirm that it corresponds to the frequency table given in the worked example.

2 Complete the tally table of runner's times in the worked example and fill in the frequency table below.

Time	Female	Male	Total
$30 < t \leq 35$			
$35 < t \leq 40$			
$40 < t \leq 45$			
$45 < t \leq 50$			
$50 < t \leq 55$			
$55 < t \leq 60$			
$60 < t \leq 65$			
$65 < t \leq 70$			
$70 < t \leq 75$			
$75 < t \leq 80$			
$80 < t \leq 85$			
$85 < t \leq 90$			
$90 < t \leq 95$			
Total			

DATA HANDLING

Summarising data

Lesson 36

Learning Outcomes and Assessment Standards

Learning Outcome 4
Data Handling
The learner is able to collect, summarise, display and analyse data and to apply knowledge of statistics and probability to communicate, justify, predict and critically interrogate findings and draw conclusions.
NOTE: Assessment Standard 11.4.4. is the focal Assessment Standard of LO4. The other LO4 Assessment Standards serve to develop the skills that will enable learners to achieve this one.
Assessment Standard
AS 11.4.4
Critically interpret data and representations thereof (with awareness of sources of error and bias), in order to draw conclusions on problems investigated and make predictions and to critique other interpretations.
AS 11.4.2
Select, justify and use a variety of methods to summarise and display data in statistical charts and graphs inclusive of:
- Tallies.
- Tables.
- Pie charts.
- Single and compound bar graphs.
- Line and broken-line graphs.

AS 11.4.3
Understand that data can be summarised and compared in different ways by calculating, and using measures of central tendency and spread (distribution), for more than one set of data inclusive of the:
- Mean.
- Media.
- Mode.
- Range.

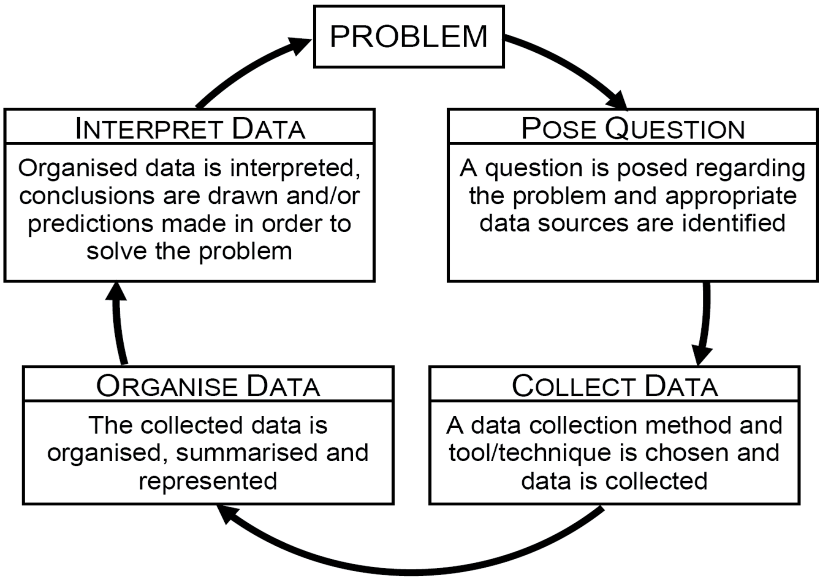
Overview

In this lesson we look at summarising data. We will use the data that we organised in the previous lesson to show the various ways of summarising data. Summarising data is part of the third stage of the data handling cycle (see data handling cycle below).

Lesson

There are different techniques that can be used to summarise data. We will look at one of these techniques in this lesson, namely: measures of central tendency. In finding the measures of central tendency, mathematicians are trying to determine one number that will best represent the set of data.

Methods and worked examples



There are three well known measures of central tendency:

- the mean;
- the median; and
- the mode.

The mean is determined by adding up all the sample values and dividing by the number of values – it is often referred to as the "average".

The median is that value which divides the ordered data into two groups of equal size. The one group contains values less than the median and the other group contains values greater than the median.

The mode is the most frequently occurring value in the data. It is possible for there to be two or more modes in a set of data.

These three measures of central tendency are known as summary statistics as they summarise the data into a single value.

Worked example

Using the data provided in the previous lesson, determine the mean, median and mode of the ages of the first 25 runners who finished the race.

Solution

The mean:

Total ages $= 719$

Mean $= 719 \div 25$

$= 28{,}76$ years

If we say that the mean of the 25 runners is 28,76 years old then people can compare their own age to this age.

The median:

In order to determine the median the data set must first be arranged in ascending or descending order.

16 19 19 20 22 23 24 24 26 26 26 26 27 27 28 29 30 31 32 34 37 38 42 43 50

The median is the middle value of the data (arranged in order). In this case, there are 25 ages so the middle age will be the 13th age (boxed value) – in this case 27 years of age. This means that if your age is below 27 years of age then you are in the younger half of the field and if your age is greater than 27 years of age then you are in the older half of the field.

If the number of values in the data set is even then the median is determined by adding the two middle values and dividing by 2.

The mode:

16 19 19 20 22 23 24 24 26 26 26 26 27 27 28 29 30 31 32 34 37 38 42 43 50

The most frequently occurring age in the above data set is 26 years of age.

A summary of the measures of central tendency for this example is therefore:

- mean: 29 years
- median: 27 years
- mode: 26 years

Additional examples

INDIVIDUAL

FORMATIVE ASSESSMENT

In this example we will return to the two runners that we met in the previous lesson and see if these summary statistics can help solve their problem. The girl is represented by "MADJ" of Ravensmead and she completed the race in 59 min and 12 s. The boy is "RANE" of Metropolitan and he completed the race in 54 mins.

Determine the mean, median and modal times for the male and female runners in the race.

Solution

The challenge in this question is that we have grouped the data.

The mode: This is the score that occurs the most often.

To answer this question we can look at the frequency table that we developed in the previous lesson (see below).

The greatest number of women finished in the interval 55 to 60 seconds. We can either say that the modal group is 55 to 60 second or choose the midpoint 57.5 to represent the group. Notice how MADJ's time falls in the modal group.

Time	FEMALE	MALE	TOTAL
t ≤ 35		6	6
35 < t ≤ 40		16	16
40 < t ≤ 45	2	22	24
45 < t ≤ 50	11	51	62
50 < t ≤ 55	10	38	48
55 < t ≤ 60	28 (mode)	43	71
60 < t ≤ 65	27 (median)	23	50
65 < t ≤ 70	20	14	34
70 < t ≤ 75	4	6	10
75 < t ≤ 80	15	7	22
80 < t ≤ 85	11	9	20
85 < t ≤ 90	6	1	7
90 < t ≤ 95	1		1
TOTAL	135	236	371

The median: There are 135 female runners so the middle value will be the 68th value which falls into the $60 < t \leq 65$ group or 62,5 minutes.

MADJ's time of 59 min 12 sec is faster than the median time which means that she finished in the top half of the group.

The mean: This is the most difficult statistic to calculate when there is grouped data.

Middle score	Time	Female	Total for group
	t ≤ 35		
	35 < t ≤ 40		
42,5	40 < t ≤ 45	2	42,5 × 2 = 85
47,5	45 < t ≤ 50	11	47,5 × 11 = 522,5
52,5	50 < t ≤ 55	10	52,5 × 10 = 525
57,5	55 < t ≤ 60	28	57,5 × 28 = 1 610
62,5	60 < t ≤ 65	27	62,5 × 27 = 1 687,5
67,5	65 < t ≤ 70	20	67,5 × 20 = 1 350
72,5	70 < t ≤ 75	4	72,5 × 4 = 290
77,5	75 < t ≤ 80	15	77,5 × 15 = 1 162,5
82,5	80 < t ≤ 85	11	82,5 × 11 = 907,5
87,5	85 < t ≤ 90	6	87,5 × 6 = 525
92,5	90 < t ≤ 95	1	92,5 × 1 = 92,5
	TOTAL	135	8 757,5

$$\begin{aligned} \text{Mean} &= 8\,757{,}5 \text{ min} \div 135 \text{ runners} \\ &= 64{,}87 \text{ min} \\ &\approx 65 \text{ min} \end{aligned}$$

MADJ's time of 59 min and 12 s means that she in a time that is faster than the average time.

A summary of the measures of central tendency for the women in the race is:

- mean: 65 min
- median: 62,5 min
- mode: 57,5 min

Activity

INDIVIDUAL

SELF ASSESSMENT

SUMMATIVE ASSESSMENT

1 Using the frequency table in the worked example determine the following for the men runners. Compare RANE’s performance with each of the statistics.

   1.1 The mean

   1.2 The median

   1.3 The mode

2. Using the frequency table in the worked example determine the following for all the runners.

   2.1 The mean

   2.2 The median

   2.3 The mode

Lesson 37

DATA HANDLING

Representing data

Learning Outcomes and Assessment Standards

Learning Outcome 4
Data Handling
The learner is able to collect, summarise, display and analyse data and to apply knowledge of statistics and probability to communicate, justify, predict and critically interrogate findings and draw conclusions.
Assessment Standard
AS 11.4.2
Select, justify and use a variety of methods to summarise and display data in statistical charts and graphs inclusive of:

- Tallies.
- Tables.
- Pie charts.
- Single and compound bar graphs.
- Line and broken-line graphs.

Overview

In this lesson we are still dealing with the third stage in the data handling cycle i.e. organising data. We will focus on representing the organised and summarised data.

Lesson

The measures of central tendency give you one way of summarising data. Another more visual way of summarising data is to draw graphs – we refer to this as representing the data.

Methods and worked examples

There are a large number of different graphs to choose from:

- pie charts;
- bar graphs;
- histograms;
- line and broken line graphs.

Among the bar graphs and histogram graphs we get:

- single bar graphs;
- multiple bar graphs; and
- compound bar graphs.

As with so much of data handling, every time you make a choice of which representation to use, you impact the way in which that the data will ultimately be interpreted. Each type of graph has their advantages and disadvantages.

Worked example

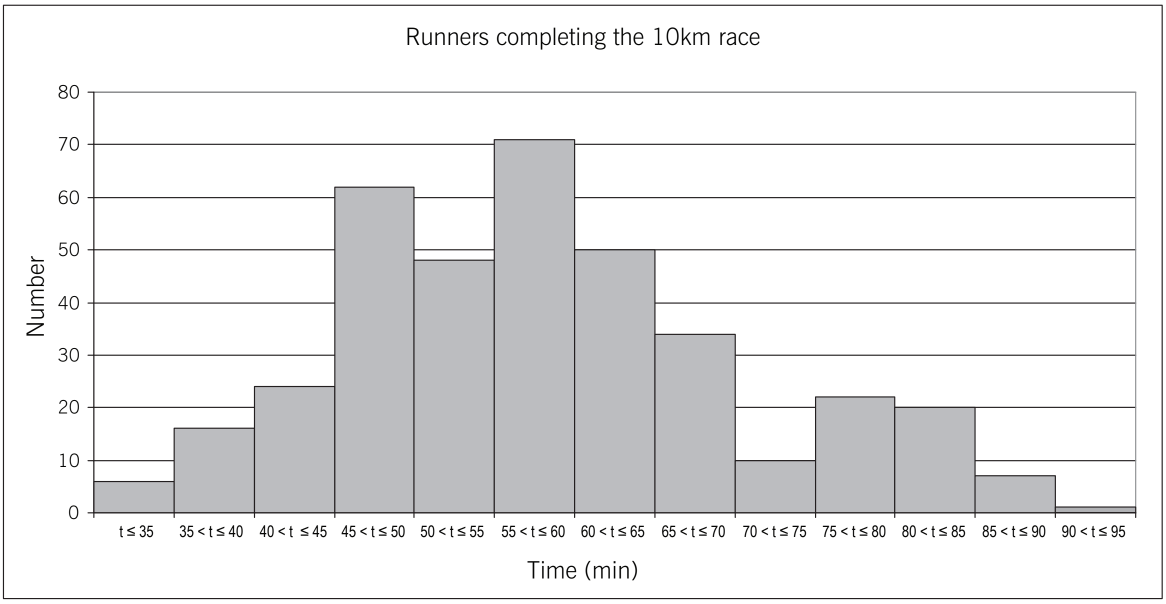
INDIVIDUAL

Represent the frequency of the finishing times of all the runners by means of a bar graph.

FORMATIVE ASSESSMENT

Solution:

Using the frequency table from the previous lesson we get:



Note: The graph has the following features:

- the graph has a title;
- both axes are labeled and units are stated where appropriate;
- divisions on both axes are evenly spaced; and
- both axes have an appropriate scale.

Activity

INDIVIDUAL

1. Draw a double bar graph to represent the finishing times of the male and female runners.

SUMMATIVE ASSESSMENT

Lesson 38

DATA HANDLING

Interpreting data

Learning Outcomes and Assessment Standards

Learning Outcome 4
Data Handling
The learner is able to collect, summarise, display and analyse data and to apply knowledge of statistics and probability to communicate, justify, predict and critically interrogate findings and draw conclusions.
NOTE: Assessment Standard 11.4.4. is the focal Assessment Standard of LO4. The other LO4 Assessment Standards serve to develop the skills that will enable learners to achieve this one.
Assessment Standard
AS 11.4.4
Critically interpret data and representations thereof (with awareness of sources of error and bias), in order to draw conclusions on problems investigated and make predictions and to critique other interpretations.
AS 11.4.1
Investigate a problem on issues such as those related to:
- social, environmental and political factors.
- people's opinions.
- human rights and inclusivity by:
- collecting or finding data by appropriate methods (e.g. interviews, questionnaires, the use of data bases) suited to the purpose of drawing conclusions to the questions.
- using appropriate statistical methods.
- selecting a representative sample from a population with due sensitivity to issues relating to bias.
- comparing data from different sources and samples.

AS 11.4.2
Select, justify and use a variety of methods to summarise and display data in statistical charts and graphs inclusive of:
- Tallies.
- Tables.
- Pie charts.
- Single and compound bar graphs.
- Line and broken-line graphs.

AS 11.4.3
Understand that data can be summarised and compared in different ways by calculating, and using measures of central tendency and spread (distribution), for more than one set of data inclusive of the:
- Mean.
- Media.
- Mode.
- Range.

AS 11.4.6
Demonstrate an awareness of how it is possible to use data in different ways to justify opposing conclusions.

Overview

This lesson deals with the last stage of the data handling cycle – interpreting data.

LC DVD

Lesson

In this final stage of the data handling cycle we need to decide what we have learnt about the problem. By representing data in a particular way it is possible to create an impression that can be misleading. As mathematically literate people it is important to be aware of the ways in which data can be represented and misrepresented and to interpret information in the light of all that we have learnt over the past seven lessons. Choices that are made at each stage of the data handling cycle impact on the results and the impression created.

Methods and worked examples

We will continue with our example of the boy and girl runners. The boy and girl ran in the same 10km race and the boy beat the girl. However, the girl claimed that she is the better runner because she did better amongst the female runners than the boy did amongst the male runners. Our problem is to establish the validity or not of the girl's claim.

Worked example

INDIVIDUAL

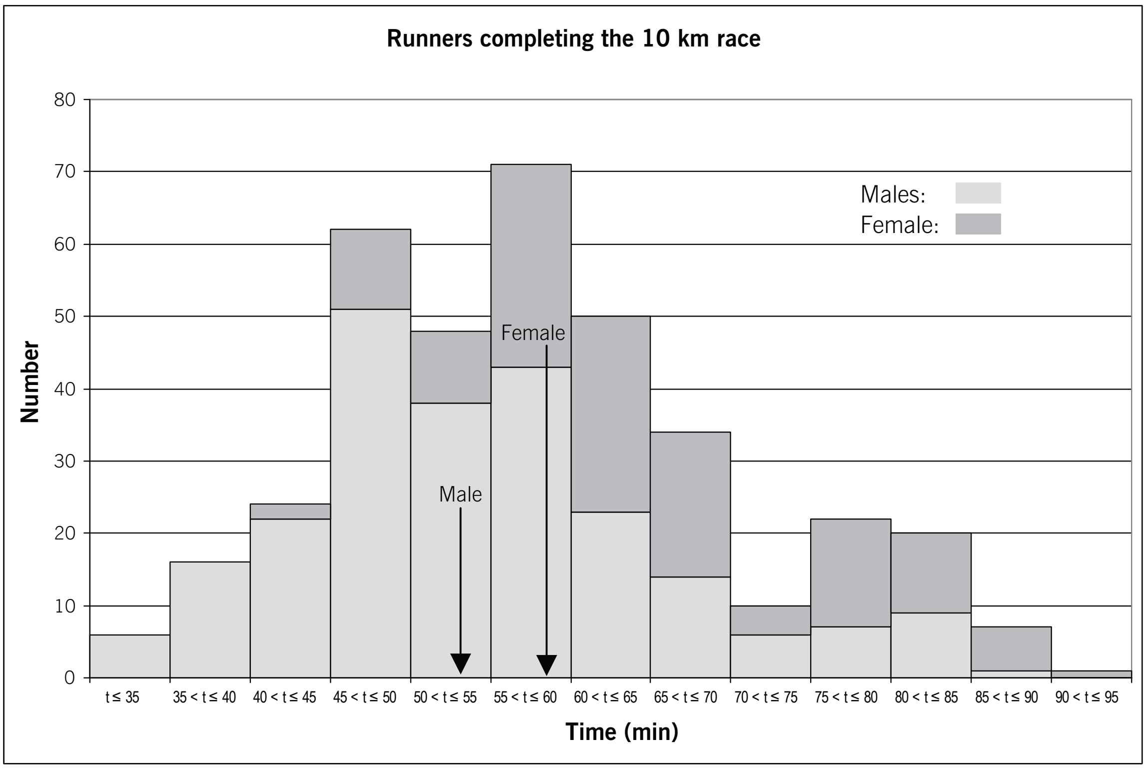
FORMATIVE ASSESSMENT

	FEMALE	MALE	TOTAL
Mean	64,9	54,3	58,1
Mode	58	48	58
Median	62,5	52,5	57,5

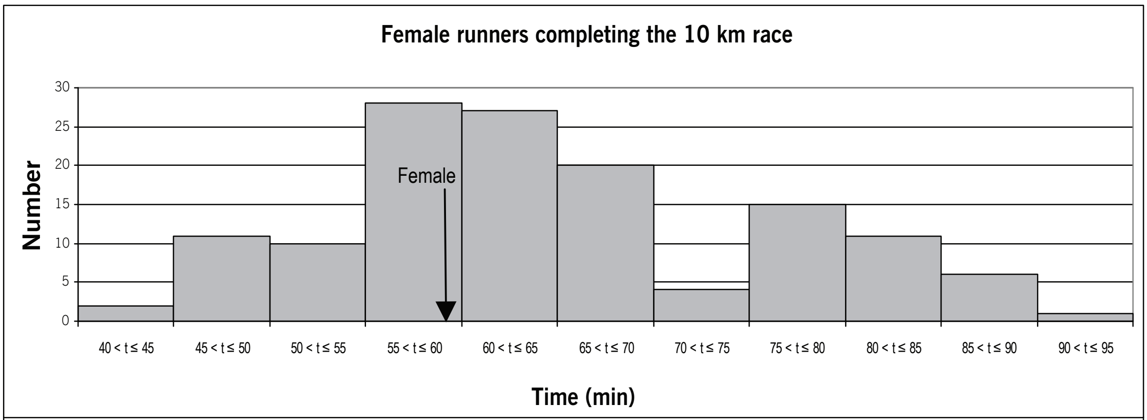
(1) Discuss the boy's and girl's race results in relation to the measures of central tendency that we calculated in lesson 36 (see table alongside). The girl's finishing time was – 59 min and 12s. The boy's finishing time was – 54 min. Ignore the mode as it is not really relevant to our problem.

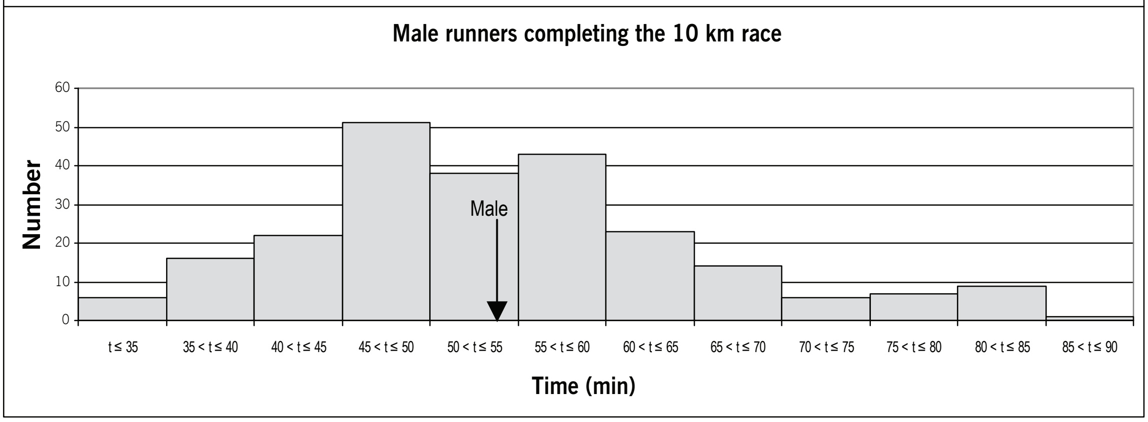
(2) The boy and the girl drew different graphs to represent the race results (see graphs below). Why do you think they used these representations? Discuss.

The boy drew a compound bar graph to represent the results:



The girl drew two separate bar graphs and placed them one above the other in order to compare them:





Solution

(1) If we compare the boys and girls results with the overall field we observe the following. The median for the whole field is 57,5 min and the mean or average is 58 min, this means that the boy with a time of 54 min is above average and is in the faster half of the field. The girl with a time of 59 min is below average and in the slower half of the field.

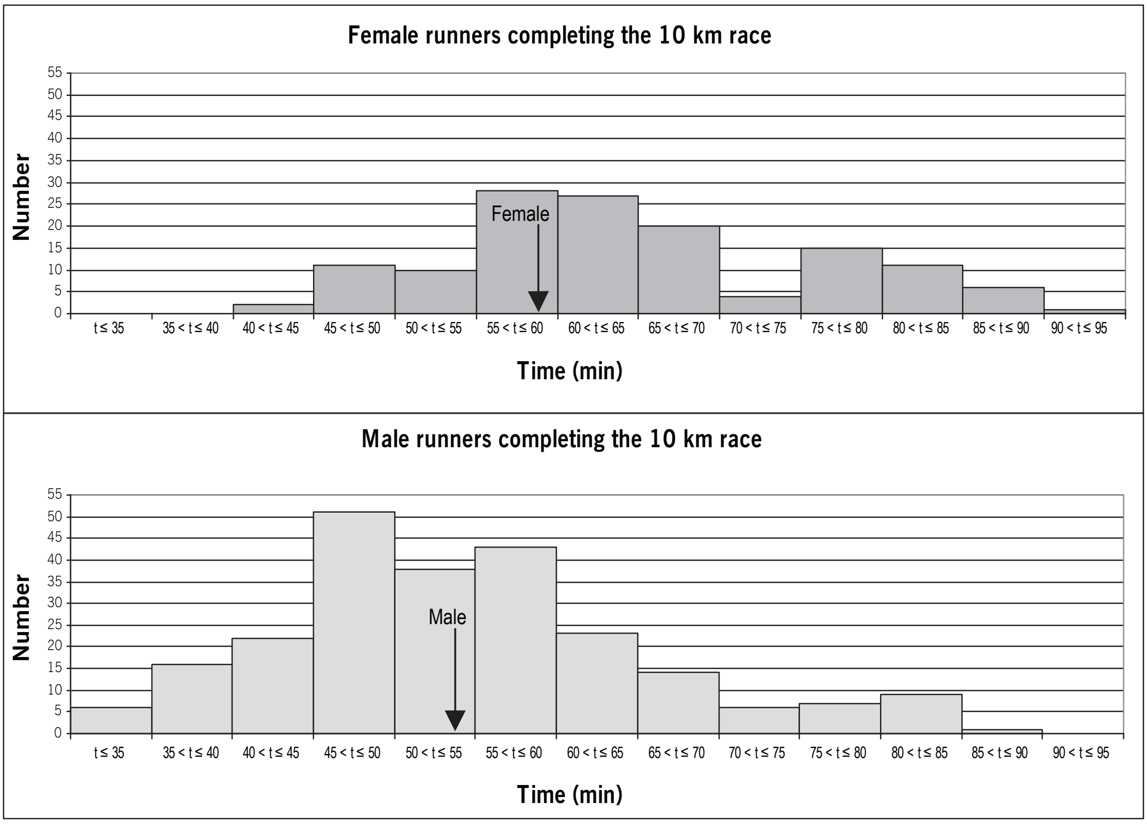
However, if we consider the male and female results separately we find the following. If we look at the men's results, we can see that the median is 52,5 min and the mean or average is 54,3 min. The boy in our example has a time of 54 min which means that he is slightly above average and he comes in the slower half of the field. If we look at the female's results, we can see that the median is 62,5 min and the mean or average is 65 min. The girl in our example has a time of 59 min which means that she is above average and she comes in the faster half of the field. From these comparisons, it looks as if the girl has done better amongst the females than the boy has amongst the males.

(2) Boy's graph: The compound bar graph shows the trends of the whole field and the males, but it is not easy to see the trend for the females. This representation makes the boy look better than the girl.

Girl's graph: The girl used two bar graphs which she placed one on top of the other. This gives the impression that the girl actually beat the boy. She did this by making the starting points of the horizontal axis different i.e. the male runner's graph starts at a time interval of $t \leq 35$ min whereas the female graph starts at a time interval of $40 < t \leq 45$ min. Although that is correct because there are no female runners in the time interval $t \leq 35$ min, by putting them on top of each other, she creates the impression that the males and the females finished at the same time. She also gives the impression that there were more female runners

than male runners by using different scales on the vertical axis. The heights of the vertical axes are the same but the female axis goes up to 30 runners whereas the male vertical scale goes to 60 runners. It gives the impression of more female because there appears to be more pink colour on the graph than blue.

A more honest representation would be as follows:



Activity

INDIVIDUAL

SUMMATIVE ASSESSMENT

1. Use the race results supplied in lesson 35 to complete the following:

   1.1 Complete in the tally table for the senior runners only:

Time	Female	Male
$30 < t \leq 35$		
$35 < t \leq 40$		
$40 < t \leq 45$		
$45 < t \leq 50$		
$50 < t \leq 55$		
$55 < t \leq 60$		
$60 < t \leq 65$		
$65 < t \leq 70$		
$70 < t \leq 75$		
$75 < t \leq 80$		
$80 < t \leq 85$		
$85 < t \leq 90$		
$90 < t \leq 95$		

1.2 Complete in the frequency table for the senior runners only:

Time	**Female**	**Male**	**Total**
$30 < t \leq 35$			
$35 < t \leq 40$			
$40 < t \leq 45$			
$45 < t \leq 50$			
$50 < t \leq 55$			
$55 < t \leq 60$			
$60 < t \leq 65$			
$65 < t \leq 70$			
$70 < t \leq 75$			
$75 < t \leq 80$			
$80 < t \leq 85$			
$85 < t \leq 90$			
$90 < t \leq 95$			
Total			

1.3 Use the frequency table to determine:

(A) The median for the females; males and the whole field.

(B) The mean or average for the females; males and the whole field.

1.4 Draw a bar graph to represent the finishing times of all the senior runners. Fill in on your graph where the boy and girl runners would fit in.

1.5 If both the runners are seniors, discuss the boy's and girl's race results in relation to the measures of central tendency determined in 1.3.

DATA HANDLING

Representing data 1

Lesson 39

Learning Outcomes and Assessment Standards

Learning Outcome 4
Data Handling
The learner is able to collect, summarise, display and analyse data and to apply knowledge of statistics and probability to communicate, justify, predict and critically interrogate findings and draw conclusions.
NOTE: Assessment Standard 11.4.4. is the focal Assessment Standard of LO4. The other LO4 Assessment Standards serve to develop the skills that will enable learners to achieve this one.
Assessment Standard
AS 11.4.4
Critically interpret data and representations thereof (with awareness of sources of error and bias), in order to draw conclusions on problems investigated and make predictions and to critique other interpretations.
AS 11.4.2
Select, justify and use a variety of methods to summarise and display data in statistical charts and graphs inclusive of:

- Tallies.
- Table.
- Pie charts.
- Single and compound bar graphsline and broken-line graphs.

AS 11.4.6
Demonstrate an awareness of how it is possible to use data in different ways to justify opposing conclusions.

Overview

In this lesson we will look at another way of representing data – the pie chart.

Lesson

Pie charts are circular charts that display data as slices of a pie and are used to summarise data where the parts add up to a whole e.g. in the example of boy and girl runners, you could draw a pie chart to represent the different running clubs participating in the race.

Unless the parts or components add up to a whole, the chart is meaningless.

There are times when it is inappropriate to use a pie chart and this could be where error and bias can occur.

Methods and worked examples

Advantages of using a pie chart:

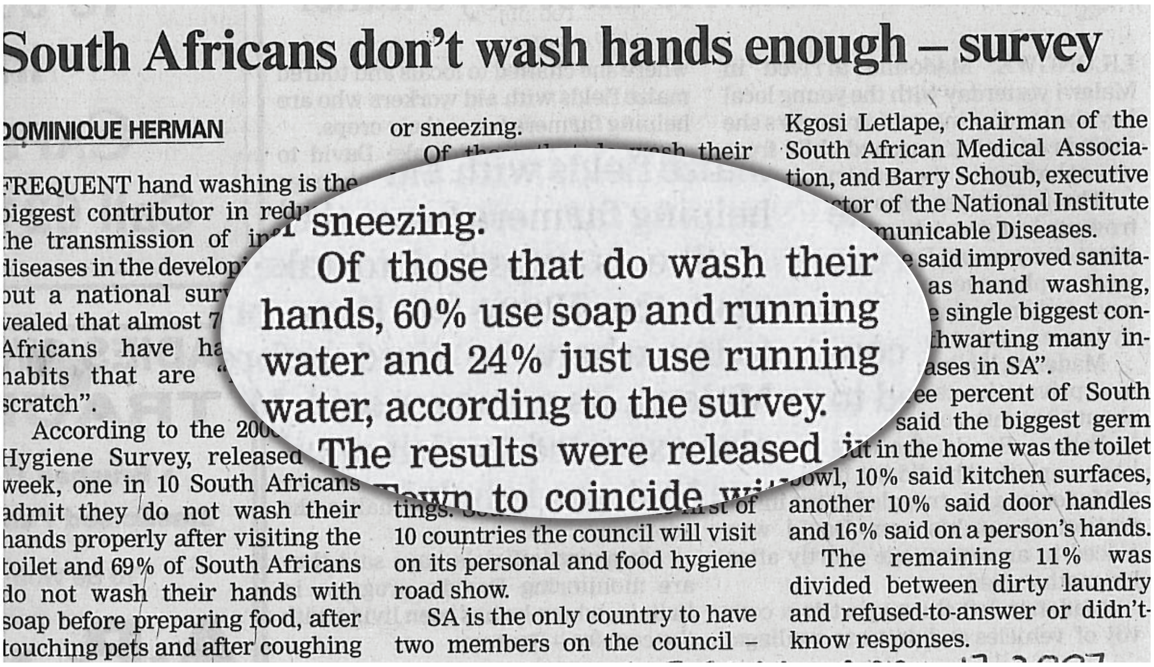
- They provide an excellent visual concept of the whole.
- They provide a clear comparison of the different components of the whole.

Disadvantages of using a pie chart:

- Comparing pie charts can be very difficult, because the components are compared to each other and not to an absolute value.
- They can be difficult to read if there are too many categories
- They are difficult to understand without labels especially if the segments are similar in size

Worked example

We considered this article in an earlier lesson.



South Africans don't wash hands enough – survey

DOMINIQUE HERMAN

FREQUENT hand washing is the biggest contributor in redu... the transmission of in... diseases in the develop... but a national sur... vealed that almost 7... Africans have ha... habits that are "... scratch".

According to the 200... Hygiene Survey, released ... week, one in 10 South Africans admit they do not wash their hands properly after visiting the toilet and 69% of South Africans do not wash their hands with soap before preparing food, after touching pets and after coughing or sneezing.

Of those that do wash their hands, 60% use soap and running water and 24% just use running water, according to the survey.

The results were released in ... Town to coincide wi... tings. ... first of 10 countries the council will visit on its personal and food hygiene road show.

SA is the only country to have two members on the council – Kgosi Letlape, chairman of the South African Medical Association, and Barry Schoub, executive ...tor of the National Institute ...municable Diseases.

...e said improved sanita... as hand washing, ...e single biggest con... thwarting many in...ases in SA".

...ee percent of South ... said the biggest germ ...t in the home was the toilet bowl, 10% said kitchen surfaces, another 10% said door handles and 16% said on a person's hands.

The remaining 11% was divided between dirty laundry and refused-to-answer or didn't-know responses.

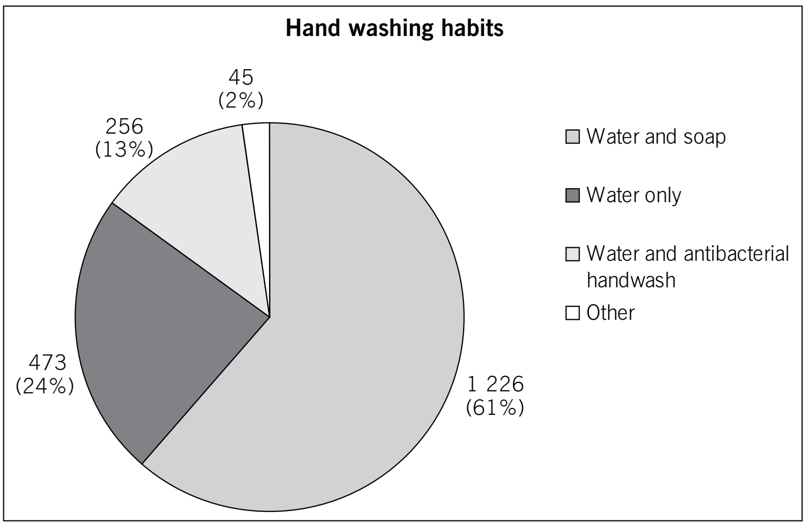
The actual research results from the research study are shown below:

When you wash your hands which of the following do you usually use?

	Actual value	Percentage
Running water and soap	1 226	60%
Running water only	473	24%
Running water and anti-bacterial handwash	256	13%
Other	45	3%
Total	2 000	100%

Draw a pie chart of the above set of data.

Solution



Additional examples:

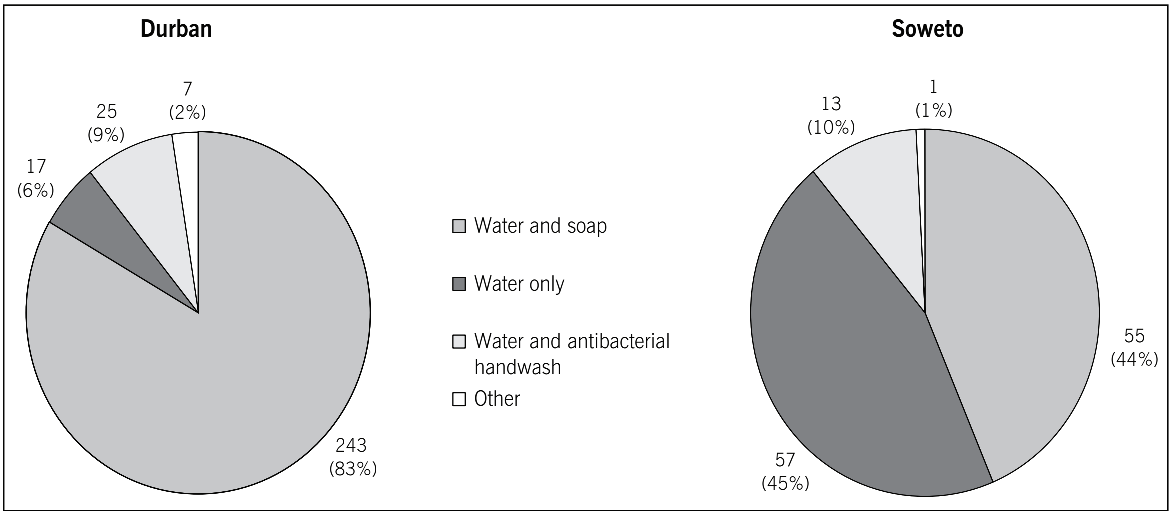
PAIRS

FORMATIVE ASSESSMENT

Two pie charts and the data used in drawing the pie charts are given below. Discuss the choice of using pie charts to compare the two places.

When you wash your hands which of the following do you usually use?

When you wash your hands which of the following do you usually use?		
	Durban	Soweto
Running water and soap	243	55
Running water only	17	57
Running water and anti-bacterial handwash	25	13
Other	7	1
Total	292	126



Solution

The most striking difference between the two sets of numbers is that the number of respondents from Durban is nearly four times the number of respondents from Soweto. The pie charts cannot show this.

Activity

INDIVIDUAL

SUMMATIVE ASSESSMENT

1. The profit made by Highlands Primary and High Schools' tuckshops for the month of March are shown below:

Profit made on the following items	Primary School	High School
Sweets	R326	R578
Juice	R530	R166
Chips	R243	R754
Hot dogs	R788	R973
Doughnuts	R254	R1 279
Total		

1.1 Calculate the percentage profit of the total profit for each item.

1.2 Draw pie charts of the profits for the two tuckshops for the month of March.

1.3 Draw a double bar graph of the profits for the two tuckshops.

1.4 Discuss the advantages and disadvantages of the graphs you have drawn in representing the data given.

Lesson 40

DATA HANDLING

Representing data II

Learning Outcomes and Assessment Standards

Learning Outcome 4
Data Handling
The learner is able to collect, summarise, display and analyse data and to apply knowledge of statistics and probability to communicate, justify, predict and critically interrogate findings and draw conclusions.
NOTE: Assessment Standard 11.4.4. is the focal Assessment Standard of LO4. The other LO4 Assessment Standards serve to develop the skills that will enable learners to achieve this one.
Assessment Standards
AS 11.4.4
Critically interpret data and representations thereof (with awareness of sources of error and bias), in order to draw conclusions on problems investigated and make predictions and to critique other interpretations.
AS 11.4.2
Select, justify and use a variety of methods to summarise and display data in statistical charts and graphs inclusive of:
- Tallies.
- Tables.
- Pie charts.
- Single and compound bar graphs.
- Line and broken-line graphs.

AS 11.4.6
Demonstrate an awareness of how it is possible to use data in different ways to justify opposing conclusions.

Overview

In the previous lesson we focused on pie charts. In this lesson we will focus on bar graphs and look at their advantages and disadvantages as well as how they can be manipulated to give a biased impression.

LESSON

Bar graphs use bars to represent how often elements within a category occur. The length of each bar is proportional to the number it represents. When representing categorical data we leave spaces between the bars as opposed to grouped numerical data where we usually make the bars touch each other.

Methods and worked examples

Advantages of using a bar graph:

- They are very useful for data comparison.
- They are usually simple to read and understand.

Disadvantages of using a bar graph:

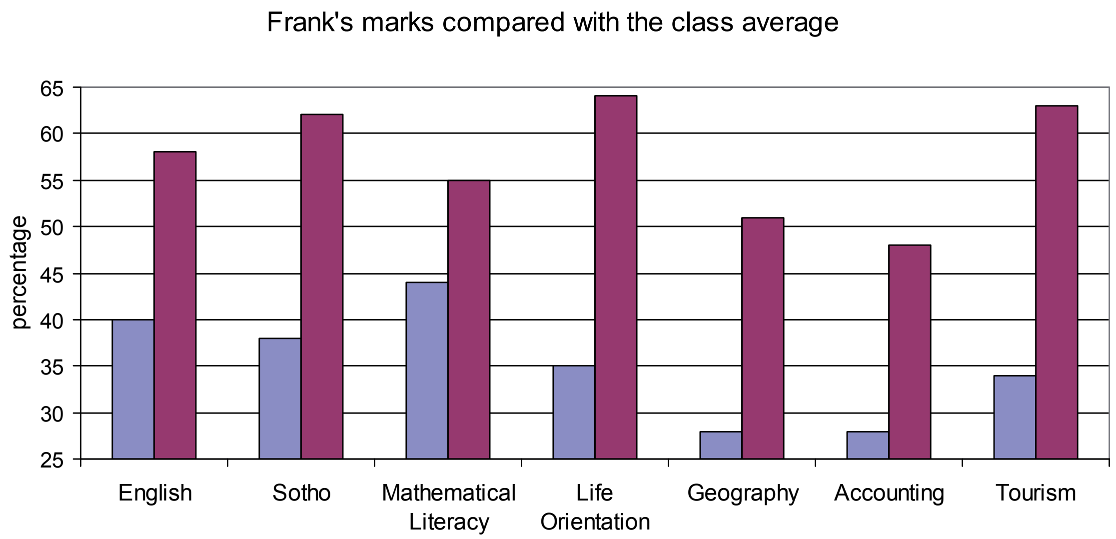
- They become difficult to read if there is too much information.
- They are easy to manipulate to give a particular impression i.e. it is easy to introduce bias.

Worked example

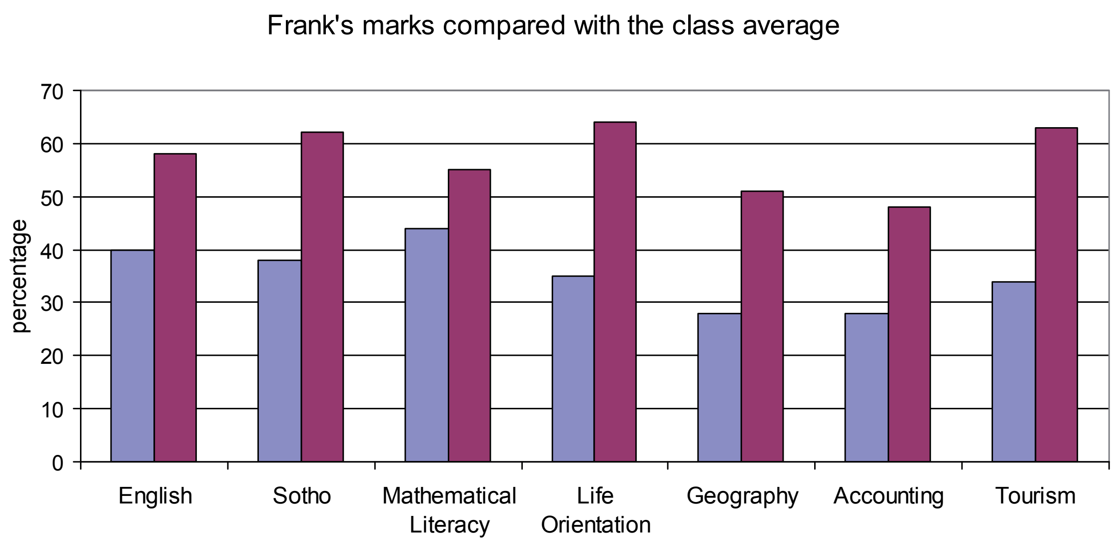
Frank's results, the class average and two representations of these results are given below. His mother's representation gives the impression that Frank has done very badly compared to the class average. Discuss how she achieved this.

Report card for Frank		
	Frank	Class average
English	40	58
Sotho	38	62
Mathematical Literacy	44	55
Life orientation	35	64
Geography	28	51
Accounting	28	48
Tourism	34	63

His mother's graph:



Another representation:



Frank's mother used her graph to exaggerate the difference between Frank's marks and the class average. She used a standard trick to create this impression. She moved the horizontal axis so that the bar no longer represented the actual value. Her graph started at 25 and not zero. This exaggerates the difference between the bars. In the second representation the length of the bars represent the actual occurrences and so the difference is not exaggerated. It still shows that Frank performed well below class average in every subject.

INDIVIDUAL

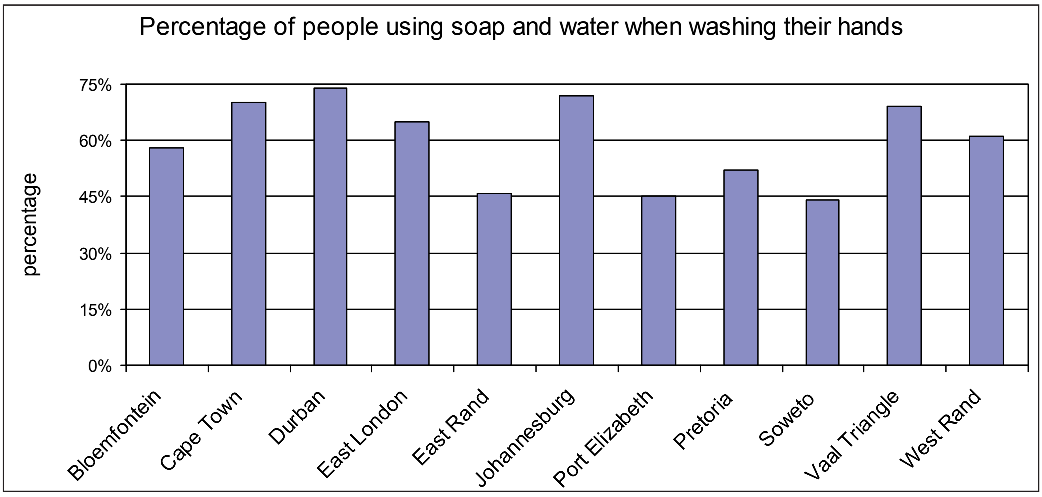
FORMATIVE ASSESSMENT

Consider the following examples:

Data from the health survey we worked with in lesson 39 and a graph representing this data is shown below. What impression does the graph give and how has the graph been manipulated in order to create this impression? Draw a graph to represent the data in a more accurate way.

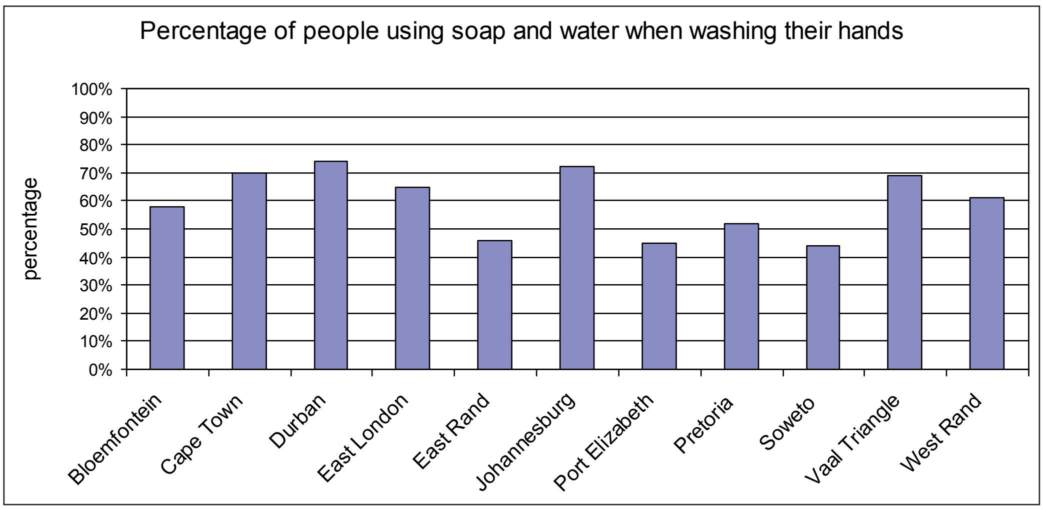
When you wash your hands which of the following do you usually use?

Answer: Running water and soap.			
Bloemfontein	58%	Johannesburg	72%
Cape Town	70%	Pretoria	52%
Durban	74%	Soweto	44%
East London	65%	Vaal Triangle	69%
East Rand	46%	West Rand	61%



Solution

It gives the impression that nearly all of the people in Durban who wash their hands, use soap and water. The overall impression is more than half of the people in each town wash their hands with soap and water.



The above graph gives a much clearer picture of the actual situation. Durban still comes out on top but you can see that not all people wash their hands with soap and water.

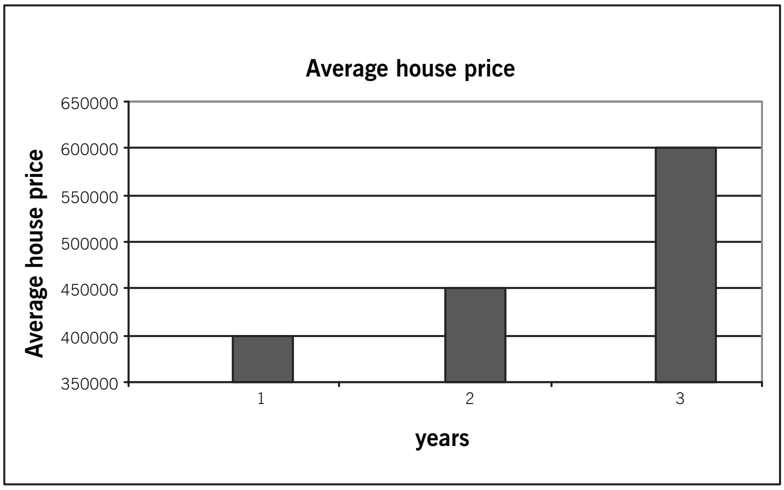
Note: (1) City names are categorical so the bars are kept separate.
(2) You can change the order of categorical data without misrepresenting it.
(3) By changing the vertical scale to less than 100%, one changes the impression you get of the data. This introduces what is called bias.

Activity

INDIVIDUAL

SUMMATIVE ASSESSMENT

1. The following graph shows how the average price of houses increased from 2004 to 2006. The article was titled: "Massive increase in the house prices over the past 3 years"



What is wrong with this representation of the data? How should the data have been represented?

2. What impression do you get about the house price increase from this representation?
3. How was this impression achieved?
Draw a more accurate account of the data.

Lesson 41

DATA HANDLING

Sampling data I

Learning Outcomes and Assessment Standards

Learning Outcome 4
Data Handling
The learner is able to collect, summarise, display and analyse data and to apply knowledge of statistics and probability to communicate, justify, predict and critically interrogate findings and draw conclusions.
NOTE: Assessment Standard 11.4.4. is the focal Assessment Standard of LO4. The other LO4 Assessment Standards serve to develop the skills that will enable learners to achieve this one.
Assessment Standard
AS 11.4.4
Critically interpret data and representations thereof (with awareness of sources of error and bias), in order to draw conclusions on problems investigated and make predictions and to critique other interpretations.
AS 11.4.1
Investigate a problem on issues such as those related to:

- Social, environmental and political factors.
- People's opinions.
- Human rights and inclusivity by:
- Collecting or finding data by appropriate methods (e.g. interviews, questionnaires, the use of data bases) suited to the purpose of drawing conclusions to the questions.
- Using appropriate statistical methods.
- Selecting a representative sample from a population with due sensitivity to issues relating to bias.
- Comparing data from different sources and samples.

AS 11.4.6
Demonstrate an awareness of how it is possible to use data in different ways to justify opposing conclusions.

Overview

In the previous two lessons we explored how the choice of representations and the decisions made in developing representations of data can be sources of both error and/or bias.

LC DVD

Lesson

There are other sources of bias and a mathematically literate person should be aware of these as the critically interpret data and representations thereof.

The above statement is talking about us being critical of all of the decisions taken during the data handling cycle and exploring how each of these could have led to some error and/or bias. In this lesson we are going to focus on sources of error and bias resulting from the selection of a sample.

Methods and worked examples

There are two concepts that are important to know when dealing with sampling.

A population:

- The collection of all the items about which we want to know something.
- Need not be only human beings e.g. it could be cars in a car park.

A sample:

- The part of the population selected with the purpose of making conclusions about the population.

- Samples should be as representative of the population as possible.

When selecting a sample from a population, we need to take into account that not all populations are homogeneous for example in a co-educational school where there are both boys and girls in the population.

Different ways of sampling are used depending on the nature of the population e.g.

- For a quality control person in a canned bean factory, it might be enough to test every 100th tin to ensure the quality of the beans.
- In a class of boys and girls, it would be important to get the opinion of both groups.
- For a large population, it is important to ensure that each member of the population has as good a chance of being selected for the sample as any other member of the population – this is the principle of called random sampling.

In this lesson we will look at two sampling techniques:

- Convenience sampling; and
- Random sampling.

Convenience sampling:

- A sample is determined by choosing items/members from the population arbitrarily, that is, in an unstructured (convenient) manner.
- This method is often used in social science research.

Random sampling:

- The sample is determined using a list of random numbers to select the members of the population which will form the sample.
- This method is intended to select members from the population in such a way that every member of the population has an equally likely chance of being selected fro the sample.

Worked example

Below is a list of people who have been buying goods at a hardware store over a period of time. For each person we know the first three letters of their name; their gender (male or female); the age group to which they belong and the amount of money that they spent in the store.

	Name	Gender	Age	Spend
1	PAT	M	45 - 60	R330,67
2	SPE	M	30 - 45	R244,00
3	WAN	F	30 - 45	R782,00
4	JOH	M	30 - 45	R262,00
5	COL	M	45 - 60	R506,67
6	MAR	M	15 - 30	R178,67
7	CHR	M	< 15	R112,67
8	SEA	M	15 - 30	R194,67
9	HAN	F	15 - 30	R432,00
10	SAN	F	30 - 45	R692,00
11	ELA	F	30 - 45	R734,00
12	ASH	M	30 - 45	R302,00

13	BAR	M	15 - 30	R186,67
14	ZOL	M	15 - 30	R132,00
15	DEL	F	30 - 45	R728,00
16	KAL	F	15 - 30	R788,00
17	CED	M	45 - 60	R338,67
18	SEA	M	30 - 45	R264,00
19	THE	F	30 - 45	R824,00
20	NIC	F	30 - 45	R842,00
21	MAN	M	15 - 30	R213,33
22	PIE	M	45 - 60	R392,00
23	DEO	M	30 - 45	R236,00
24	EVE	F	30 - 45	R650,00
25	TRE	M	30 - 45	R262,00
26	LIZ	F	15 - 30	R428,00
27	FRA	F	15 - 30	R308,00
28	ANG	F	15 - 30	R376,00
29	HEI	M	15 - 30	R164,00
30	ASH	F	< 15	R156,00
31	EVA	F	30 - 45	R686,00
32	PED	M	15 - 30	R146,67
33	PAU	M	30 - 45	R222,00
34	THE	M	30 - 45	R324,00
35	DAV	M	30 - 45	R240,00
36	AND	F	15 - 30	R424,00
37	GER	M	30 - 45	R212,00
38	HEN	M	30 - 45	R282,00
39	MAR	F	30 - 45	R662,00
40	COL	F	30 - 45	R776,00
41	SHA	M	30 - 45	R298,00
42	SAM	F	30 - 45	R722,00
43	CHE	F	30 - 45	R656,00
44	HEN	M	45 - 60	R272,00
45	DAN	M	30 - 45	R228,00
46	FRI	M	30 - 45	R300,00
47	RYA	M	15 - 30	R149,33
48	BAD	M	45 - 60	R306,67
49	ROS	F	30 - 45	R704,00
50	TON	M	45 - 60	R381,33
51	SHE	F	30 - 45	R488,00
52	NAI	F	15 - 30	R500,00
53	YUS	M	30 - 45	R272,00
54	CAR	M	15 - 30	R153,33
55	DEO	F	30 - 45	R602,00
56	GER	M	30 - 45	R244,00
57	HEI	F	15 - 30	R400,00
58	MUN	M	< 15	R112,67
59	COB	M	30 - 45	R274,00
60	THA	M	15 - 30	R148,00
61	MAR	M	30 - 45	R184,00
62	HEL	F	15 - 30	R316,00
63	JAM	M	15 - 30	R146,67

64	GER	M	30 - 45	R240,00
65	ANN	F	45 - 60	R1 096,00
66	LOA	F	30 - 45	R668,00
67	BET	F	30 - 45	R860,00
68	JAC	F	30 - 45	R740,00
69	MAR	M	15 - 30	R170,67
70	I	M	45 - 60	R336,00
71	HER	F	15 - 30	R344,00
72	FRE	F	30 - 45	R626,00
73	RUT	F	30 - 45	R872,00
74	J	M	30 - 45	R298,00
75	ROE	F	15 - 30	R500,00
76	HAN	F	15 - 30	R500,00
77	COL	M	30 - 45	R278,00
78	FRA	M	30 - 45	R272,00
79	DEW	M	15 - 30	R186,67
80	AND	F	30 - 45	R764,00
81	SUN	F	30 - 45	R740,00
82	MIC	M	< 15	R100,00
83	ROG	M	30 - 45	R228,00
84	L	M	30 - 45	R284,00
85	MIK	M	45 - 60	R328,00
86	ERI	M	30 - 45	R268,00
87	LUS	F	30 - 45	R746,00
88	THE	F	15 - 30	R432,00
89	RIC	M	15 - 30	R174,67
90	ANN	F	60+	R1 380,00
91	ALE	F	30 - 45	R1 076,00
92	DAP	F	45 - 60	R1 480,00
93	DAM	M	15 - 30	R152,00
94	W	M	30 - 45	R228,00
95	SOP	F	30 - 45	R794,00
96	CHA	F	45 - 60	R720,00
97	RAS	M	30 - 45	R268,00
98	BRE	M	30 - 45	R432,00
99	PAU	M	30 - 45	R288,00
100	LIZ	F	15 - 30	R404,00
101	ULL	F	45 - 60	R1 128,00
102	MGW	M	< 15	R94,67
103	MAR	F	30 - 45	R800,00
104	JES	F	15 - 30	R432,00
105	STE	M	30 - 45	R238,00
106	JUN	F	30 - 45	R632,00
107	CAR	F	30 - 45	R842,00
108	LIN	F	15 - 30	R540,00
109	MAR	M	30 - 45	R262,00
110	OLW	F	45 - 60	R944,00
111		M	30 - 45	R276,00
112	CAT	F	30 - 45	R740,00
113	CHA	M	30 - 45	R278,00
114	PAU	M	30 - 45	R272,00

115	AMA	F	30 - 45	R644,00
116	ALT	F	30 - 45	R860,00
117	STE	M	30 - 45	R298,00
118	JAK	M	30 - 45	R226,00
119	ALA	M	30 - 45	R244,00
120	H	M	30 - 45	R254,00
121	ISM	M	30 - 45	R318,00
122	SAN	F	30 - 45	R764,00
123	AFR	M	45 - 60	R309,33
124	PIX	F	45 - 60	R864,00
125	JEA	F	30 - 45	R866,00
126	R	M	30 - 45	R216,00
127	STI	M	30 - 45	R310,00
128	CHA	M	15 - 30	R152,00
129		M	30 - 45	R248,00
130	MIC	M	30 - 45	R230,00
131	JOH	M	60+	R460,00
132	STE	M	30 - 45	R336,00
133	EMM	F	15 - 30	R584,00
134	IAN	M	30 - 45	R242,00
135	VAU	M	30 - 45	R266,00
136	ROG	M	15 - 30	R149,33
137	PAU	M	30 - 45	R276,00
138	WOU	M	30 - 45	R290,00
139	D	M	30 - 45	R266,00
140	DAN	F	< 15	R146,00
141	MON	F	30 - 45	R524,00
142	PAU	M	30 - 45	R232,00
143	DAV	M	15 - 30	R220,00
144	DAW	F	45 - 60	R800,00
145	DIO	M	30 - 45	R204,00
146	LIN	F	45 - 60	R1 056,00
147	LEE	M	15 - 30	R132,00
148	BRI	M	45 - 60	R344,00
149	HAN	F	30 - 45	R752,00
150	KEV	M	45 - 60	R312,00
151	CAT	F	15 - 30	R344,00
152	REG	M	30 - 45	R246,00
153	EUG	M	15 - 30	R208,00
154	A	M	30 - 45	R246,00
155	RIA	F	30 - 45	R578,00
156	JAN	M	45 - 60	R338,67
157	C	M	45 - 60	R328,00
158	PAT	M	30 - 45	R308,00
159	H	F	30 - 45	R596,00
160	ROB	M	45 - 60	R344,00
161	SIP	M	< 15	R94,00
162	KEN	M	30 - 45	R290,00
163	KEI	M	30 - 45	R342,00
164	RUB	F	30 - 45	R770,00
165	BRE	M	15 - 30	R198,67

166	KEN	M	30 - 45	R258,00
167	MTS	F	15 - 30	R300,00
168	LIE	F	15 - 30	R744,00
169	GID	M	30 - 45	R242,00
170	HAZ	F	30 - 45	R722,00
171	THE	F	30 - 45	R806,00
172	SCH	M	15 - 30	R162,67
173	EVE	M	45 - 60	R402,67
174	JAC	M	15 - 30	R160,00
175	DON	M	30 - 45	R188,00
176	LUN	M	30 - 45	R260,00
177	ISA	M	15 - 30	R133,33
178	LUT	M	< 15	R97,33
179	C	M	45 - 60	R317,33
180	S	M	30 - 45	R324,00
181	CEC	F	45 - 60	R848,00
182	KER	F	15 - 30	R376,00
183	SUE	F	45 - 60	R848,00
184	AAR	M	30 - 45	R236,00
185	PIE	M	30 - 45	R252,00
186	TIM	M	45 - 60	R402,67
187	JOH	M	30 - 45	R188,00
188	MAR	F	30 - 45	R746,00
189	FRI	M	30 - 45	R220,00
190	NEL	M	15 - 30	R146,67
191	JEA	M	30 - 45	R244,00
192	GAR	M	15 - 30	R169,33
193	MEL	F	15 - 30	R424,00
194	REG	F	30 - 45	R968,00
195	NOZ	F	30 - 45	R680,00
196	AJ	M	30 - 45	R244,00
197	MAL	M	30 - 45	R234,00
198	DAV	M	30 - 45	R274,00
199	C	F	30 - 45	R770,00
200	ELS	F	30 - 45	R728,00
201	NAN	F	45 - 60	R744,00
202	ALA	F	15 - 30	R308,00
203	ROB	M	45 - 60	R370,67
204	STE	M	30 - 45	R300,00
205	WAS	F	30 - 45	R752,00
206	NIN	F	30 - 45	R848,00
207	JAM	M	15 - 30	R181,33
208	AND	M	45 - 60	R349,33
209	LUY	M	15 - 30	R152,00
210	FOR	M	15 - 30	R140,00
211	L	M	30 - 45	R230,00
212	PAU	M	30 - 45	R248,00
213	MOE	M	30 - 45	R312,00
214	DIA	F	45 - 60	R800,00
215	K	M	45 - 60	R346,67
216	ANT	M	45 - 60	R456,00

217	COR	F	15 - 30	R520,00
218	WEN	F	30 - 45	R806,00
219	GHO	F	45 - 60	R1 096,00
220	JOH	M	30 - 45	R268,00
221	JAC	M	30 - 45	R236,00
222	EBR	M	30 - 45	R312,00
223	M	F	15 - 30	R380,00
224	MIC	F	30 - 45	R782,00
225	EST	F	30 - 45	R656,00
226	ANT	M	45 - 60	R429,33
227	EBR	M	15 - 30	R190,67
228	NIC	F	15 - 30	R428,00
229	JAC	F	30 - 45	R704,00
230	AJ	M	30 - 45	R362,00
231	PET	M	30 - 45	R246,00
232	RAK	M	30 - 45	R290,00
233	JUL	F	30 - 45	R836,00
234	JEN	F	30 - 45	R1 076,00
235	LIE	F	15 - 30	R412,00
236	CAS	M	60+	R413,33
237	ALI	F	30 - 45	R716,00
238	LAU	F	30 - 45	R602,00
239	CHR	M	45 - 60	R320,00
240	JON	M	45 - 60	R360,00
241	RIC	M	30 - 45	R274,00
242	ISA	M	15 - 30	R168,00
243	MAG	F	30 - 45	R686,00
244	LIZ	F	30 - 45	R698,00
245	W	M	45 - 60	R552,00
246	JIM	M	45 - 60	R349,33
247	MIR	F	15 - 30	R392,00
248	AND	M	15 - 30	R162,67
249	LES	M	45 - 60	R394,67
250	AMI	M	30 - 45	R378,00
251	JO	F	30 - 45	R770,00
252	FID	M	15 - 30	R233,33
253	DIC	M	45 - 60	R413,33
254	LIN	F	30 - 45	R770,00
255	SHA	F	15 - 30	R620,00
256	CHA	F	30 - 45	R740,00
257	MOO	M	30 - 45	R194,00
258	WAL	M	45 - 60	R472,00
259	GUY	M	30 - 45	R282,00
260	MAR	F	30 - 45	R578,00
261	GAV	M	30 - 45	R262,00
262	LIZ	F	30 - 45	R668,00
263	R	M	30 - 45	R324,00
264	RET	M	15 - 30	R166,67
265	GER	M	45 - 60	R360,00
266	PET	F	30 - 45	R854,00
267	PIE	M	30 - 45	R278,00

268	AFZ	M	30 - 45	R248,00
269	LEI	F	15 - 30	R496,00
270	MIC	F	30 - 45	R830,00
271	JAM	M	30 - 45	R202,00
272	HIL	F	30 - 45	R764,00
273	AND	M	30 - 45	R320,00
274	JOH	M	60+	R513,33
275	KIM	F	30 - 45	R656,00
276	CHR	M	30 - 45	R244,00
277	PRO	F	15 - 30	R512,00

We have summarised this population and know the following about it:

Age (in years)	F	M	Total	**% of population**
< 15	2	6	**8**	**3%**
15 – 30	31	34	**65**	**23%**
30 – 45	66	90	**156**	**56%**
45 – 60	13	31	**44**	**16%**
60+	1	3	**4**	**2%**
Total	**113**	**164**	**277**	
% of population	41%	59%		

As we test our different sampling techniques we will want to compare the characteristics of our samples against these characteristics to see how representative the sample is.

Determine a sample (sample size = 25) from the population using each of the following techniques (which are described below).

1. Convenience sampling;
2. Random sampling

Analyse your sample in the same way as we analysed the population and discuss how representative each sample is when compared to the population.

INDIVIDUAL

FORMATIVE ASSESSMENT

Solution

1. Convenient sampling

If you take the first 25 people on the list you get the following results:

Age (in years)	F	M	Grand total	% of sample
< 15		1	**1**	**4%**
15 – 30	2	5	**7**	**28%**
30 – 45	7	6	**13**	**52%**
45 – 60		4	**4**	**16%**
60+				0%
Grand total	9	16	25	
% of sample	36%	64%		

The sample was not a bad reflection of the population. There is one group (60+) that is not represented at all and the male/female spread is out by about 5%.

2. Random sampling

Below is a list of random numbers that we will use to select our sample. This list consists of a list of randomly generated three digit numbers – a computer generated these. All of the people in our population have been numbered from 1 to 277. We now highlight the first 25 numbers on the random number list that correspond to numbers in the population.

355	327	495	792	696	687	484	361
816	1	452	477	717	603	137	700
851	782	774	876	884	829	840	837
853	424	556	408	896	443	905	25
672	519	2	466	562	190	627	13
842	509	171	430	186	267	783	899
252	779	715	499	473	849	904	554
352	910	196	595	35	413	169	479
553	906	1	861	411	812	303	380
489	952	481	226	215	483	845	284
277	790	367	678	849	369	694	580
899	391	532	650	817	563	508	313
762	752	358	705	34	851	900	663
906	147	835	180	296	378	342	535
837	362	955	294	740	209	979	439
216	347	917	737	114	569	619	37
243	948	781	237	612	379	594	331
988	739	90	432	425	423	284	281
538	895	718	540	201	471	804	373
967	523	614	665	734	11	848	616
970	6	912	32	856	542	775	849
513	831	81	308	443	263	352	96
541	147	958	111	219	366	461	992
70	814	298	334	635	475	109	587
740	765	99	140	273	92	671	7
487	163	25	438	356	181	702	397
85	464	324	458	788	565	62	278
198	968	237	518	430	895	68	913
519	64	40	874	725	898	681	803
606	175	982	787	995	971	977	470
737	52	3	62	459	146	159	365
478	952	469	937	466	739	861	650
884	862	522	790	541	469	131	68
82	760	621	976	117	538	84	724
30	239	506	189	267	435	268	678
120	512	373	4	501	792	772	415
69	298	64	972	642	450	880	894
591	146	960	345	448	141	2	540
788	126	908	751	198	626	918	680
112	459	30	633	965	542	447	979
327	812	625	730	186	362	895	678
382	968	261	931	405	630	514	580

440	231	805	485	704	919	422	611
387	867	256	284	539	907	131	106
262	497	100	106	917	39	956	619
163	421	25	241	987	955	289	699
195	380	202	686	0	467	788	732
36	251	356	38	442	352	566	785

We now go to the population list and highlight the 25 people that correspond to the random numbers. The analysed results are as follows:

Age (in years)	F	M	Total	% of sample
< 15				0%
15 – 30	1	5	**6**	**24%**
30 – 45	5	10	**15**	**60%**
45 – 60	1	3	**4**	**16%**
60+				0%
Total	7	18	25	
% of sample	28%	72%		

This sample had two groups (< 15 and 60+) missing and furthermore, the male/female spread does not compare very well with the population. It would seem that the random sampling method led to a less representative sample than the convenience method. Although the convenience sample was, in this case, more representative it is important not to think that all convenience samples will always be more representative!

Activity

LC INDIVIDUAL

LC SELF ASSESSMENT

1. Use the population list in the worked example above to do the following activities. For each of the following, select and analyse the sample specified, fill in the table provided and then discuss how representative the sample is when compared to the population.

   1.2 Use the convenience method of sampling to select a sample of 15 people.

Age (in years)	F	M	Total	% of sample
< 15				
15 – 30				
30 – 45				
45 – 60				
60+				
Total				
% of sample				

   1.3 Use the random method of sampling to select a sample of 15 people. You can start anywhere on the list of random numbers to select your numbers.

Age (in years)	F	M	Total	% of sample
< 15				
15 – 30				
30 – 45				
45 – 60				
60+				
Total				
% of sample				

1.4 Use the convenience method of sampling to select a sample of 40 people.

Age (in years)	F	M	Total	% of sample
< 15				
15 – 30				
30 – 45				
45 – 60				
60+				
Total				
% of sample				

1.5 Use the random method of sampling to select a sample of 40 people. You can start anywhere on the list of random numbers to select your numbers.

Age (in years)	**F**	**M**	**Total**	**% of sample**
< 15				
15 – 30				
30 – 45				
45 – 60				
60+				
Total				
% of sample				

1.6 Consider the samples in the worked example as well as the samples that you have determined and discuss which method and what sample size led to the most representative sample.

DATA HANDLING

Sampling data II

Lesson 42

Learning Outcomes and Assessment Standards

Learning Outcome 4
Data Handling
The learner is able to collect, summarise, display and analyse data and to apply knowledge of statistics and probability to communicate, justify, predict and critically interrogate findings and draw conclusions.
NOTE: Assessment Standard 11.4.4. is the focal Assessment Standard of LO4. The other LO4 Assessment Standards serve to develop the skills that will enable learners to achieve this one.
Assessment Standard
AS 11.4.4
Critically interpret data and representations thereof (with awareness of sources of error and bias), in order to draw conclusions on problems investigated and make predictions and to critique other interpretations.
AS 11.4.1
Investigate a problem on issues such as those related to:

- Social, environmental and political factors.
- People's opinions.
- Human rights and inclusivity by:
- Collecting or finding data by appropriate methods (e.g. interviews, questionnaires, the use of data bases) suited to the purpose of drawing conclusions to the questions.
- Using appropriate statistical methods.
- Selecting a representative sample from a population with due sensitivity to issues relating to bias.
- Comparing data from different sources and samples.

AS 11.4.6
Demonstrate an awareness of how it is possible to use data in different ways to justify opposing conclusions.

Overview

In earlier lessons we explored how the choice of representations and the decisions made in developing the representation can be sources of both error and/or bias.

In our previous lesson we started exploring how the choice of the sample used in the data handling process can also contribute to bias in the findings. In this lesson we continue our exploration of different forms of data sampling.

Lesson

Methods and worked examples

In order to successfully make conclusions about the whole population from a sample we need to ensure that the sample is as representative as possible. In the previous lesson we looked at two sampling techniques:

- Convenience sampling; and
- Random sampling.

In this lesson we will look at two more sampling techniques:

- Systematic sampling; and
- Stratified sampling.

Systematic sampling:

- A sample is determined by firstly deciding on the sample size (s) that we want from a population with size (n) and then choosing every $(n \div s)^{th}$ member of the population (which is randomly arranged) starting at some randomly (e.g. using a random number table) determined point.

Stratified sampling:

- A sample is determined in such a way that each group in the population is represented in the sample in the same proportion that it exists in the population. The sample from each group is selected using the random sample selection technique

INDIVIDUAL

FORMATIVE ASSESSMENT

Worked example

Refer to the list of people who have been buying goods at a hardware store over a period of time supplied in lesson 41.

Determine a sample (sample size = 25) from the population using each of the following techniques (which are described below).

3. Systematic sampling;
4. Stratified sampling

Analyse your sample in the same way as we analysed the population (see lesson 41) and discuss how representative each sample is when compared to the population.

Solution

3. Systematic sampling:

   We need to select every 277 ÷ 25 person which is every 11,08th person. This means that we will select every 11th person. We could start anywhere on our list but for our example we will start at the seventh person. The results can be summarised as follows:

Age (in years)	F	M	Total	**% of sample**
< 15		1	**1**	**4%**
15 – 30	3	6	**9**	**36%**
30 – 45	3	9	**12**	**48%**
45 – 60	2	1	**3**	**12%**
60+				0%
Total	8	17	25	
% of sample	32%	68%		

   The sample is reasonably representative of the population. There is one group (60+) that is not represented at all and the male/female spread is also not as representative as we might like.

4. Stratified sampling

   In each of our three samples so far the percentage of women in the sample has been less than the percentage of women in the population. If we want to ensure that each stratum (group) is represented in the sample in the same proportion that it exists in the population then we use a method of sampling called stratified sampling.

   There are 113 females out of 277 which is approximately 41% and 164 males out of 277 which is approximately 59%. This means that we must have 41% of 25 females in our sample. This is approximately 10,25 females – let's say 10 females

   This means that we need 15 males. We now need to identify these males and females using one of the other methods we have worked with. In our example we will use random sampling.

Below is a list of random numbers that we will use to select our sample.

314	80	241	200	46	111	43	236	12	4	630	182	268	240	42	96	261	111	259	83
40	242	132	176	67	196	269	44	64	96	817	222	269	167	119	98	104	218	253	108
335	42	237	55	158	130	3	276	140	47	951	219	182	86	129	208	69	90	194	171
991	62	239	137	64	256	234	34	98	132	656	34	5	237	215	37	197	19	277	68
110	168	247	117	94	83	23	71	126	85	787	1	224	75	48	55	14	121	46	226
460	75	267	225	194	118	221	193	267	152	499	185	65	207	69	110	6	162	213	277
670	217	183	59	11	266	140	27	86	58	937	18	240	276	125	173	203	113	32	225
828	152	210	77	59	272	234	37	111	11	485	163	223	258	213	177	62	26	130	138
81	227	174	9	14	260	196	254	219	223	726	255	98	103	70	140	231	235	201	190
842	130	112	141	243	79	179	161	130	222	882	44	30	109	141	202	163	37	265	41
29	214	7	236	8	108	133	72	32	107	948	182	145	13	265	189	111	13	239	47
769	107	113	80	84	234	115	32	218	28	549	11	267	74	259	143	169	253	256	223
687	43	136	222	58	250	181	116	204	17	103	256	51	95	159	271	5	15	63	99
707	84	41	172	143	147	232	219	219	107	832	243	3	27	18	201	120	260	235	216
556	228	156	123	57	135	46	39	34	156	706	265	38	118	235	269	241	50	256	157
546	111	116	225	117	49	212	72	103	178	47	83	46	145	265	114	160	147	30	87
529	211	199	16	174	131	20	98	38	248	293	249	225	225	88	122	209	100	113	238
391	136	69	168	69	203	180	31	160	141	627	95	23	127	255	76	226	265	107	48
923	74	271	26	121	58	75	142	104	153	964	264	80	48	262	63	26	48	183	166
647	228	200	214	95	199	262	176	152	135	170	123	137	159	121	208	16	95	224	130
257	141	159	198	234	208	59	59	170	274	77	49	146	271	193	114	118	45	133	135
520	120	73	162	147	233	244	139	117	138	297	74	48	251	45	167	157	88	32	266
764	262	217	242	113	136	71	275	76	87	154	65	128	71	209	154	87	120	253	185
972	33	254	131	86	146	147	254	204	122	963	236	211	221	95	25	244	237	192	209

We now go to the population list which has been sorted by gender below and highlight the 10 females and 15 males that correspond to the random numbers in the table.

Females:

	Name	Gender	Age	Spend
1	DAN	F	< 15	R146,00
2	ASH	F	< 15	R156,00
3	MTS	F	15 - 30	R300,00
4	FRA	F	15 - 30	R308,00
5	ALA	F	15 - 30	R308,00
6	HEL	F	15 - 30	R316,00
7	HER	F	15 - 30	R344,00
8	CAT	F	15 - 30	R344,00
9	ANG	F	15 - 30	R376,00
10	KER	F	15 - 30	R376,00
11	M	F	15 - 30	R380,00
12	MIR	F	15 - 30	R392,00
13	HEI	F	15 - 30	R400,00
14	LIZ	F	15 - 30	R404,00
15	LIE	F	15 - 30	R412,00
16	MEL	F	15 - 30	R424,00
17	AND	F	15 - 30	R424,00
18	NIC	F	15 - 30	R428,00
19	LIZ	F	15 - 30	R428,00
20	HAN	F	15 - 30	R432,00
21	JES	F	15 - 30	R432,00
22	THE	F	15 - 30	R432,00
23	LEI	F	15 - 30	R496,00
24	NAI	F	15 - 30	R500,00
25	ROE	F	15 - 30	R500,00
26	HAN	F	15 - 30	R500,00
27	LIE	F	15 - 30	R744,00
28	PRO	F	15 - 30	R512,00
29	COR	F	15 - 30	R520,00
30	LIN	F	15 - 30	R540,00
31	EMM	F	15 - 30	R584,00
32	SHA	F	15 - 30	R620,00
33	KAL	F	15 - 30	R788,00
34	SHE	F	30 - 45	R488,00
35	MON	F	30 - 45	R524,00
36	RIA	F	30 - 45	R578,00
37	MAR	F	30 - 45	R578,00
38	H	F	30 - 45	R596,00
39	DEO	F	30 - 45	R602,00
40	LAU	F	30 - 45	R602,00
41	FRE	F	30 - 45	R626,00
42	JUN	F	30 - 45	R632,00
43	AMA	F	30 - 45	R644,00
44	EVE	F	30 - 45	R650,00
45	KIM	F	30 - 45	R656,00
46	EST	F	30 - 45	R656,00
47	CHE	F	30 - 45	R656,00
48	MAR	F	30 - 45	R662,00
49	LIZ	F	30 - 45	R668,00

	Name	Gender	Age	Spend
50	LOA	F	30 - 45	R668,00
51	NOZ	F	30 - 45	R680,00
52	EVA	F	30 - 45	R686,00
53	MAG	F	30 - 45	R686,00
54	SAN	F	30 - 45	R692,00
55	LIZ	F	30 - 45	R698,00
56	ROS	F	30 - 45	R704,00
57	JAC	F	30 - 45	R704,00
58	ALI	F	30 - 45	R716,00
59	SAM	F	30 - 45	R722,00
60	HAZ	F	30 - 45	R722,00
61	DEL	F	30 - 45	R728,00
62	ELS	F	30 - 45	R728,00
63	ELA	F	30 - 45	R734,00
64	JAC	F	30 - 45	R740,00
65	CHA	F	30 - 45	R740,00
66	CAT	F	30 - 45	R740,00
67	SUN	F	30 - 45	R740,00
68	LUS	F	30 - 45	R746,00
69	MAR	F	30 - 45	R746,00
70	HAN	F	30 - 45	R752,00
71	WAS	F	30 - 45	R752,00
72	AND	F	30 - 45	R764,00
73	SAN	F	30 - 45	R764,00
74	HIL	F	30 - 45	R764,00
75	JO	F	30 - 45	R770,00
76	LIN	F	30 - 45	R770,00
77	RUB	F	30 - 45	R770,00
78	C	F	30 - 45	R770,00
79	COL	F	30 - 45	R776,00
80	WAN	F	30 - 45	R782,00
81	MIC	F	30 - 45	R782,00
82	SOP	F	30 - 45	R794,00
83	MAR	F	30 - 45	R800,00
84	WEN	F	30 - 45	R806,00
85	THE	F	30 - 45	R806,00
86	THE	F	30 - 45	R824,00
87	MIC	F	30 - 45	R830,00
88	JUL	F	30 - 45	R836,00
89	NIC	F	30 - 45	R842,00
90	CAR	F	30 - 45	R842,00
91	NIN	F	30 - 45	R848,00
92	PET	F	30 - 45	R854,00
93	BET	F	30 - 45	R860,00
94	ALT	F	30 - 45	R860,00
95	JEA	F	30 - 45	R866,00
96	RUT	F	30 - 45	R872,00
97	REG	F	30 - 45	R968,00
98	JEN	F	30 - 45	R1 076,00
99	ALE	F	30 - 45	R1 076,00

	Name	Gender	Age	Spend
100	CHA	F	45 - 60	R720,00
101	NAN	F	45 - 60	R744,00
102	DAW	F	45 - 60	R800,00
103	DIA	F	45 - 60	R800,00
104	SUE	F	45 - 60	R848,00
105	CEC	F	45 - 60	R848,00
106	PIX	F	45 - 60	R864,00
107	OLW	F	45 - 60	R944,00
108	LIN	F	45 - 60	R1 056,00
109	ANN	F	45 - 60	R1 096,00
110	GHO	F	45 - 60	R1 096,00
111	ULL	F	45 - 60	R1 128,00
112	DAP	F	45 - 60	R1 480,00
113	ANN	F	60+	R1 380,00

Males:

	Name	Gender	Age	Spend
1	SIP	M	< 15	R94,00
2	MGW	M	< 15	R94,67
3	LUT	M	< 15	R97,33
4	MIC	M	< 15	R100,00
5	MUN	M	< 15	R112,67
6	CHR	M	< 15	R112,67
7	ZOL	M	15 - 30	R132,00
8	LEE	M	15 - 30	R132,00
9	ISA	M	15 - 30	R133,33
10	FOR	M	15 - 30	R140,00
11	PED	M	15 - 30	R146,67
12	JAM	M	15 - 30	R146,67
13	NEL	M	15 - 30	R146,67
14	THA	M	15 - 30	R148,00
15	ROG	M	15 - 30	R149,33
16	RYA	M	15 - 30	R149,33
17	LUY	M	15 - 30	R152,00
18	CHA	M	15 - 30	R152,00
19	DAM	M	15 - 30	R152,00
20	CAR	M	15 - 30	R153,33
21	JAC	M	15 - 30	R160,00
22	SCH	M	15 - 30	R162,67
23	AND	M	15 - 30	R162,67
24	HEI	M	15 - 30	R164,00
25	RET	M	15 - 30	R166,67
26	ISA	M	15 - 30	R168,00
27	GAR	M	15 - 30	R169,33
28	MAR	M	15 - 30	R170,67
29	RIC	M	15 - 30	R174,67
30	MAR	M	15 - 30	R178,67
31	JAM	M	15 - 30	R181,33
32	BAR	M	15 - 30	R186,67
33	DEW	M	15 - 30	R186,67

	Name	Gender	Age	Spend
34	EBR	M	15 - 30	R190,67
35	SEA	M	15 - 30	R194,67
36	BRE	M	15 - 30	R198,67
37	EUG	M	15 - 30	R208,00
38	MAN	M	15 - 30	R213,33
39	DAV	M	15 - 30	R220,00
40	FID	M	15 - 30	R233,33
41	JOH	M	30 - 45	R188,00
42	DON	M	30 - 45	R188,00
43	MOO	M	30 - 45	R194,00
44	JAM	M	30 - 45	R202,00
45	DIO	M	30 - 45	R204,00
46	GER	M	30 - 45	R212,00
47	R	M	30 - 45	R216,00
48	FRI	M	30 - 45	R220,00
49	PAU	M	30 - 45	R222,00
50	JAK	M	30 - 45	R226,00
51	ROG	M	30 - 45	R228,00
52	W	M	30 - 45	R228,00
53	DAN	M	30 - 45	R228,00
54	L	M	30 - 45	R230,00
55	MIC	M	30 - 45	R230,00
56	PAU	M	30 - 45	R232,00
57	MAL	M	30 - 45	R234,00
58	AAR	M	30 - 45	R236,00
59	JAC	M	30 - 45	R236,00
60	DEO	M	30 - 45	R236,00
61	STE	M	30 - 45	R238,00
62	GER	M	30 - 45	R240,00
63	DAV	M	30 - 45	R240,00
64	GID	M	30 - 45	R242,00
65	IAN	M	30 - 45	R242,00
66	JEA	M	30 - 45	R244,00
67	CHR	M	30 - 45	R244,00
68	AJ	M	30 - 45	R244,00
69	ALA	M	30 - 45	R244,00
70	GER	M	30 - 45	R244,00
71	SPE	M	30 - 45	R244,00
72	A	M	30 - 45	R246,00
73	PET	M	30 - 45	R246,00
74	REG	M	30 - 45	R246,00
75	PAU	M	30 - 45	R248,00
76	J	M	30 - 45	R248,00
77	AFZ	M	30 - 45	R248,00
78	PIE	M	30 - 45	R252,00
79	H	M	30 - 45	R254,00
80	KEN	M	30 - 45	R258,00
81	LUN	M	30 - 45	R260,00
82	MAR	M	30 - 45	R262,00
83	GAV	M	30 - 45	R262,00

	Name	Gender	Age	Spend
84	JOH	M	30 - 45	R262,00
85	TRE	M	30 - 45	R262,00
86	SEA	M	30 - 45	R264,00
87	D	M	30 - 45	R266,00
88	VAU	M	30 - 45	R266,00
89	RAS	M	30 - 45	R268,00
90	JOH	M	30 - 45	R268,00
91	ERI	M	30 - 45	R268,00
92	FRA	M	30 - 45	R272,00
93	YUS	M	30 - 45	R272,00
94	PAU	M	30 - 45	R272,00
95	RIC	M	30 - 45	R274,00
96	DAV	M	30 - 45	R274,00
97	COB	M	30 - 45	R274,00
98	p	M	30 - 45	R276,00
99	PAU	M	30 - 45	R276,00
100	PIE	M	30 - 45	R278,00
101	CHA	M	30 - 45	R278,00
102	COL	M	30 - 45	R278,00
103	HEN	M	30 - 45	R282,00
104	GUY	M	30 - 45	R282,00
105	L	M	30 - 45	R284,00
106	PAU	M	30 - 45	R288,00
107	WOU	M	30 - 45	R290,00
108	KEN	M	30 - 45	R290,00
109	RAK	M	30 - 45	R290,00
110	STE	M	30 - 45	R298,00
111	SHA	M	30 - 45	R298,00
112	J	M	30 - 45	R298,00
113	FRI	M	30 - 45	R300,00
114	STE	M	30 - 45	R300,00
115	ASH	M	30 - 45	R302,00
116	MAR	M	30 - 45	R184,00
117	PAT	M	30 - 45	R308,00
118	STI	M	30 - 45	R310,00
119	MOE	M	30 - 45	R312,00
120	EBR	M	30 - 45	R312,00
121	BRE	M	30 - 45	R432,00
122	ISM	M	30 - 45	R318,00
123	AND	M	30 - 45	R320,00
124	S	M	30 - 45	R324,00
125	R	M	30 - 45	R324,00
126	THE	M	30 - 45	R324,00
127	STE	M	30 - 45	R336,00
128	KEI	M	30 - 45	R342,00
129	AJ	M	30 - 45	R362,00
130	AMI	M	30 - 45	R378,00
131	HEN	M	45 - 60	R272,00
132	BAD	M	45 - 60	R306,67
133	AFR	M	45 - 60	R309,33

	Name	Gender	Age	Spend
134	KEV	M	45 - 60	R312,00
135	C	M	45 - 60	R317,33
136	CHR	M	45 - 60	R320,00
137	MIK	M	45 - 60	R328,00
138	C	M	45 - 60	R328,00
139	P	M	45 - 60	R330,67
140	I	M	45 - 60	R336,00
141	JAN	M	45 - 60	R338,67
142	CED	M	45 - 60	R338,67
143	ROB	M	45 - 60	R344,00
144	BRI	M	45 - 60	R344,00
145	K	M	45 - 60	R346,67
146	JIM	M	45 - 60	R349,33
147	AND	M	45 - 60	R349,33
148	GER	M	45 - 60	R360,00
149	JON	M	45 - 60	R360,00
150	ROB	M	45 - 60	R370,67
151	TON	M	45 - 60	R381,33
152	PIE	M	45 - 60	R392,00
153	LES	M	45 - 60	R394,67
154	TIM	M	45 - 60	R402,67
155	EVE	M	45 - 60	R402,67
156	DIC	M	45 - 60	R413,33
157	ANT	M	45 - 60	R429,33
158	ANT	M	45 - 60	R456,00
159	WAL	M	45 - 60	R472,00
160	COL	M	45 - 60	R506,67
161	W	M	45 - 60	R552,00
162	CAS	M	60+	R413,33
163	JOH	M	60+	R460,00
164	JOH	M	60+	R513,33

The summarised results are as follows:

Age (in years)	F	M	Total	% of sample
< 15		1	1	4%
15 – 30	2	1	3	12%
30 – 45	7	10	17	68%
45 – 60	1	3	4	16%
60+				0%
Total	10	18	25	
% of sample	40%	60%		

Although this sample has one group (60+) missing, the male/female spread corresponds very well with the population – which is by design! While the male/female spread corresponds very well with the population, the age groups still doesn't. Of course we could treat the age groups as groups or strata as well and then use this same technique again and in so doing we would ensure that the age groups are represented in the sample in the same proportion as what they exist in the population.

GROUPS

SELF ASSESSMENT

Activity

1. Use the population list in the worked example above to do the following activities. For each of the following, select and analyse the sample specified, fill in the table provided and then discuss how representative the sample is when compared to the population.

   1.1 Use the systematic method of sampling to select a sample of 15 people.

Age (in years)	F	M	Total	% of sample
< 15				
15 – 30				
30 – 45				
45 – 60				
60+				
Total				
% of sample				

   1.2 Use the stratified method of sampling to select a sample of 15 people. You must use the list which has been organized in females and males. You can start anywhere on the list of random numbers to select your numbers.

Age (in years)	F	M	Total	% of sample
< 15				
15 – 30				
30 – 45				
45 – 60				
60+				
Total				
% of sample				

   1.3 Use the systematic method of sampling to select a sample of 40 people.

Age (in years)	F	M	Total	% of sample
< 15				
15 – 30				
30 – 45				
45 – 60				
60+				
Total				
% of sample				

   1.4 Use the stratified method of sampling to select a sample of 40 people. You must use the list which has been organized in females and males. You can start anywhere on the list of random numbers to select your numbers.

Age (in years)	F	M	Total	% of sample
< 15				
15 – 30				
30 – 45				
45 – 60				
60+				
Total				
% of sample				

1.5 Consider the samples in the worked example as well as the samples that you have determined and discuss which method and what sample size led to the most representative sample.

Lesson 43

DATA HANDLING

Sampling data III

Learning Outcomes and Assessment Standards

Learning Outcome 4
Data Handling
The learner is able to collect, summarise, display and analyse data and to apply knowledge of statistics and probability to communicate, justify, predict and critically interrogate findings and draw conclusions.
NOTE: Assessment Standard 11.4.4. is the focal Assessment Standard of LO4. The other LO4 Assessment Standards serve to develop the skills that will enable learners to achieve this one.
Assessment Standard
AS 11.4.4
Critically interpret data and representations thereof (with awareness of sources of error and bias), in order to draw conclusions on problems investigated and make predictions and to critique other interpretations.
AS 11.4.1
Investigate a problem on issues such as those related to:
- Social, environmental and political factors.
- People's opinions.
- Human rights and inclusivity by:
- Collecting or finding data by appropriate methods (e.g. interviews, questionnaires, the use of data bases) suited to the purpose of drawing conclusions to the questions.
- Using appropriate statistical methods.
- Selecting a representative sample from a population with due sensitivity to issues relating to bias.
- Comparing data from different sources and samples.

AS 11.4.6
Demonstrate an awareness of how it is possible to use data in different ways to justify opposing conclusions.

Overview

In the previous two lessons we have looked at four techniques of sampling:

- Convenience sampling – choosing items arbitrarily and in an unstructured manner.
- Random sampling – using a list of random numbers to identify the members of the population that will be selected for the sample.
- Systematic sampling – choosing every $(n \div s)^{th}$ member of the population where n is the population size and s is the sample size.
- Stratified sampling – ensuring each group within the population is represented in the same proportion that it exists in the population.

The purpose of sampling is to make conclusions about the whole – population.

In this lesson we will compare the different samples that we determined using the different techniques in terms of their range, mean and median.

LC DVD

Lesson

Methods and worked examples

For our population – the people who have been buying things at a hardware store over a period of time – we have determined a sample using each technique and we have summarised the sample in terms of the number of males and females as well as the number of people in each of the different age groups. Doing this enabled us to get a sense of how representative of the population each sample was.

It is important to note that in the example that we are working on we know everything about the whole population. We did this so that we can learn about sampling. However, in reality we would not know everything about a population – if we did there would be no reason to select a sample!

Worked example

INDIVIDUAL

FORMATIVE ASSESSMENT

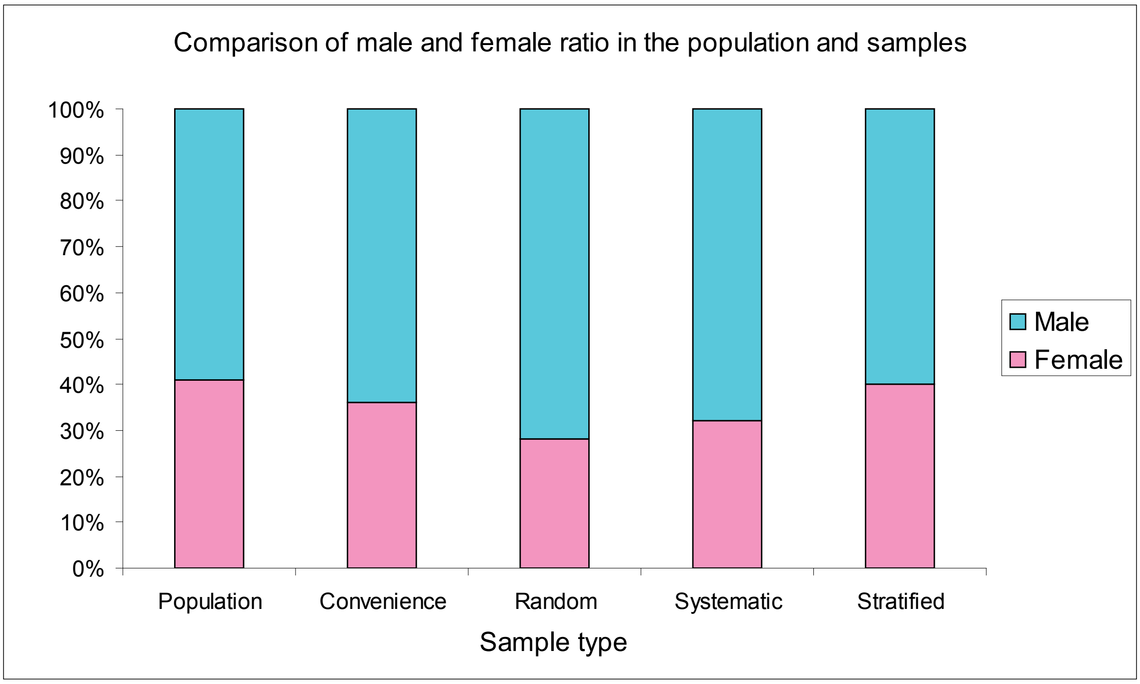
The hardware owner could be interested in a description of his clientele for a number of reasons:

- By knowing whether the store's customers are male or female, old or young the owner would know what kinds of products to keep.
- By knowing how much customers typically spend – the so called basket value – and how many customers come through the door the owner could budget the store's anticipated cash-flow.

1. Draw a compound bar graph to represent the male/female ratio for the population and all four sampling techniques. What conclusions can you make?
2. Calculate the minimum and maximum, mean and median amount of money spent by the customers for the population and the samples. (Should be done as a group activity)
3. Represent the above statistics on a graph. Draw a vertical line from the minimum to the maximum values. Use a round dot to represent the mean and a diamond to represent the median amount.
4. Compare the graphs.

Solution

1. Compound bar graph:



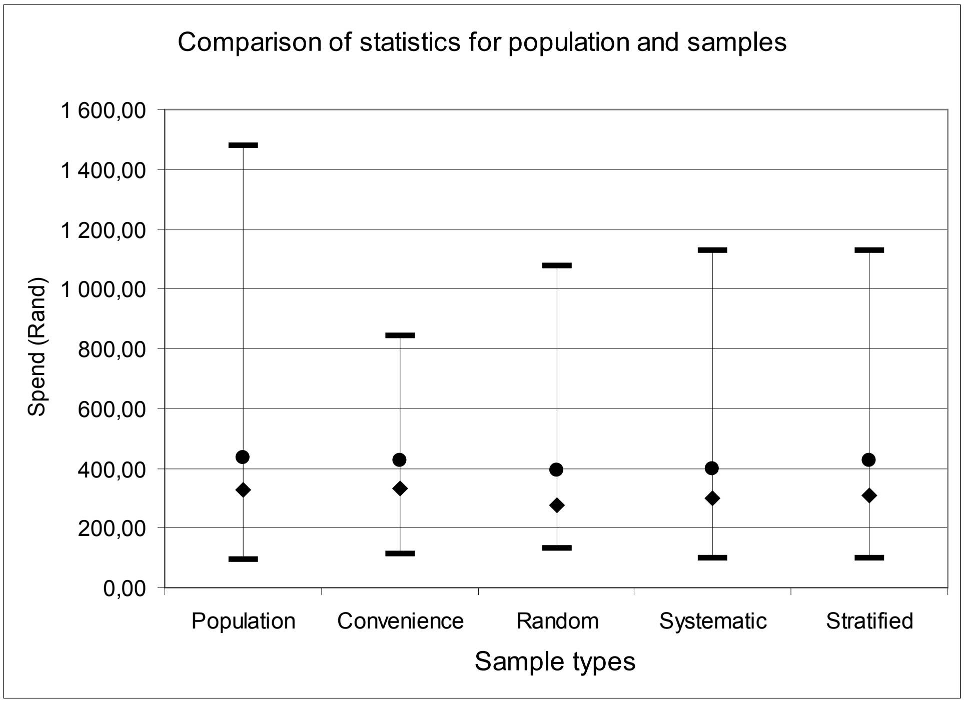
Although all of the samples produced by the various techniques show that there are more male customers than there are females customers. Only the stratified sample has males and females in the same ratio as the ratio of males and females in the population. We would anticipate that the stratified sample would give the most accurate representation of male/female ratio because the technique is designed to do that.

2. Mean = $\frac{\text{total amount spent}}{\text{number of customers}}$

Median = value which divides the ordered data into two groups of equal size. In other words half of the people in the population spent more than this amount and half spent less than this amount

	Population	Convenience	Random	Systematic	Stratified
Minimum	R94,00	R112,67	R132,00	R97,33	R97,33
Mean	R433,91	R425,12	R394,35	R399,17	R426,83
Median	R328,00	R330,67	R276,00	R298,00	R308,00
Maximum	R1 480,00	R842,00	R1 076,00	R1 128,00	R1 128,00

3. Graphs showing minimum, maximum, mean and median.



4. In almost all of the cases we get a very similar mean and median but the range varies quite a bit from one sample to the next. The important point to be made is that while we are able to compare the sample with the population in the case of this activity in reality the hardware owner would not be able to do so and if he had used the convenience sampling method, he might be under the impression that the range within which his customers spend is: ≈ R110 to ≈ R840.

Consider the following:

The article appeared in the newspaper and reports on a survey that established that South African men spend more on facial cosmetics than any other men in the world.

If we want to critique the findings we need to interrogate the data cycle that led to the finding.

When we read the article carefully we discover that the results are based on a survey of 25 000 internet users world wide and it turns out that 2 000 of those are from South Africa. However, Stats SA website states that the 2001 census revealed that less than 9% of South Africans have access to the internet in their homes. The number of Black households with computers is even lower – it's less than 2%. So a survey that relies on a sample of people with access to the internet is unlikely to be representative of the South African population.

SA men go big on image, says poll

SOUTH African males have embraced the world of metrosexuality — with more men spending time and money on their appearance.

An international consumer survey by the Nielsen Company claims that South African men — along with the Portuguese and Greeks — are world leaders when it comes to caring for their appearance, with 94% of South African respondents claiming that men have become more interested in personal grooming.

The reasons given vary from hopes of attracting a new partner to looking good for a spouse and feeling good about themselves.

Nielsen product manager Meave Beckett-Leech said the survey of 25 000 Internet users across the world found that South African men spent more on beauty products and treatments than any other.

Activity

GROUPS

SELF ASSESSMENT

1. Use the data for the female customers to complete the following:
   - 1.1 Calculate the minimum and maximum, mean and median amount of money spent by the female customers for the population and the samples you have determined in lessons 41 and 42.
   - 1.2 Represent the above statistics on a graph as in the worked example.
   - 1.3 Compare the graphs.
2. Use the data for the male customers to complete the following:
   - 2.1 Calculate the minimum and maximum, mean and median amount of money spent by the male customers for the population and the samples you have determined in lessons 41 and 42.
   - 2.2 Represent the above statistics on a graph as in the worked example.

     Compare the graphs.

Lesson 44

DATA HANDLING

Likelihood I

Learning Outcomes and Assessment Standards

Learning Outcome 4
Data Handling
The learner is able to collect, summarise, display and analyse data and to apply knowledge of statistics and probability to communicate, justify, predict and critically interrogate findings and draw conclusions.
Note: Assessment Standard 11.4.4: is the focal Assessment Standard of LO4. The other LO4 Assessment Standards serve to develop the skills that will enable learners to achieve this one.
Assessment Standard
AS 11.4.1
Investigate a problem on issues such as those related to:

- Social, environmental and political factors.
- People's opinions.
- Human rights and inclusivity by:
- Collecting or finding data by appropriate methods (e.g. interviews, questionnaires, the use of data bases) suited to the purpose of drawing conclusions to the questions.
- Using appropriate statistical methods.
- Selecting a representative sample from a population with due sensitivity to issues relating to bias.
- Comparing data from different sources and samples.

AS 11.4.4
Critically interpret data and representations thereof (with awareness of sources of error and bias) in order to draw conclusions on problems investigated and make predictions and to critique other interpretations.
AS 11.4.5
Work with simple notions of likelihood/probability in order to make sense of statements involving these notions.
AS 11.4.6
Demonstrate an awareness of how it is possible to use data in different ways to justify opposing conclusions.

Overview

Will it rain? Who will win the match? How old will I become? How reliable is that product? All these questions show a desire to predict the future.

In the 17th century this desire to be able to predict the future gave rise to a branch of mathematics known as probability.

In the next three lessons we will focus on the topic of probability or likelihood.

We define probability as:

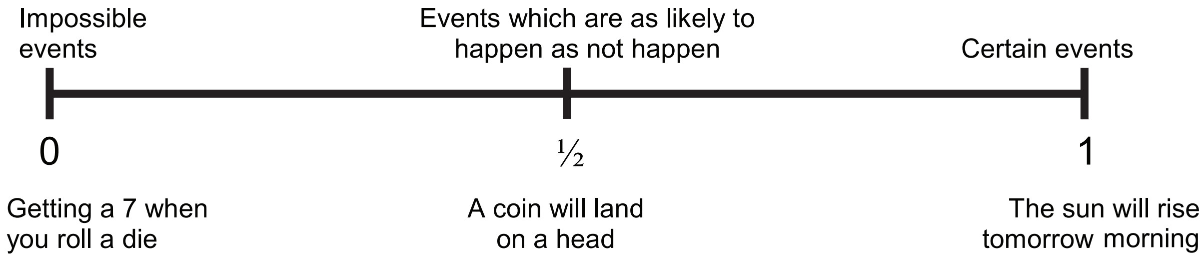
- the measure of the likelihood of an event occurring – a measure that ranges from 0 to 1.

Lesson
Methods and worked examples

We say that events which:

- cannot happen (impossible events), have a probability of 0;
- will definitely happen (certain events), have a probability of 1; and
- are just as likely to happen as they are likely not to happen, have a likelihood of 12 or 0,5 or 50%.

These numbers give rise to what we call the probability scale.



When the curriculum speaks about us developing the ability to make sense of statements involving notions of likelihood, it is referring to the fact that we, in many parts of our lives, meet up with statements involving expressions of likelihood and we should understand how these impact on our lives.

There are two ways of developing expressions of likelihood:

- theoretical expressions of likelihood – determined by studying the structure of a situation e.g. the probability the you will get a head if you toss a coin is $\frac{1}{2}$, and
- experimental expressions of likelihood – determined by gathering a very large number of data, analysing it and then predicting the future based on the results on the analysis e.g. drug trials in medicine.

Worked example

The table below compares different contraceptive methods, in terms of their effectiveness in preventing pregnancy and in terms of their effectiveness in preventing sexually transmitted diseases.

Contraceptive method	Effectiveness in preventing pregnancies	Effectiveness in preventing STDs
Male condom	86%	Effective
Female condom	80%	Very effective
Combined pill	95%	Nil
Abstinence	100%	Very effective

(1) Compare the effectiveness of the male and female condom.

(2) What does 86% effective in preventing pregnancies mean?

(3) What does 95% effective in preventing pregnancies mean?

(4) Show where these percentages would fit onto the probability scale.

(5) Where would these figures come from?

Solution

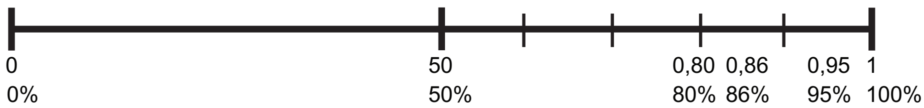
(1) Although the male condom is more effective in preventing pregnancies than the female condom is, the female condom is more effective in the prevention of STDs than the male condom is.

(2) "86% effective in preventing pregnancy" means that out of every 100 couples using the male condom correctly as their form of contraception for one year, 14 (100 minus 86) might still fall pregnant.

NOTE: (refer also to question 5): it is important to stress that this does not mean that if we take a random sample 100 people using the male contraceptive as their form of contraception and monitor them over one year then exactly 14 of them will nonetheless fall pregnant. Instead it means that if we took a very large sample of such people and monitored them over many years then the pregnancy rate would average out at approximately 14 couples out of every 100 couples falling pregnant every year.

(3) It means that for every 100 women who use the pill as their form of contraception for one year, 5 (100 minus 95) might still fall pregnant. See note above.

(4)



(5) Based on data collected for a large number of people and over a long period of time, researchers have collected enough data to be able to conclude that if the conditions remain the same, then in the future, and for a large number of people the effectiveness of the contraceptive methods will be as in the table.

NOTE: The researchers are unable to tell us for whom the method will fail and for whom it will be effective. They can only predict that based on the evidence that they have collected that the method will have a certain success rate for a large number of people.

INDIVIDUAL

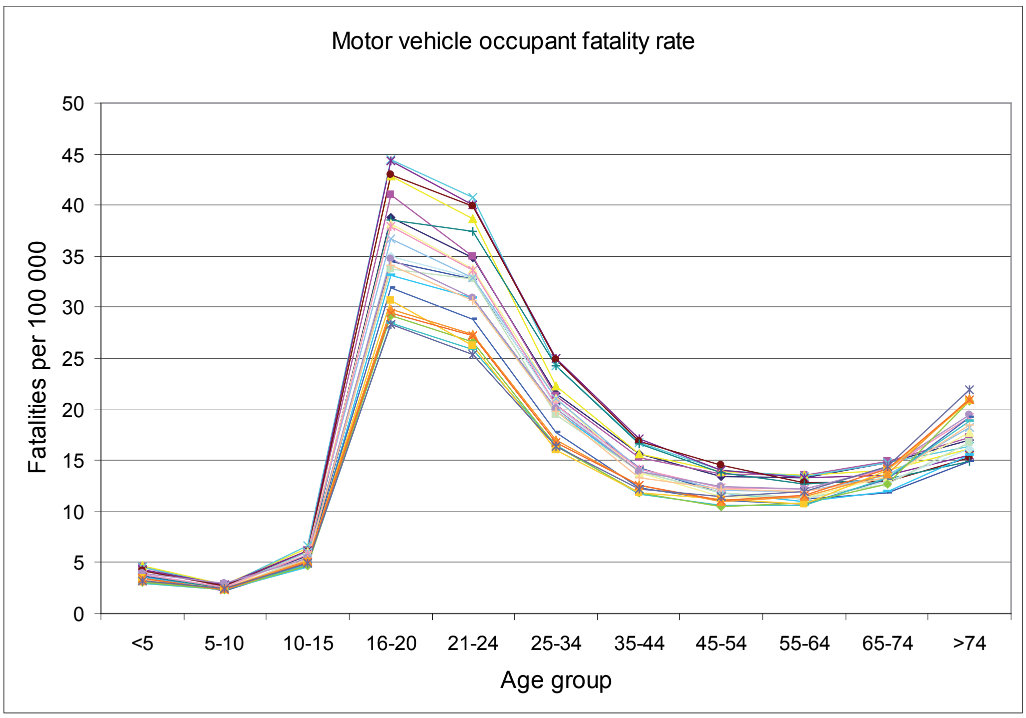
FORMATIVE ASSESSMENT

Consider the following example:

(1) The graph below shows the statistics on motor vehicle fatalities for the years 1975 to 1997 in the United States.

The horizontal axes shows the different age groups and the vertical axis shows the number of deaths in each age group per 100 000 people in the age groups.

Each line represents the values for a different year.



The reason the values on the vertical axis are expressed as a rate – deaths per 100 000 – is because we want to look for trends across the age groups. If we plotted the actual number of deaths then we might not see the trend as clearly because we would not be taking the number of people in each age group into account – we are trying to compare like with like.

Discuss any trends shown by the graph and what predictions you can make for the future.

Solution

What we observe from those graphs is that although the actual values may change from year to year, the trend is that the people most at risk from dying in a motor vehicle accident are the people in the 16–20 (remember that people in the United States start driving motor vehicles at age 16) and 21 to 24 year old age groups. The graphs also show that risk of dying in a motor vehicle accident increase for people over ≈ 65–70 years of age.

It is reasonable to say, given that we have data for more than 30 years, that we can anticipate that in the future people in the age group 16–24 will continue to be at the greatest risk of dying in a motor vehicle accident. We can say the likelihood or probability of dying in a motor car accident is greater if you are between 16 and 24 years old.

Do you know: Car insurance premium for people in the age group 18–25 is greater than for all other age groups and some car hire companies will not even rent cars to people under the age of 25?

Notice that the data does not tell us who will die in a car accident or when they will die, it only tells us that people in certain age groups are at a greater risk than others and that we can expect approximately so many deaths per 100 000.

The experimental probability helps us to predict what will happen across the population over time and based on that knowledge we can make decisions – in this case to increase the insurance premiums for people in certain age groups.

Activity

PAIRS

SELF ASSESSMENT

1. Read the following newspaper articles and answer the questions below:

Reduce the risk of death by flu by having a flu injection

It's that time of year again when many of us come down with "a bug", a cold, the flu or even pneumonia. Because we spend more time indoors and close to other people, there is more opportunity for the "bugs" to spread.

Children often get up to 6 to 8 colds a year, while adults, may average 2–4 colds a year. And the flu typically causes over 100 000 hospitalisations, and more than 20 000 deaths each year.

But there are steps you can take to reduce the risk of you and your family becoming sick this year. Key to reducing the number of colds is to wash your hands, and to wash them often.

To prevent the flu, there is a vaccine available. Studies have shown that the vaccine is 70%-90% effective in preventing the disease in healthy young adults. The vaccine can also reduce hospitalisation by about 70%, and death by 85%, when taken by elderly people who do not reside in a nursing homes. Even among nursing home residence, the flu vaccine can reduce the risk of hospitalisation by about 50%, reduce the risk of pneumonia by 50%-60%, and reduce the risk of death by 80%.

1.1 What word in the title of the article indicates that the article is about expressions of likelihood?

1.2 "Studies have shown that the vaccine is 70%–90% effective in preventing the disease in healthy young adults", is an expression of likelihood. Explain what is meant by this.

1.3 Do you think that you should have a flu injection? Substantiate your answer.

2 Read the following newspaper articles and answer the questions below:

Folate may increase the chance of twins

New research has shown that folate supplements, widely recommended for women planning pregnancy, may increase the chances of having twins.

Professor Judith Lumley, from the Centre for the Study of Mothers' Health at La Trobe University examined three folate supplement trials and found women who took folate before falling pregnant had a 40% greater chance of having twins.

"The increased risk of having twins and the greater health problems they face could make recommending folate a dilemma," she said.

2.1 What word in the title of the article indicates that the article is about expressions of likelihood?

2.2 Why do you think Prof Lumley said "could make recommending folate a dilemma"?

2.3 The likelihood of having twins for women not taking supplements such a folate has been determined to about 1,4%. What does this mean? Use a sample size of 1000 to substantiate your answer.

2.4 How would an increase of 40% with the use of folate impact on the number of twins – again use a sample size of 1 000 to illustrate your answer.

2.5 Comment on Prof Lumley's remarks in light of your calculations.

DATA HANDLING

Likelihood II

Lesson 45

Learning Outcomes and Assessment Standards

Learning Outcome 4
Data Handling
The learner is able to collect, summarise, display and analyse data and to apply knowledge of statistics and probability to communicate, justify, predict and critically interrogate findings and draw conclusions.
NOTE: Assessment Standard 11.4.4. is the focal Assessment Standard of LO4. The other LO4 Assessment Standards serve to develop the skills that will enable learners to achieve this one.
Assessment Standard
AS 11.4.4
Critically interpret data and representations thereof (with awareness of sources of error and bias), in order to draw conclusions on problems investigated and make predictions and to critique other interpretations.
AS 11.4.1
Investigate a problem on issues such as those related to:

- Social, environmental and political factors.
- People's opinions.
- Human rights and inclusivity by:
- Collecting or finding data by appropriate methods (e.g. interviews, questionnaires, the use of data bases) suited to the purpose of drawing conclusions to the questions.
- Using appropriate statistical methods.
- Selecting a representative sample from a population with due sensitivity to issues relating to bias.
- Comparing data from different sources and samples.

AS 11.4.5
Work with simple notions of likelihood/probability in order to make sense of statements involving these notions.
AS 11.4.6
Demonstrate an awareness of how it is possible to use data in different ways to justify opposing conclusions.

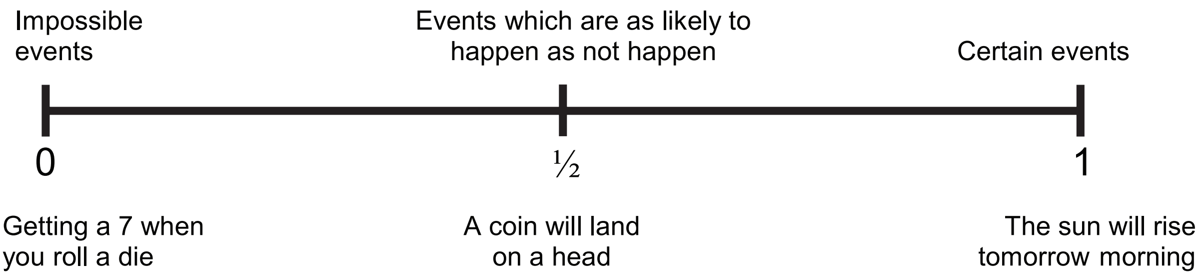
Overview

In the previous lesson we defined probability as the measure of the likelihood of an event occurring and said that probability is expressed using numbers between 0 and 1.

We also said that there are two ways of developing expressions of likelihood:

- theoretical expressions of likelihood – determined by studying the structure of a situation e.g. the probability the you will get a head if you toss a coin is $\frac{1}{2}$; and
- experimental expressions of likelihood – determined by gathering a very large amount of data, analysing it and then predicting the future based on the results on the analysis e.g. drug trials in medicine.

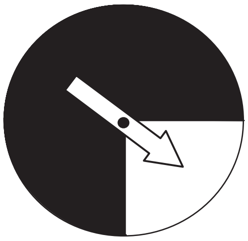
The probability scale which is shown below indicates the measures of probability:



We have already dealt with experimental probability in the previous lesson so we will focus on theoretical probability in this lesson.

Lesson

Methods and worked examples



We will be using spinners to explain theoretical probability. On the left is an example of a spinner.

In the spinner alongside there are only two outcomes:

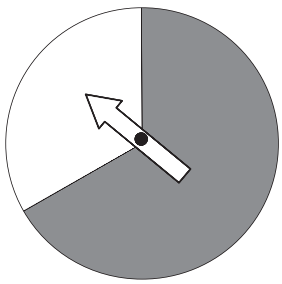
- either the spinner lands on a shaded region; or
- the spinner lands on a un-shaded region.

If we were to use this spinner in a game of chance we would want to be able to predict how often it would land on the shaded region and how often it would land on the un-shaded portion. From the structure of the above spinner we could say that it would land on the shaded region $\frac{3}{4}$ of the time. We call this the theoretical probability or likelihood.

Expressions of likelihood talk about the long term – about many events – and not about the outcome of the next event. Because the outcome of the spinner is completely random, we simply cannot predict the outcome for any single event. But we can, and with great confidence, predict the outcome for a very large number of events.

Worked example

The questions/instructions that follow are based on the spinner below:



(1) What is the probability or likelihood of the spinner landing on the shaded region?

(2) Spin the spinner 6 times and record the results using a tally table.

(3) Repeat spinning in groups of 6 for another 5 times i.e. so you end up with 6 groups of 6 spins.

(4) Calculate the cumulative frequency of spins landing in the shaded region.

(5) Draw a graph of the cumulative number of spins with the cumulative frequency for the theoretical probability and the experimental probability.

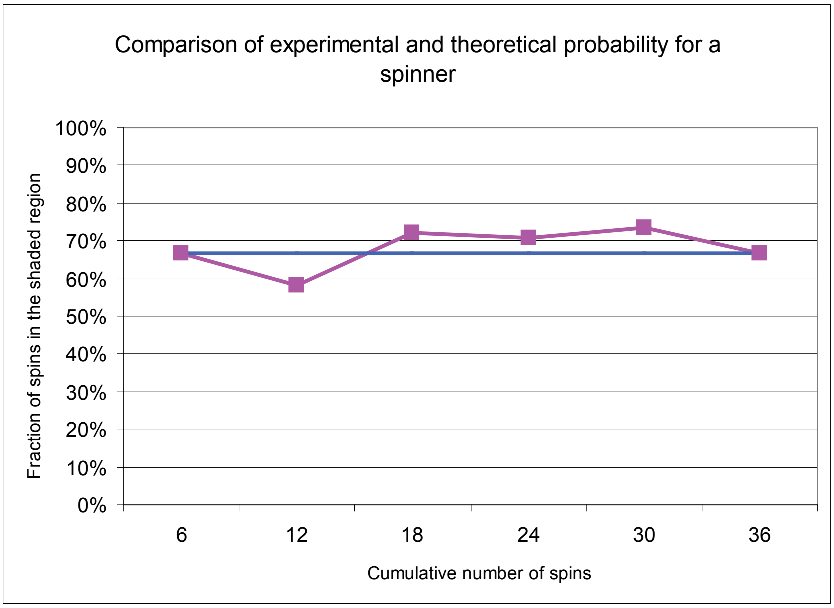
Solution

(1) The shaded area measures 240°. Therefore we would expect the spinner to land in the shaded area 240 out of 360 times. We can simplify that to 23.

(2 – 4)

Shaded area		Unshaded		Cumulative frequency for shaded
	$\frac{4}{6} = \frac{2}{3}$		$\frac{1}{3}$	$\frac{2}{3} = 67\%$
	$\frac{3}{6} = \frac{1}{2}$		$\frac{3}{6} = \frac{1}{2}$	$\frac{7}{12} = 59\%$
	$\frac{6}{6} = 1$		0	$\frac{13}{18} = 72\%$
	$\frac{4}{6} = \frac{2}{3}$		$\frac{1}{3}$	$\frac{17}{24} = 71\%$
	$\frac{5}{6}$		$\frac{1}{6}$	$\frac{22}{30} = 73\%$
	$\frac{2}{6} = \frac{1}{3}$		$\frac{2}{3}$	$\frac{24}{36} = 67\%$

(5)



Note: As the number of spins increases so the fraction of spins landing in the shaded region gets closer and closer to what we predicted. We could call the fraction based on the actual results the experimental probability and what we now observe is that as the number of spins increases so the experimental probability begins to approach the theoretical probability.

Activity

GROUPS

SELF ASSESSMENT

1. Make a spinner of your own, out of cardboard, which has a different size shaded region to the one we used in the worked example.
   - 1.1 Spin the spinner and record the results for every 6 spins using a tally table as in the worked example.
   - 1.2 Calculate the cumulative frequencies.
   - 1.3 Draw a graph to show the comparison of the theoretical probability with the experimental probability.
   - 1.4 Discuss what you observed.

Lesson 46

DATA HANDLING

Likelihood III

Learning Outcomes and Assessment Standards

Learning Outcome 4
Data Handling
The learner is able to collect, summarise, display and analyse data and to apply knowledge of statistics and probability to communicate, justify, predict and critically interrogate findings and draw conclusions.
NOTE: Assessment Standard 11.4.4. is the focal Assessment Standard of LO4. The other LO4 Assessment Standards serve to develop the skills that will enable learners to achieve this one.

Assessment Standard
AS 11.4.4
Critically interpret data and representations thereof (with awareness of sources of error and bias), in order to draw conclusions on problems investigated and make predictions and to critique other interpretations.
AS 11.4.1
Investigate a problem on issues such as those related to:
- Social, environmental and political factors.
- People's opinions.
- Human rights and inclusivity by:
  — collecting or finding data by appropriate methods (e.g. interviews, questionnaires, the use of data bases) suited to the purpose of drawing conclusions to the questions.
- Using appropriate statistical methods.
- Selecting a representative sample from a population with due sensitivity to issues relating to bias.
- Comparing data from different sources and samples.

AS 11.4.5
Work with simple notions of likelihood/probability in order to make sense of statements involving these notions.
AS 11.4.6
Demonstrate an awareness of how it is possible to use data in different ways to justify opposing conclusions.

Overview

We have been exploring likelihood over the last two lessons and have said that likelihood is the branch of mathematics concerned with predicting the future.

Lesson

We have defined probability as the measure of the likelihood of an event occurring; and have said that probability is expressed using numbers between 0 and 1.

We also said that there are two distinct ways in which we can develop expressions of likelihood:

- theoretical expressions of likelihood which we dealt with in the previous lesson – these are determined by studying the structure of a situation e.g. the probability the you will get a head if you toss a coin is 12; and
- experimental expressions of likelihood – determined by gathering a very large amount of data, analysing it and then predicting the future based on the results on the analysis e.g. drug trials in medicine.

There is another context in which people like to make predictions: sport. Not only do people like to predict the outcome of sporting events but people win and lose a great deal of money by betting on the results.

In the case of betting on sports events bookmakers offer different odds based on their predictions of who will win or lose.

People may think that because of those odds that predictions with respect to the outcome of a sports event are similar to predictions about the spinner, however, the big difference between the sports event and the spinner is the nature of the variables involved.

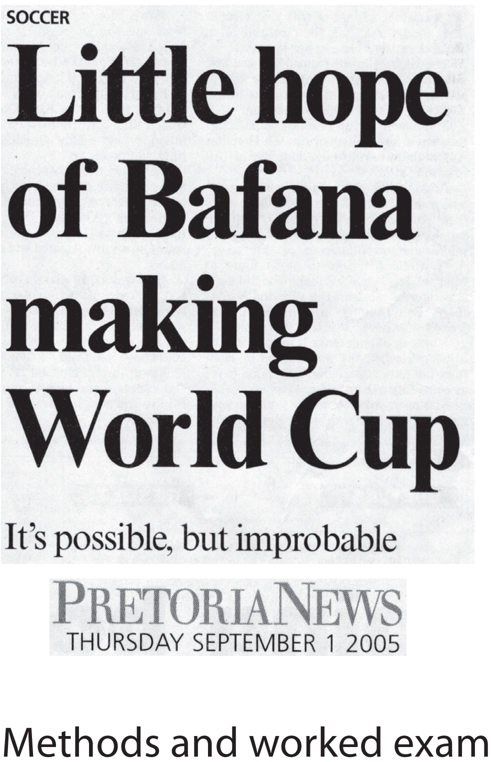
When we spin a spinner, the outcome is completely random, there is neither skill involved in the actual spinning of the spinner or form as in the fitness of the players.

A sport team's form may vary from one season to the next depending on who is playing for the side, or on who is coaching the side, injuries and other factors all impact on the outcome of the game.

The conditions of each sports game are unique, whereas with the spinner the conditions are basically the same from one spin to the next and the outcome is completely random.

Therefore, because the conditions change from one game to the next we don't have either history to make an "experimental" prediction or structure to make a "theoretical" prediction.

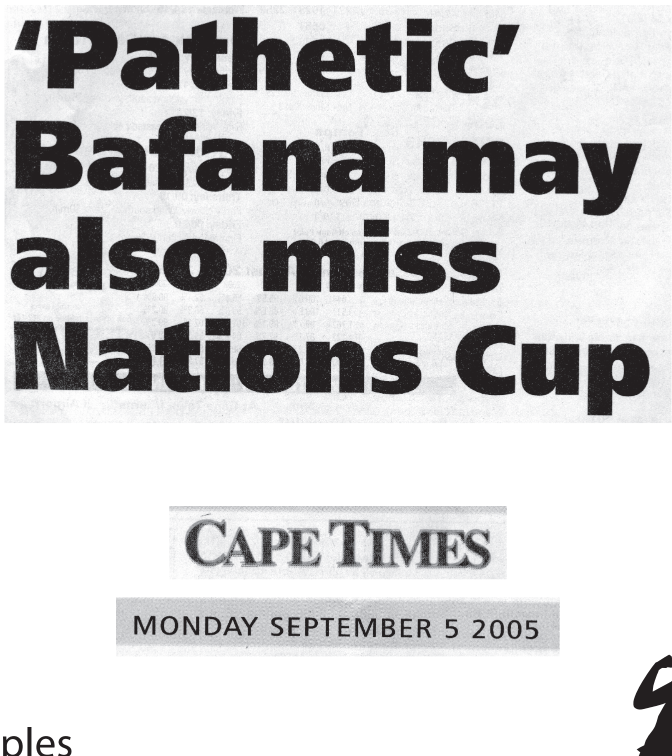
So, although the headlines below look and read like predictions and they use the language of probability they are in fact only an analysis of a situation.


SOCCER

Little hope of Bafana making World Cup

It's possible, but improbable

PRETORIA NEWS
THURSDAY SEPTEMBER 1 2005


'Pathetic' Bafana may also miss Nations Cup

CAPE TIMES
MONDAY SEPTEMBER 5 2005

Methods and worked examples

In this lesson we are going to conduct our own analysis of a sporting event.

INDIVIDUAL

FORMATIVE ASSESSMENT

Worked example

The following information comes from one of the group competitions in the lead up to the 2006 World Cup soccer tournament.

Below is the log table for Group G at the start of the group competition.

The four teams in the group were France, Korea, Switzerland and Togo. Each team had to play every other team in the group once earning 3 points for a win, 1 point for a draw and no points for losing.

The top two teams in each group qualify for the knock-out round.

Each team must play three games and because they earn three points for a win, the most points that any team can earn is 9.

Log table

Group G

Team	MP	W	D	L	GF	GA	Pts
France							
Korea							
Switzerland							
Togo							

The number of games played in this group will be: 4 teams $\times$ 3 games $=$ 12 games

However each game involves 2 teams so the number of games will be
$12 \div 2 = 6$ games

Below is the fixture list and the results of the first matches.

Fixture List

Group G

13-Jun-06	Stuttgart	FRA:SUI	0 : 0
13-Jun-06	Frankfurt	KOR:TOG	2 : 1
18-Jun-06	Leipzig	FRA:KOR	
19-Jun-06	Dortmund	TOG:SUI	
23-Jun-06	Cologne	TOG:FRA	
23-Jun-06	Hanover	SUI:KOR	

(1) Using the two results provided in the fixture list, fill in the log table.

(2) Can each team still win the group at this stage in the competition? Explain your answer.

(3) According to the fixture list, France and Korea drew their game one goal each and Togo lost to Switzerland 0 goals to 2. Fill in these results in the fixture list and the log table.

(4) Can each country still win? Explain your answer.

(5) What are the three possible outcomes in the match between Korea and Switzerland?

(6) Discuss how the outcomes of the game between Korea and Switzerland could affect France's chance of being top of this group. Discuss all three of the possible outcomes mentioned in question (5).

(7) The results of the final matches were as follows: France beat Toga 2:0 and Switzerland won their match two goals to nil. Fill in these results in the fixture list and the log table. Who was on the top of this group and who came second?

Solution

(1) Log table after first round of matches:

Team	MP	W	D	L	GF	GA	Pts
France	1	-	1	-	-	-	1
Korea	1	1	-	-	2	1	3
Switzerland	1	-	1	-	-	-	1
Togo	1	-	-	1	1	2	0

(2) Yes, they do. Since each team still has two games to play each team can potentially earn 6 points so each team can theoretically still come first or second in the group.

(3) Fixture list and log table after the second round of matches:

18-Jun-06	Leipzig	FRA:KOR	1:1
19-Jun-06	Dortmund	TOG:SUI	0:2

Team	MP	W	D	L	GF	GA	Pts
France	2	—	2	-—	1	1	2
Korea	2	1	1	—	3	2	4
Switzerland	2	1	1	—	2	—	4
Togo	2	—	—	2	1	4	0

(4) Each country must still play one more game and the maximum number of points they can get from that game is 3. It is clear that Togo can no longer catch up to Korea and Switzerland and cannot qualify for the next round – but there is a consolation: Togo could, however, still place above France. Korea; Switzerland and France still have a chance of being top of this group.

(5) Korea wins, Switzerland wins or the game is drawn.

(6) If either Korea or Switzerland wins they will earn 3 points and have seven in total making it impossible for France to win the group. However, by winning their game France would still place second. If the game between Korea and Switzerland is drawn then they each get 1 point taking them to a total of 5 points and if France wins their game they will also be on 5 points and the group leader will be decided on goal difference.

(7) Fixture list and log table after the last round of matches:

18-Jun-06	Leipzig	FRA:KOR	1:1
19-Jun-06	Dortmund	TOG:SUI	0:2

Team	MP	W	D	L	GF	GA	Pts
France	3	1	2	—	3	1	5
Korea	3	1	1	1	3	4	4
Switzerland	3	2	1	—	4	—	7
Togo	3	—	—	3	1	6	0

Switzerland was top of the group and France came second.

PAIRS

SELF ASSESSMENT

Activity

1. Below is the Group E fixture list for the same tournament:

FIFA WORLD CUP GERMANY 2006

Fixture List

Group G

12-Jun-06	Hanover	ITA:GHA	2:0
12-Jun-06	Gelsenkirchen	USA:CZE	0:3
17-Jun-06	Kaiserlauten	ITA:USA	
17-Jun-06	Cologne	CZE:GHA	
22-Jun-06	Hamburg	CZE:ITA	
22-Jun-06	Nuremberg	USA:GHA	

1.1 Using the two results provided in the fixture table, fill in the log table below:

Log Table

Group E

Team	MP	W	D	L	GF	GA	Pts
Italy							
Ghana							
USA							
Czechoslovakia							

1.1 Does each team still have a chance of winning the group at this stage in the competition? Explain your answer.

1.2 According to the fixture list, Italy and the USA drew their match one all and Ghana beat Czechoslovakia two nil. Fill in these results on both the fixture list and the log table.

1.3 Can each country still win and who could come second? Explain your answer in detail giving as many possible scenarios as possible.

1.4 The results of the final matches were as follows: Italy beat Czechoslovakia 2:0 and Ghana beat the USA 2:1. Fill in these results on the fixture list and the log table. Who was top of Group E and who came second?

DATA HANDLING

Summary

Lesson 47

Learning Outcomes and Assessment Standards

Learning Outcome 4
Data Handling
The learner is able to collect, summarise, display and analyse data and to apply knowledge of statistics and probability to communicate, justify, predict and critically interrogate findings and draw conclusions.
Note: Assessment Standard 11.4.4. is the focal Assessment Standard of LO4. The other LO4 Assessment Standards serve to develop the skills that will enable learners to achieve this one.
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Investigate a problem on issues such as those related to:
- Social, environmental and political factors.
- People's opinions.
- Human rights and inclusivity by:
- Collecting or finding data by appropriate methods (e.g. interviews, questionnaires, the use of data bases) suited to the purpose of drawing conclusions to the questions.
- Using appropriate statistical methods.
- Selecting a representative sample from a population with due sensitivity to issues relating to bias.
- Comparing data from different sources and samples.

AS 11.4.2
Select, justify and use a variety of methods to summarise and display data in statistical charts and graphs inclusive of:
- Tallies.
- Tables.
- Pie charts.
- Single and compound bar graphs.
- Line and broken-line graphs.

AS 11.4.3
Understand that data can be summarised and compared in different ways by calculating, and using measures of central tendency and spread (distribution), for more than one set of data inclusive of the:
- Mean.
- Median.
- Mode.
- Range.

AS 11.4.5
Work with simple notions of likelihood/probability in order to make sense of statements involving these notions.
AS 11.4.6
Demonstrate an awareness of how it is possible to use data in different ways to justify opposing conclusions.

Overview

Over the last 15 lessons we have been working on the topic of Data Handling. In this lesson we are going to summarise the key concept that we hope you have learnt.

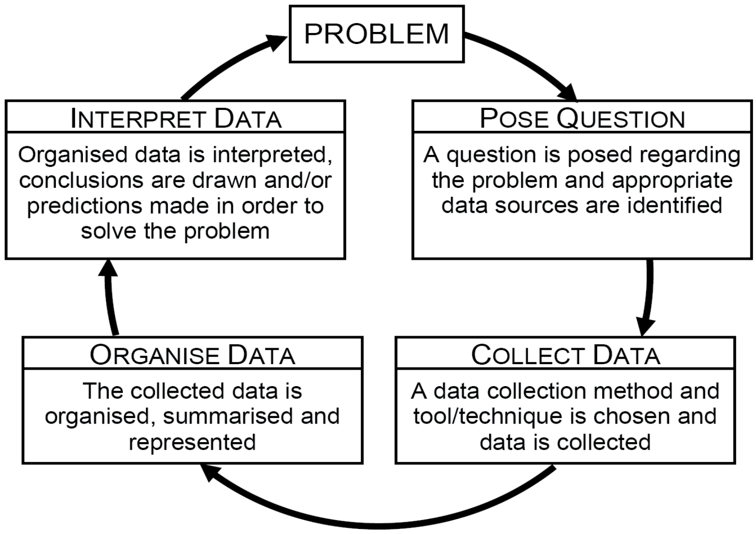
Lesson

Methods and worked examples

Data handling

The main message that we have been developing throughout these lessons is that information is generated from data through a process, a process we have called the Data Handling Cycle. It is important that you understand what happens at each stage of the cycle because at each stage the researcher makes choices and these choices impact on the conclusions that can be drawn from the process. This means that you need to understand how those choices can impact on the conclusions and this will in turn enable you to critique information with which you are presented.

The data handling cycle is shown below.



At the heart of the cycle is the problem – something that we'd like to investigate.

<u>Stage 1:</u>

Having established the problem and expressed it by means of a question, we need to identify appropriate data sources. Identifying appropriate data sources may involve sampling. Sampling is necessary when we are unable to survey the entire population.

Definitions:

A population:

- The collection of all the items about which we want to know something (Note: this need not be only human beings e.g. it could be cars in a car park).

A sample:

- The selection of part (sample) of the population that we choose with the purpose of making conclusions about the population.

The purpose of taking a sample is to make conclusions about the population which you cannot survey. Crucial to the success of the research process is ensuring that this sample is as representative of the population as possible.

There are different techniques of sampling (in our lessons we looked at):

- Convenience sampling
- Random sampling

- Systematic sampling
- Stratified sampling

The choice of sampling technique, the size of the sample and many other factors will impact on the conclusions we can make (a) about the population and (b) from the study itself.

Stage 2:

Having determined the question we want to ask, the next challenge is to collect the data. There are different ways of doing so. These include:

- Observations;
- Questionnaires;
- Interviews;
- Surveys

The data collection technique that we use can have a huge impact on both the validity of the data we collect and the conclusions that we can draw.

This point is illustrated by the hand washing survey which we referred to in an earlier lesson. In one part of the study participants were interviewed about their hand washing habits. In the American version of the study some 95% of respondents said that they washed their hands after using a public bathroom. However, when the researchers actually observed people in public restrooms they found that only 83% of the people washed their hands.

The advantages and disadvantages of each method are given below:

Observation:

This means actually watching something happen e.g. counting people who wash their hands.

- Advantages:
    - Actual observation of a phenomenon.
- Limitations:
    - Time consuming
- Application:
    - Recording the results of an experiment
    - Counting events.

Interviews:

- Advantages:
    - Allows the interviewer to clarify responses to questions.
    - High participation rate.
- Limitations:
    - Time consuming
    - Influence of interviewer.
    - Reliability of responses.
- Application:
    - Small scale detailed/in-depth investigations.

Questionnaires/surveys:

- Advantages:
    - Permits anonymity which may contribute to more sincere (honest) responses.
    - Time efficient.
- Limitations:
    - Possible misinterpretation of questions by respondents.
    - Unwillingness of respondents to complete the questionnaire.
- Application:
    - Large scale investigations.

Using available information:

- Advantages:
    - The information already exists.
- Limitations:
    - The existing data may not be sufficiently detailed.
- Application:
    - Looking for historical trends.

Stage 3:

Organising data involves at least three different steps:

- Organising data;
- Summarizing data; and
- representing the data

There are two different types of data to take note of when organising data:

- Categorical data – data with labels rather than numbers e.g. the club that a runner belongs to or the sex of a person; and
- Quantitative or numerical data. There are two types of numerical data:
    - Discrete data – data that can only have a countable value e.g. the number of children in a family;
    - Continuous data – data that can have any value in a range of values e.g. the height of a person.

The kind of data we have impacts on the way we organise data. For example we cannot group categorical data but without grouping it is impossible to organise continuous data – there would simply be too many different values.

In the same way, the kind of data that we have will also impact on the way we represent it. For example when we use a bar graph to represent categorical data we can draw the bars in any order and we leave gaps between them, but when representing continuous numerical data we use a histogram and we pay attention to the order of the groups and we ensure that the bars touch each other.

Having organised the data we want to summarise it – we want to take a large amount of data and use one or two numbers to represent all of it.

At this stage in the curriculum we use three statistics to summarise the data:

- the mean;

- the median; and
- the mode.

The <u>mean</u> is determined by adding up all the sample values and dividing by the number of values – it is often referred to as the “average”.

The <u>median</u> is that value which divides the ordered data into two groups of equal size. The one group contains values less than the median and the other group contains values greater than the median.

The <u>mode</u> is the most frequently occurring value in the data. It is possible for there to be two or more modes in a set of data.

The three above mentioned measures of central tendency are known as summary statistics as they summarise the data into a single value.

We use graphs to represent the data. Not only can we use different graphs to create desired impressions but we are also able to exaggerate (misrepresent) those impressions by the way in which we draw the graph.

We expect you to be able to draw and interpret the following kinds of graphs:

- pie charts;
- bar graphs;
- histograms; and
- line and broken line graphs.

Among the bar graphs and histogram graphs we get:

- single bar graphs;
- multiple bar graphs; and
- compound bar graphs.

<u>Stage 4:</u>

The final stage of the data handling cycle is interpreting the data that we have collected, organised, summarised and represented in order to answer the question that we posed in the first place. It is important that as much as you should be able to collect data, summarise and represent it, you should be able to critique and interpret it. This means that you should be aware of sources of error and bias in order to draw conclusions on the problem being investigated, make predictions and to critique other interpretations.

Likelihood

In the last three lessons on Data Handling we turned our attention to a related topic: likelihood or probability. Probability is that branch of mathematics which is concerned with trying to make predictions about the future.

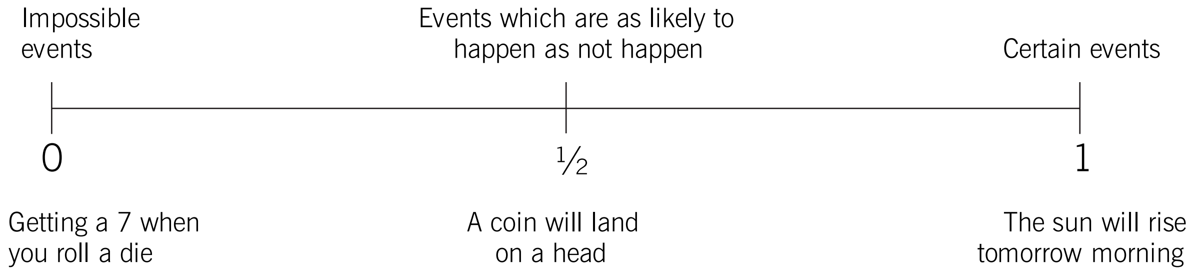
We define probability as:

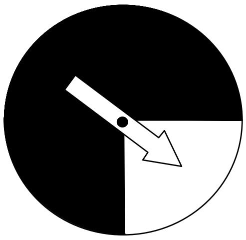
- the measure of the likelihood of an event occurring – a measure that ranges from 0 to 1.

We say that events which:

- cannot happen (impossible events), have a probability of 0;
- will definitely happen (certain events), have a probability of 1; and
- are just as likely to happen as they are likely not to happen, have a likelihood of 12 or 0,5 or 50%.

These numbers give rise to what we call the probability scale. See below.





There are two ways of developing expressions of likelihood:

- Theoretical expressions of likelihood – determined by studying the structure of a situation. We used the spinner alongside to illustrate theoretical probability. We said that the likelihood of landing on a shaded region is 34.
- experimental expressions of likelihood – determined by gathering a very large amount of data, analysing it and then predicting the future based on the results on the analysis. We illustrated this by looking at the data on road deaths in the USA. We saw how the data formed a trend and how that trend influenced insurance premiums and car hire rental policies.

In both cases it is very important to understand that our predictions can only predict long term trends. In the case of the fatalities we can only predict that in future there will be more deaths within a certain age group than some other one – but we cannot predict who will die. In the case of the spinner we are able to predict that three quarters of a very large number of spins will land in the shaded region – we can make no prediction about the next spin or about any spin for that matter. And even if the spinner lands in the shaded region three times in a row we still have no greater confidence about the next spin.

PAIRS

PROJECT

Activity

1. Collect at least three newspaper/magazine articles that report on surveys. For each article comment on evidence of the four stages of the data handling cycle described in the report.
2. Collect at least one newspaper/magazine article which includes an expression of likelihood. Comment on the following:
   - 1.1 The type of likelihood – experimental/theoretical/misuse of language.
   - 1.2 The implication of the expression; in other words what is meant by the expression of likelihood or how it may impact on ones behaviour.

SPACE, SHAPE AND MEASUREMENT

Reading maps

Lesson 48

Learning Outcomes and Assessment Standards

Learning Outcome 3
Space, Shape and Measurement
The learner is able to measure using appropriate instruments, to estimate and calculate physical quantities, and to interpret, describe and represent properties of and relationships between 2-dimensional shapes and 3-dimensional objects in a variety of orientations and positions.
Assessment Standard
AS 11.3.4
Use grids and maps, and compass directions, in order to:

- Determine locations.
- Describe relative positions.

Overview

This is the start of our study of a new topic – space; shape and measurement. This lesson is the first of four lessons that will deal with the various stages of planning a trip.

In this lesson we will look at the skill of reading a map.

Lesson

Methods and worked examples

There are lots of different maps that are available to us. A road atlas of South Africa has a different page for each province and in the index it lists place names i.e. the names of towns and cities. It does not show street names. You will need a more detailed map if you want to find a particular street in a town or city. These maps are called Street Guides.

Worked example

In the problem that we deal with in the video; Frank needs to get from his home to his work everyday and we use an appropriate map to determine a route between his home and work. The addresses are for the problem are as follows:

<u>Home</u>

Number 13

NY 123

Guguletu

<u>Work</u>

Number 43

Leo Road

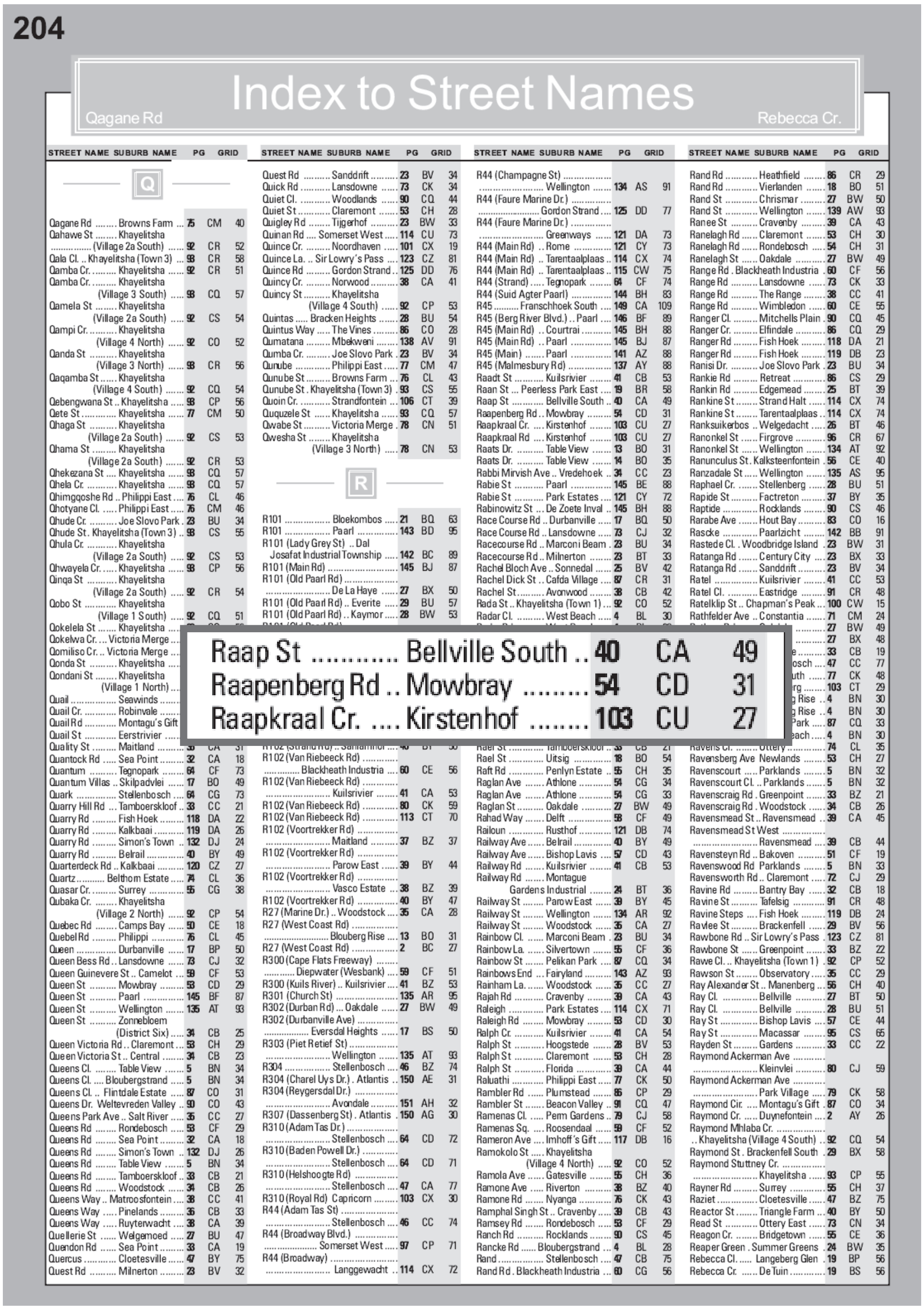
Elfindale

Solution

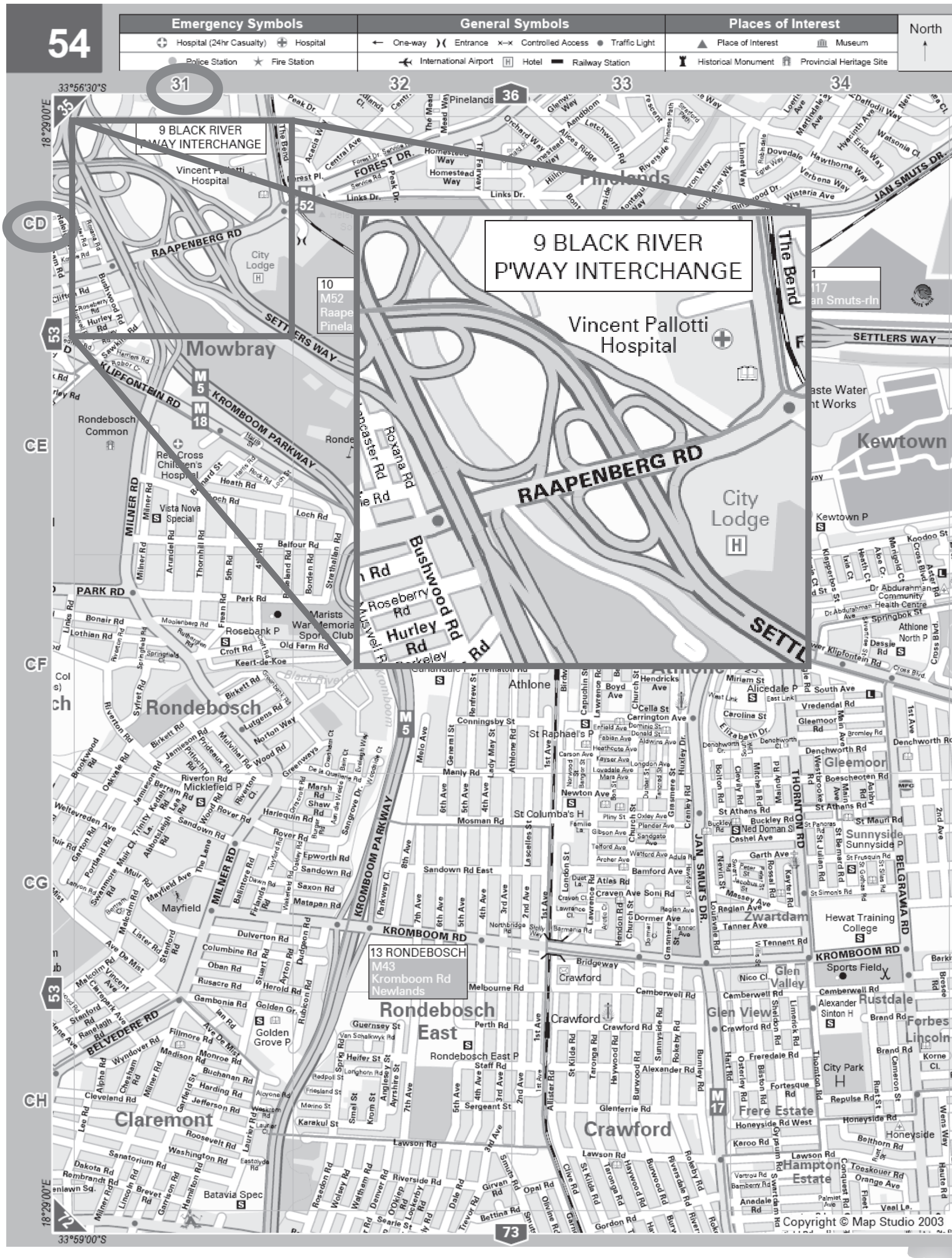
Rather than supplying all of the pages from the map book, we will illustrate the steps used to complete this task for a different road: Raapenberg Rd.

The use of the word appropriate in the question is quite deliberate as Frank will need to use a street guide of Cape Town in order to solve his problem.

To start out with you would look in the index of the map book to find out where a street is. Below is an example of a page out of the Cape Town Street Guide. The highlighted section shows you what information you are given: (a) the suburb that the street is found in: Mowbray; (b) the page number (bold) of the map book where you will find the street: page 54; and (c) the coordinates for the grid position on that page: CD 31.

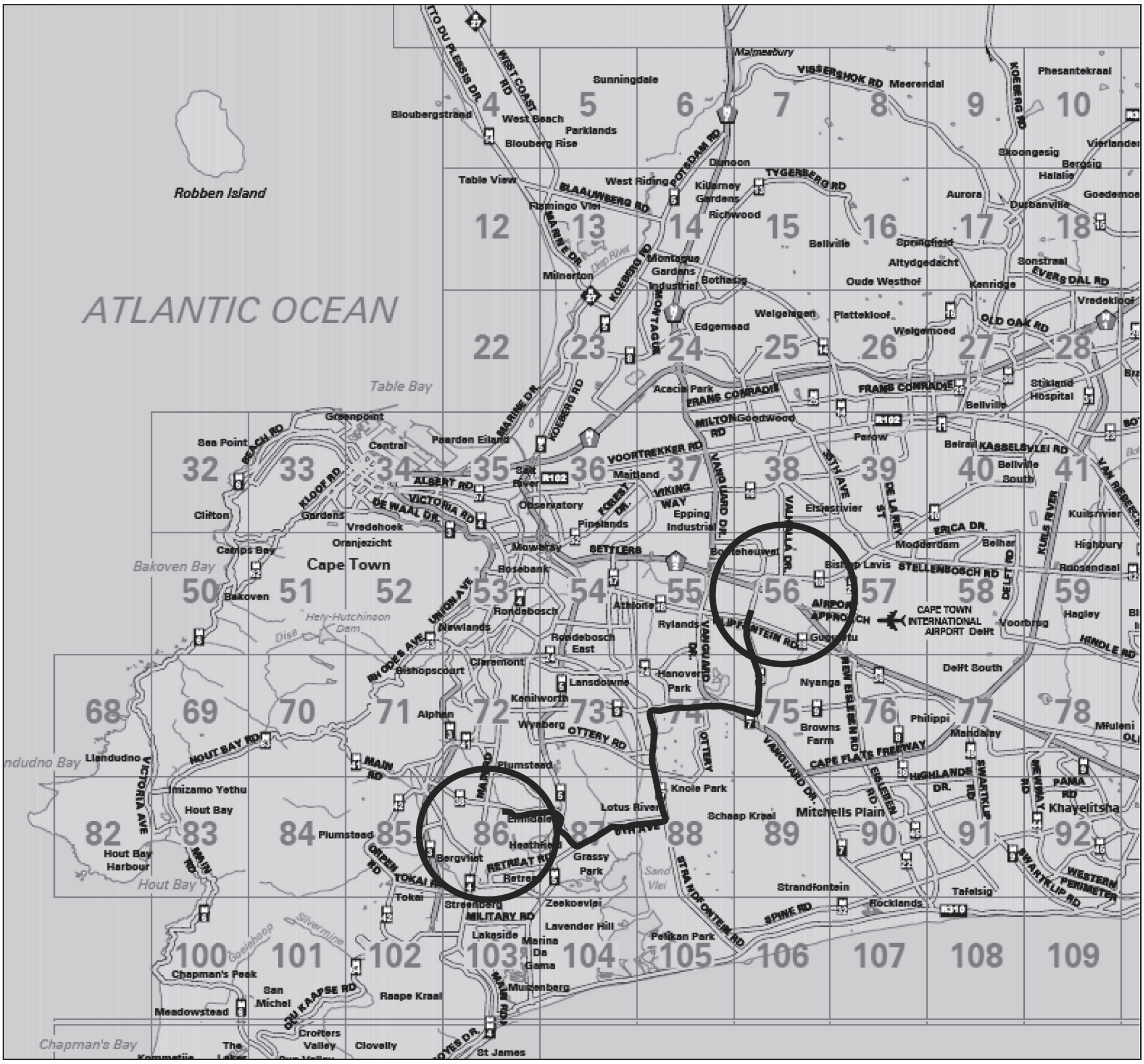


Below is page 54 from the street guide. The letters of the grid reference are found going down the left hand side of the page (vertical) and the number of the grid reference are found along the top of the page (horizontal).



Raapenberg road has a grid reference of CD 31. These references have been circled on the map above. This means that Raapenberg Road is found in the top left hand square. Although we cannot know the exact position of the house because this map does not show such detail the road is clearly visible in the block – especially in the enlarged version.

Having used the street guide to determine the location of his house and place of work, he needs to get a sense of where the two places are relative to each other. This is best done using the map at the front of the book (shown below) as this shows the relation of the pages to each other and hence of the two places relatives to each other. Frank's home is on page 56 and his place of work page 86.



Using summary page (at the front of the book) Frank can get a sense of the relative positions of his house and place of work and can use the main roads (shown on this map) to get a sense of a possible route between the two places. One possible route is indicated on the map and what Frank needs to do now is go to the relevant pages in the street guide and develop a more detailed route.

A summary of the process is as follows:

- Choose an appropriate map book.
- Use the index to get the page number and grid reference.
- Use the global map in the front of the map book to get a sense of how to travel.
- Use the street maps to determine the detailed route with directions.

Activity

PAIRS

SELF ASSESSMENT

1. Use the index to locate the following streets on the street map. Write down the grid reference and suburb and mark the street on the map.
   1.1 Ranelagh Road.
   1.2 Raglan Road.
   1.3 Describe how you could possibly travel from Ranelagh Road to Raglan Road.
2. Source a street map for your town or city (you may need to make use of the local library). Determine the grid reference for your home and the homes of three other members in your class. Trace a route to each of their homes from your home.

Lesson 49

SPACE, SHAPE AND MEASUREMENT

Working with scale

Learning Outcomes and Assessment Standards

Learning Outcome 3
Space, Shape and Measurement
The learner is able to measure using appropriate instruments, to estimate and calculate physical quantities, and to interpret, describe and represent properties of and relationships between 2-dimensional shapes and 3-dimensional objects in a variety of orientations and positions.
Assessment Standard
AS 11.3.1
Solve problems in 2-dimensional and 3-dimensional contexts by:
- Estimating, measuring and calculating values which involve:
- Lengths and distances.
- Perimeters and areas of polygons.
- Volumes of right prisms and right circular cylinders.
- Surface areas of right prisms and right circular cylinders.
- Making adjustments to calculated values to accommodate measurement errors and inaccuracies due to rounding.

AS 11.3.2
Convert units of measurement between different scales and systems using conversion tables provided.
AS 11.3.3
Use and interpret scale drawings of plans to:
- estimate and calculate values according to scale.

AS 11.3.4
Use grids and maps, and compass directions, in order to:
- Determine locations.
- Describe relative positions.

Overview

In the previous lesson we looked at the process of finding a route between two places using a street guide (see pictures of the street guide in the previous lesson). In this lesson we will look at the next stage of planning a trip which involves working with scale.

Now that we have a route we need to try to work out how long it will take us to complete the journey and in order to do that we need to have a sense of how far we need to travel.

LC DVD

Lesson

Methods and worked examples

The scale of the street guide is 1 in 20 000. This means that every unit on the map corresponds to 20 0000 of those units on the ground i.e.1 cm on the map corresponds to 20 000 cm on the ground and 1 mm on the map corresponds to 20 000 mm on the ground.

We can convert these measurements to metres in the following way:

Map	Ground	
1 cm	20 000 cm	100 cm = 1 m
	200 m	1 000 m = 1 km
	0,2 km	

The grid lines on the Street Guide map are 5 cm apart so that means that they will be 5 × 20 000 cm apart on the ground. This means that the grid lines are 1 000 m apart or phrased differently the distance between the grid lines on the map corresponds to 1 km on the ground and each of the blocks corresponds to an area of 1 km by 1 km.

INDIVIDUAL

FORMATIVE ASSESSMENT

Worked example

The distance from Frank's home to his work was measured with a piece of string and found to be about 110,5 cm. Determine the traveling distance on the ground and hence estimate the time taken for the trip.

Solution

Map		Ground
110,5cm		(110,5 × 20 000) cm
		(110,5 ×0,2) km
	=	22,1 km

<u>Note</u>: You might not be sure when measuring the distance with a piece of string where Frank's house is in the road that he lives. This means that the measured distance could be inaccurate. However, in real terms this might not be too much of a problem. This is illustrated below:

If the measured distance was out by 2 cm it would translate to (2 × 200) m = 400m on the ground. 400 m out of 22,1 km is only about 2% of the distance which would not make too much of a difference to the travelling time.

However, Road atlases have a scale of 1 in 1 500 000 so a 1 cm inaccuracy in measurement translates as follows:

Map	Ground	
1 cm	1 500 000 cm	100 cm = 1 m
	15 000 m	1 000 m = 1 km
	15 km	
1 mm	1,5 km	

So a distance of 2 cm would translate to 30 km on a map in a road atlas.

Placing string along the route in itself also causes distortions and hence errors, which is why maps with larger scale often have distance tables in them.

To estimate the time taken for the journey, we must fist decide on an average speed he will be able to travel. Although the speed limit is 60 km/h you can seldom reach that speed if you consider all the stop streets, traffic lights and turns along the journey, so we are going to do two calculations; one with an average speed of 50 km/h and secondly with an average speed of 40 km/h.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Time} = \frac{\text{Distance}}{\text{speed}}$$

Calculation with average speed of 50 km/h:

$$\text{Time} = \frac{22 \text{ km}}{50 \text{ km/h}}$$

$$= 0{,}44 \text{ h}$$

$$\approx 27 \text{ min}$$

Calculation with average speed of 40 km/h:

$$\text{Time} = \frac{22 \text{ km}}{40 \text{ km/h}}$$

$$= 0{,}55 \text{ h}$$

$$\approx 33 \text{ min}$$

A difference of 10km/h has quite an impact on the time taken to complete the journey.

Note: The precision with which we work has a greater or lesser impact depending on the scale of the map and a small difference in our estimated speed will also impact quite profoundly on the time taken to complete a journey.

INDIVIDUAL

SUMMATIVE ASSESSMENT

Activity

1 The distance from Frank's home to his friend's house was measured with a piece of string and found to be about 13,6 cm. Determine the traveling distance on the ground in km and hence estimate the time taken for the trip if he walks at an average speed of 10 min/km (scale 1:20 000).

2 A street guide has a scale of 1 : 20 000 and a road map has a scale of 1 : 1 500 000. If the string, measurement was out by 0,5 cm, calculate:

2.1 What distance this error represents on the street guide?

2.2 The percentage error that the above distance makes of the distance that Frank walked.

2.3 What distance this error represents on the road atlas?

SPACE, SHAPE AND MEASUREMENT

Choosing a train

Lesson **50**

Learning Outcomes and Assessment Standards

Learning Outcome 3
Space, Shape and Measurement
The learner is able to measure using appropriate instruments, to estimate and calculate physical quantities, and to interpret, describe and represent properties of and relationships between 2-dimensional shapes and 3-dimensional objects in a variety of orientations and positions.
Assessment Standard
AS 11.3.1
Solve problems in 2-dimensional and 3-dimensional contexts by:

- estimating, measuring and calculating values which involve:
  - Lengths and distances.
  - Perimeters and areas of polygons.
  - Volumes of right prisms and right circular cylinders.
  - Surface areas of right prisms and right circular cylinders.
  - Making adjustments to calculated values to accommodate measurement errors and inaccuracies due to rounding.

AS 11.3.2
Convert units of measurement between different scales and systems using conversion tables provided.
AS 11.3.3
Use and interpret scale drawings of plans to:

- estimate and calculate values according to scale.

AS 11.3.4
Use grids and maps, and compass directions, in order to:

- Determine locations.
- Describe relative positions.

Overview

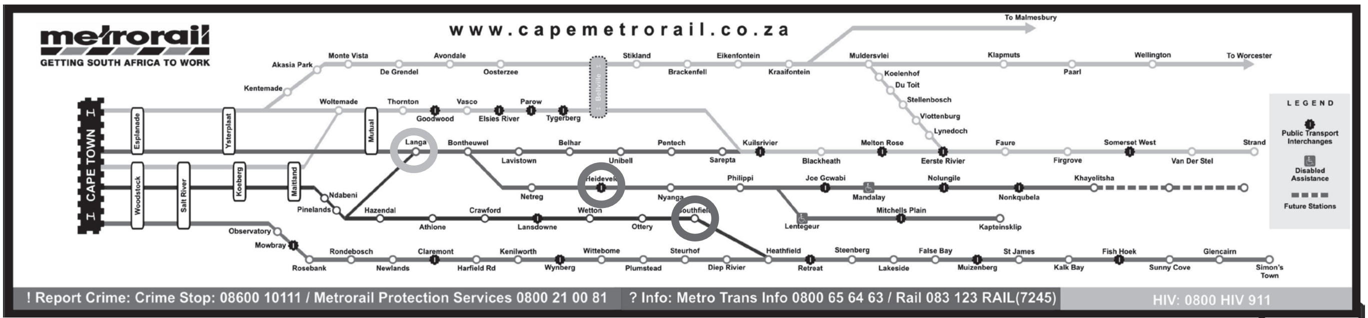
In the previous lessons we used a street guide to plan a route that you could drive between two places and we calculated the distance you would need to drive. However, if you do not have a car available you would need to investigate another form of transport. In this lesson we will look at an alternate manner of getting to work – namely choosing and taking a train.

Lesson

L C DVD

Methods and worked examples

Trains have their own maps which show the train services available. Below is an example of a train route map. It is the Cape Metro route map:



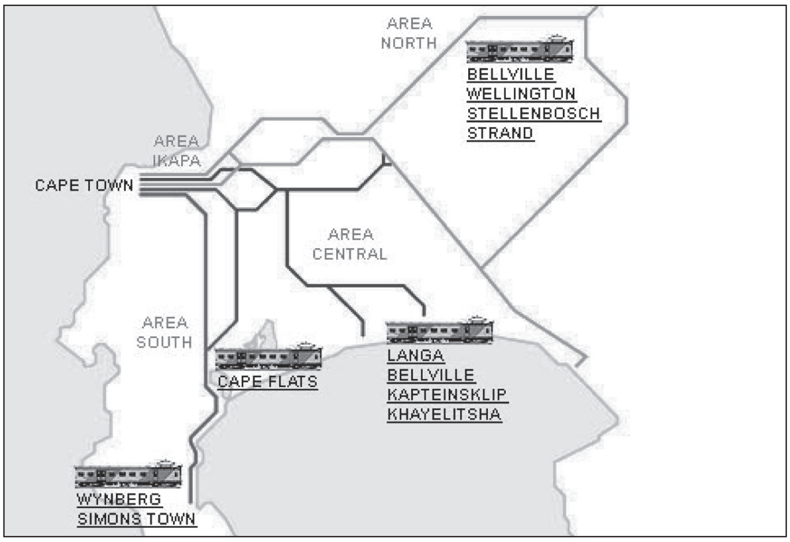
Note the following:

- There are different coloured lines – each representing a different train service.

All the routes start out from Cape Town.

- Along each line are little dots which represent the stations that the train will stop at.
- All the routes are depicted as straight lines. The route map makes no attempt to be realistic in terms of the relative geographic positions of the stations. It only lists the stations and the sequence in which they come in as economical a manner as possible i.e. a straight line

Below is another example of a route map. This map also shows the relative directions of the routes superimposed on a map of the peninsula. However, on this map the route names are given and not the stations that the train will stop at. For example: The blue route is called the Langa, Bellville, Kapteinsklip, Khayelitsha route.

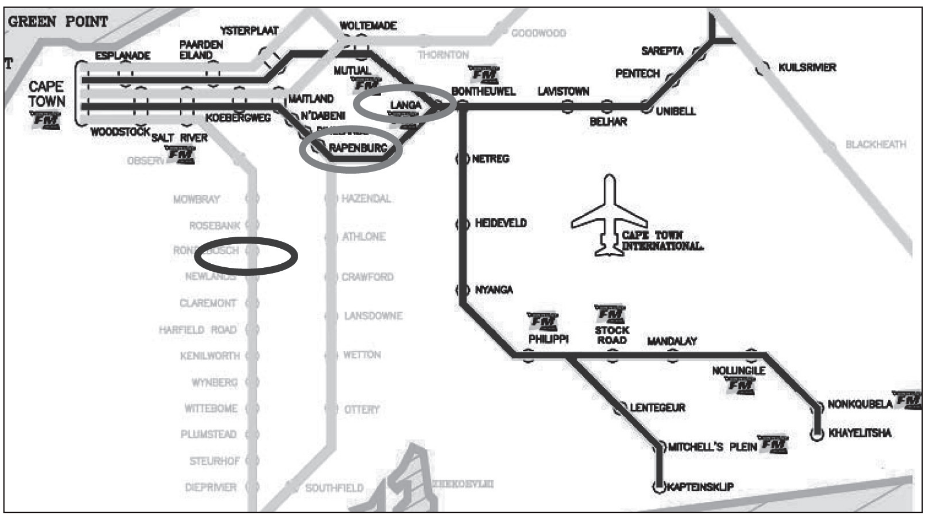


Both maps are useful in determining which train to catch.

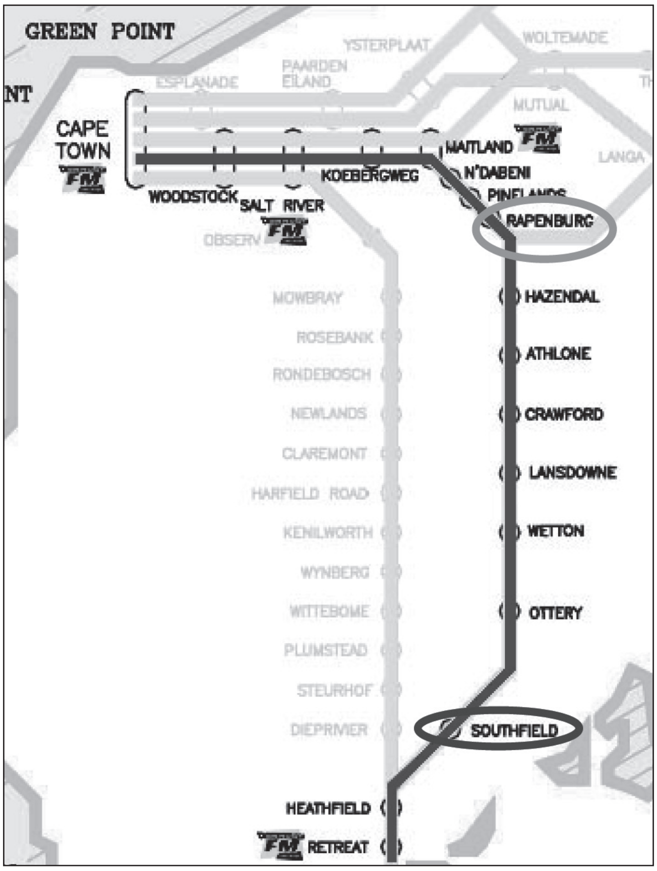
Worked example

We will continue with the problem from the previous lesson. This time we will find out how Frank can use the train to get from his house to his place of work. Use the detailed maps below to help you plan the route.

Blue Route:



Grey/red Route:



Solution

The first thing that we need to know is which station is closest to his house and which station is closest to his place of work. This can be found by looking in the Street Guide that we used in lesson 48. The station closest to his house is Heideveld Station and the station closest to work is Southfield Station.

The two stations are marked with a circle on the strip map. Heideveld (circled in red) is on the blue line and Southfield (circled in blue) is on the grey line. (This line is shown in red on the Peninsula route map).Two different coloured lines that tells us that Frank is not going to be able to go directly from the one station to the other on the same train – he will have to change services somewhere.

Frank needs to get on the blue train at Heideveld and take the train in the direction of Cape Town via Rapenberg. When it reaches Rapenburg he needs to change trains to the red line which will take him to the Southfield station.

Activity

INDIVIDUAL

SUMMATIVE ASSESSMENT

1. Use the route maps given above to determine the following routes. In each case name the route(s) that need to be taken and indicate clearly where any change of services take place. Give alternative routes if possible however only routes that are practical must be given.
   - 1.1 From Kuils River to Cape Town
   - 1.2 From Cape Town to Stellenbosch
   - 1.3 From Stellenbosch to Nyanga
   - 1.4 From Muizenberg to Stellenbosch

Lesson 51

SPACE, SHAPE AND MEASUREMENT

Using a train timetable

Learning Outcomes and Assessment Standards

Learning Outcome 3
Space, Shape and Measurement:
The learner is able to measure using appropriate instruments, to estimate and calculate physical quantities, and to interpret, describe and represent properties of and relationships between 2-dimensional shapes and 3-dimensional objects in a variety of orientations and positions.
Assessment Standard
AS 11.3.2
Convert units of measurement between different scales and systems using conversion tables provided.
AS 11.3.4
Use grids and maps, and compass directions, in order to:

- Determine locations.
- Describe relative positions.

Overview

In the previous lesson we used a train route map to work out which trains to catch from one place to another. In this lesson we will use a train timetable to decide which train to catch.

Lesson

Methods and worked examples

Train timetables are available at all train stations. Some pages from the Cape Metrorail timetable are shown below. Notice that there are two timetables for each service. This is because the trains travel in both directions. You must be careful that you select the correct timetable for your purposes.

FROM KHAYELITSHA - KAPTEINSKLIP - SAREPTA - LANGA TO CAPE TOWN

MONDAY TO FRIDAY

TRAIN NO.		9416	TWFS 9206	9004	9418	9508	TWFS 9422	9510	9006	9312	TWFS 9212
BELLVILLE				05:43					06:05		
SAREPTA				05:50					06:14		
PENTECH				05:53					06:17		
UNIBELL				05:55					06:19		
BELHAR				05:58					06:22		
LAVISTOWN				06:01					06:25		
KHAYELITSHA		05:35		..	05:45		05:55		..	06:05	
NONKQUBELA		05:38		..	05:48		05:58		..	06:08	
NOLUNGILE		05:41		..	05:51		06:01		..	06:11	
MANDALAY	A	05:45		..	05:55		06:05		..	06:15	
	D	05:46		..	05:56		06:06		..	06:16	
STOCK ROAD		05:48		..	05:58		06:08		..	06:18	
KAPTEINSKLIP		..	05:40	..	..	05:50	..	06:00	..	..	06:10
MITCHELLS PL.		..	05:43	..	..	05:53	..	06:03	..	..	06:13
LENTEGEUR	A	..	05:46	..	..	05:56	..	06:06	..	..	06:16
	D	..	05:47	..	..	05:57	..	06:07	..	..	06:17
PHILIPPI		05:51	05:51	..	06:01	06:01	06:11	06:11	..	06:20	..
NYANGA		..	05:56	..	06:06	06:06	06:16	06:16	..	..	06:23
HEIDEVELD		05:58	06:00	..	06:10	06:10	06:20	06:20	..	06:27	06:27
NETREG		..	06:04	..	06:14	06:14	06:24	06:24	..	..	06:31
BONTHEUWEL	A	06:04	06:07	06:05	06:17	06:17	06:27	06:27	06:29	06:33	06:34
	D	06:05	06:08	06:09	06:18	06:18	06:28	06:28	06:30	06:34	06:36
LANGA		06:08	06:11	06:12	06:21	06:21	06:31	06:31	06:33	06:37	06:39
MUTUAL		06:12	06:15	..	06:25	..	06:35	..	06:37	..	06:43
YSTERPLAAT		06:18	06:21	..	06:31	..	06:41	..	06:43	..	06:49
PAARDENEILAND		..	..	..	..	..	06:43	..	..	..	..
ESPLANADE		06:24	06:26	..	06:36	..	06:46	..	06:48	..	06:54
PINELANDS		..	..	06:17	..	06:26	..	06:36	..	06:42	..
NDABENI		..	..	06:19	..	06:28	..	06:38	..	06:44	..
MAITLAND		..	..	06:22	..	06:31	..	06:41	..	06:47	..
KOEBERG RD		..	..	06:24	..	06:33	..	06:43	..	06:49	..
SALT RIVER		..	..	06:27	..	06:36	..	06:46	..	06:52	..
WOODSTOCK		..	..	06:30	..	06:39	..	06:49	..	06:55	..
CAPE TOWN		06:28	06:31	06:34	06:41	06:43	06:51	06:53	06:53	07:01	06:59

TWFS : TRAIN WITHDRAWN FROM SERVICE 18 DECEMBER 2006 - 12 JANUARY 2007

FROM CAPE TOWN TO LANGA - LAVISTOWN - KAPTEINSKLIP - KHAYELITSHA

MONDAY TO FRIDAY

CAPE TOWN PLATFORM NO.		TWFS 21	19	16	20	15	18	17	19	20	16
TRAIN NO.		9219	9041	9567	9461	9317	9221	9569	9043	9463	9571
CAPE TOWN		17:02	17:10	17:10	17:15	17:22	17:25	17:30	17:36	17:40	17:43
WOODSTOCK		..	..	17:13	..	17:25	..	17:33	..	..	17:46
SALT RIVER		..	..	17:16	..	17:28	..	17:36	..	..	17:49
KOEBERG RD		..	..	17:18	..	17:30	..	17:38	..	..	17:51
MAITLAND		..	..	17:21	..	17:33	..	17:41	..	..	17:54
NDABENI		..	..	17:24	..	17:36	..	17:44	..	..	17:57
PINELANDS		..	..	17:26	..	17:38	..	17:46	..	..	17:59
ESPLANADE		17:06	17:14	..	17:19	..	17:29	..	17:40	17:44	..
PAARDENEILAND		..	..	..	..	..	..	..	..	..	..
YSTERPLAAT		17:11	17:19	..	17:24	..	17:34	..	17:45	17:49	..
MUTUAL		17:17	17:25	..	17:30	..	17:40	..	17:51	17:55	..
LANGA		17:22	17:30	17:31	17:35	17:43	17:45	17:51	17:56	18:00	18:04
BONTHEUWEL	A	17:25	17:33	17:34	17:38	17:46	17:48	17:54	17:59	18:03	18:07
	D	17:26	17:34	17:35	17:39	17:47	17:49	17:55	18:00	18:04	18:08
NETREG		17:30	..	17:38	17:42	17:50	17:53	17:58	..	18:07	18:11
HEIDEVELD		17:33	..	17:41	17:45	17:53	17:56	18:01	..	18:10	18:14
NYANGA		17:37	..	17:45	17:49	17:57	18:00	18:05	..	18:14	18:18
PHILIPPI		17:41	..	17:49	17:54	18:01	18:04	18:09	..	18:19	18:22
LENTEGEUR	A	17:46	..	17:54	..	..	18:09	18:14	..	..	18:27
	D	17:47	..	17:55	..	..	18:10	18:15	..	..	18:28
MITCHELLS PL.		17:50	..	17:58	..	..	18:13	18:18	..	..	18:31
KAPTEINSKLIP		17:53	..	18:01	..	..	18:16	18:21	..	..	18:34
STOCK ROAD			..		17:57	18:04			..	18:22	
MANDALAY	A		..		18:00	18:07			..	18:25	
	D		..		18:01	18:08			..	18:26	
NOLUNGILE			..		18:04	18:11			..	18:29	
NONKQUBELA			..		18:07	18:14			..	18:32	
KHAYELITSHA			..		18:11	18:18			..	18:36	
LAVISTOWN			17:39						18:05		
BELHAR			17:43						18:09		
UNIBELL			17:46						18:12		
PENTECH			17:49						18:15		
SAREPTA			17:52						18:18		
BELLVILLE			17:58						18:24		

TWFS : TRAIN WITHDRAWN FROM SERVICE 18 DECEMBER 2006 - 12 JANUARY 2007

RETREAT - CAPE TOWN (VIA CAPE FLATS)

MONDAYS TO FRIDAYS

TRAIN NO.	0540	0542	0544	0546	0548	0550	0552	0554	0556	0558
RETREAT	14:00	14:25	14:56	15:20						
HEATHFIELD	14:03	14:28	14:59	15:23	15:44	15:56	16:20	16:38	16:58	17:08
SOUTHFIELD	14:07	14:32	15:03	15:27	15:48	16:00	16:24	16:42	17:02	17:12
OTTERY	14:10	14:35	15:06	15:30	15:51	16:03	16:27	16:45	17:05	17:15
WETTON	14:13	14:38	15:09	15:33	15:54	16:06	16:30	16:48	17:08	17:18
LANSDOWNE	14:15	14:40	15:11	15:35	15:56	16:08	16:32	16:50	17:10	17:20
CRAWFORD	14:17	14:42	15:13	15:37	15:58	16:10	16:34	16:52	17:12	17:22
ATHLONE	14:20	14:45	15:16	15:40	16:01	16:13	16:37	16:55	17:15	17:25
HAZENDAL	14:22	14:47	15:18	15:42	16:03	16:15	16:39	16:57	17:17	17:27
PINELANDS	14:25	14:50	15:21	15:45	16:06	16:18	16:42	17:00	17:20	17:30
NDABENI	14:27	14:52	15:23	15:47	16:08	16:20	16:44	17:02	17:22	17:32
MAITLAND	14:31	14:56	15:27	15:51	16:12	16:24	16:48	17:06	17:26	17:36
KOEBERG RD	14:33	14:58	15:29	15:53	16:14	16:26	16:50	17:08	17:28	17:38
SALT RIVER	14:35	15:00	15:31	15:55	16:16	16:28	16:52	17:10	17:30	17:40
WOODSTOCK	14:39	15:04	15:35	15:59	16:20	16:32	16:56	17:14	17:34	17:44
CAPE TOWN	14:42	15:07	15:38	16:02	16:23	16:35	16:59	17:17	17:37	17:47

CAPE TOWN - RETREAT VIA CAPE FLATS

MONDAYS TO FRIDAYS

CAPE TOWN PLATFORM NO.	15	14	13	15	* 7	12	16	15	13	13
TRAIN NO.	0501	0503	0505	0507	0109	0509	0511	0513	0515	0517
CAPE TOWN	05:02	05:20	05:34	05:50	05:58	06:05	06:12	06:22	06:37	06:54
WOODSTOCK	05:05	05:23	05:37	05:53	06:01	06:08	06:15	06:25	06:40	06:57
SALT RIVER	05:08	05:26	05:40	05:56	06:04	06:11	06:18	06:28	06:43	07:00
KOEBERG RD	05:10	05:28	05:42	05:58	06:06	06:13	06:20	06:30	06:45	07:02
MAITLAND	05:12	05:30	05:44	06:00	06:08	06:15	06:22	06:32	06:47	07:04
NDABENI	05:15	05:33	05:47	06:03	06:11	06:18	06:25	06:35	06:50	07:07
PINELANDS	05:18	05:36	05:50	06:06	06:14	06:21	06:28	06:38	06:53	07:10
HAZENDAL	05:20	05:38	05:52	06:08	06:16	06:23	06:30	06:40	06:55	07:12
ATHLONE	05:23	05:41	05:55	06:11	06:19	06:26	06:33	06:43	06:58	07:15
CRAWFORD	05:25	05:43	05:57	06:13	06:21	06:28	06:35	06:45	07:00	07:17
LANSDOWNE	05:27	05:45	05:59	06:15	06:23	06:30	06:37	06:47	07:02	07:19
WETTON	05:30	05:48	06:02	06:18	06:26	06:33	06:40	06:50	07:05	07:22
OTTERY	05:32	05:50	06:04	06:20	06:28	06:35	06:42	06:52	07:07	07:24
SOUTHFIELD	05:36	05:54	06:08	06:24	06:32	06:39	06:46	06:56	07:11	07:28
HEATHFIELD	05:40	05:58	06:12	06:28	06:36	06:43	06:50	07:00	07:15	07:32
RETREAT					06:38		06:52			

* Train operates from Cape Town to Fish Hoek.

Train timetables can also be downloaded from the Internet – as shown in the episode.

Worked example

INDIVIDUAL

FORMATIVE ASSESSMENT

We will continue with the problem from the previous lesson – namely to get Frank to work by means of the train. Use the timetable provided to determine which train(s) he can to catch:

- In the morning in order for him to get to work by 07:30; and
- In the evening once he has finished work at 17:00.

Solution

In our planning of a journey we determined from the train maps in the previous lesson that in going to work Frank would need to first travel along the blue route (towards Cape Town) from Heideveld to Rapenberg and then to change trains to the red route and travel from Rapenburg to Southfield.

In order to determine which train to use, it is better to start with the end in mind. If he wants to get to work by 07:30 he will need to reach Southfield by at least 07:20 in order to have time to walk from the station to work.

Use the timetable that says 'from Cape Town to the Cape Flats'. There are two options; one arriving at 07:28 and the other at 07:11. The 07:11 would be the better choice. This means he must catch train number 515.

Next we need to see at what time train 515 departs from Rapenburg, the station where we want to join the services. The timetable does not show Rapenburg so we will have to catch the train from Pinelands instead. Train number 515 which gets to Southfield at 07:11 passes through Pinelands at 06:53.

Now we need to find a train service from Heideveld to Pinelands that will get Frank to Pinelands before 06:53. We must use the timetable that says that this is in the direction from the various stations to Cape Town.

The best option is the train that passes through Heideveld at 06:27and stops in Pinelands at 06:42, which means that Frank will have to wait for about ten minutes for the connecting train. The train number is 9312.

In the evening after work Frank will have to reverse the sequence of his morning trip. The first timetable that we need is the timetable for the Cape Flats train going in the direction of Cape Town. If he walks quickly from work he should be able to get on the one that leaves Southfield at 17:12 – train number 558: it passes through Pinelands at 17:20.

Now we need the timetable for the blue train travelling away from Cape Town. The ideal train departs from Pinelands at 17:26 and reaches Heideveld at 17:41 (train number 9567), leaving Frank with a short walk home – he should be there before 18:00. Frank has had a long day as he left home at 06:30 and returned again at 18:00.

INDIVIDUAL

SELF ASSESSMENT

Activity

1. Use the train timetables given above to determine which trains you should take in the following situations. Give more than one option if possible.
   - 1.1 You live near Philippi station and work near Woodstock station. You need to arrive at Woodstock station by at least 07:00.
   - 1.2 You can get to Woodstock station by 17:00 and want to return home.
   - 1.3 You live near Philippi station and work near Lansdowne station. You need to arrive at Lansdowne station by at least 07:00.
   - 1.4 You can get to Lansdowne station by 17:00 and want to return home.

SPACE, SHAPE AND MEASUREMENT

House plans I

Lesson 52

Learning Outcomes and Assessment Standards

Learning Outcome 3
Space, Shape and Measurement:
The learner is able to measure using appropriate instruments, to estimate and calculate physical quantities, and to interpret, describe and represent properties of and relationships between 2-dimensional shapes and 3-dimensional objects in a variety of orientations and positions.
Assessment Standard
AS 11.3.3
Use and interpret scale drawings of plans to:
- estimate and calculate values according to scale.

Overview

In the previous four lessons we worked with one representation of the world – a map. We used street maps and train route maps to plan a trip.

In this lesson we will look at another kind of representation of the world – house or building plans.

Lesson

House plans are in many ways exactly the same as street maps – they are representations of the world drawn to scale. Street maps help us to locate places in relation to each other; house plans help us to anticipate the relative positions of rooms as well as knowing their sizes and other features.

Methods and worked examples

A floor plan gives you a sense of the rooms and their relative positions, it does not, however, give you a feeling of the house itself.

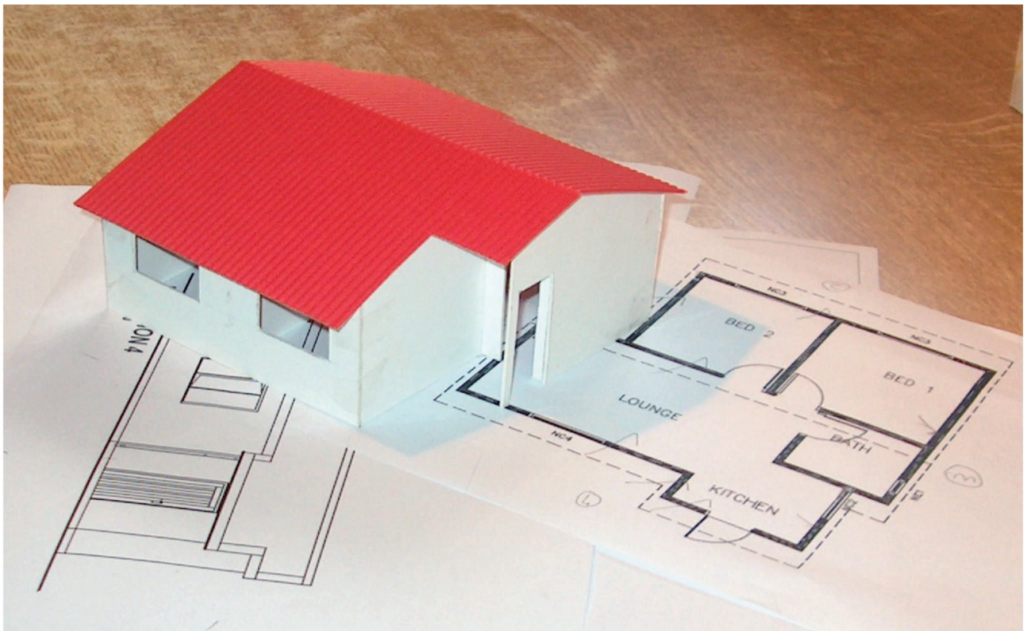
For this lesson and the two that follow it we will use house plans for houses being built as part of the Cosmo City Project outside Johannesburg. These houses have been marketed to families whose gross monthly income does not exceed R7 000. People who qualify for a mortgage bond and a Government subsidy may apply for these homes.

3 000 of these homes will be built in Cosmo City and in total more than 11 000 homes will be built there.

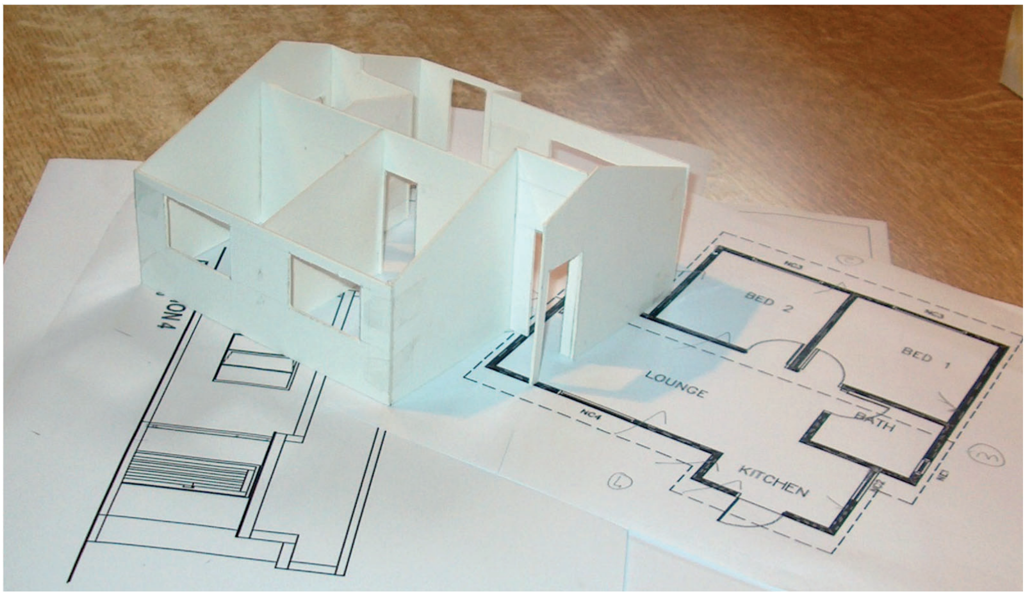
Picture 1 on the next page shows a model of a Cosmo City house and the plan of the house. The model helps us to get a sense of what the house will look like whereas the plans are more like a footprint that the house leaves.

Picture 2 shows the house without the roof. This shell will fit exactly onto the floor plan or footprint of the house.

Picture 1



Picture 2

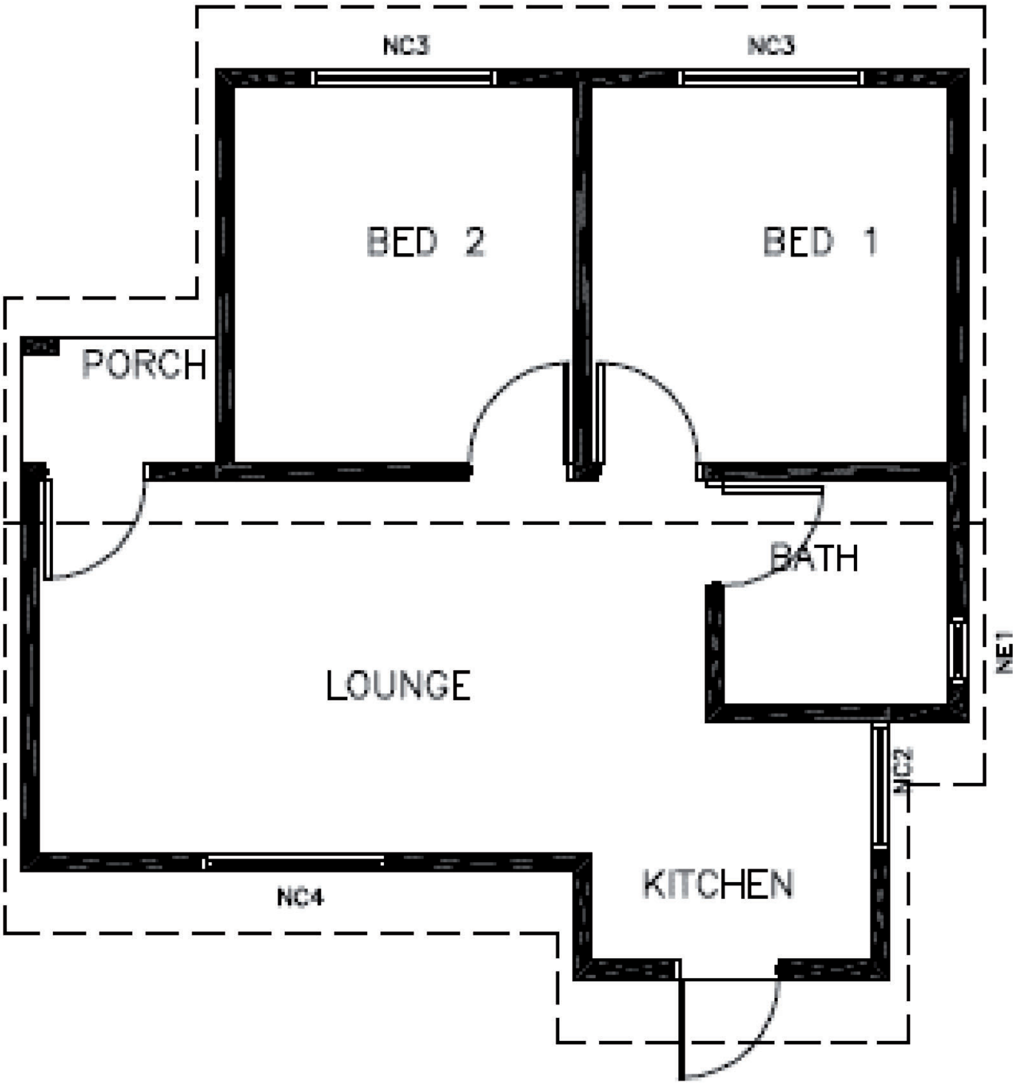


INDIVIDUAL

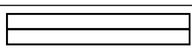
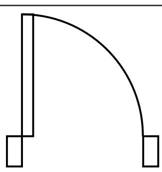
FORMATIVE ASSESSMENT

Worked example

Here is a plan of the house:

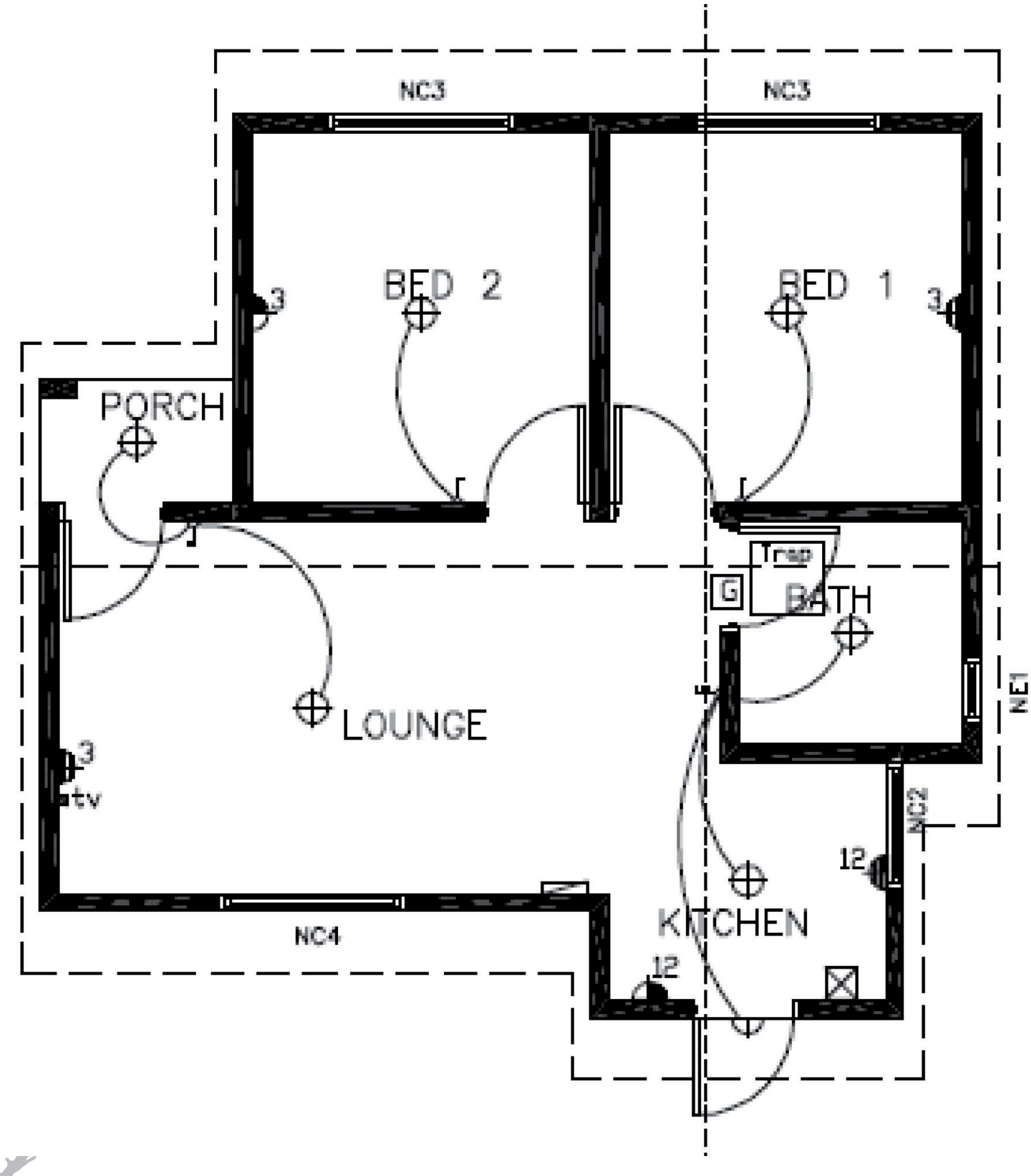


There are various drawing conventions which architects use in their plans. Find the following on the plan and circle them in the colours specified and then complete the last column of the table.

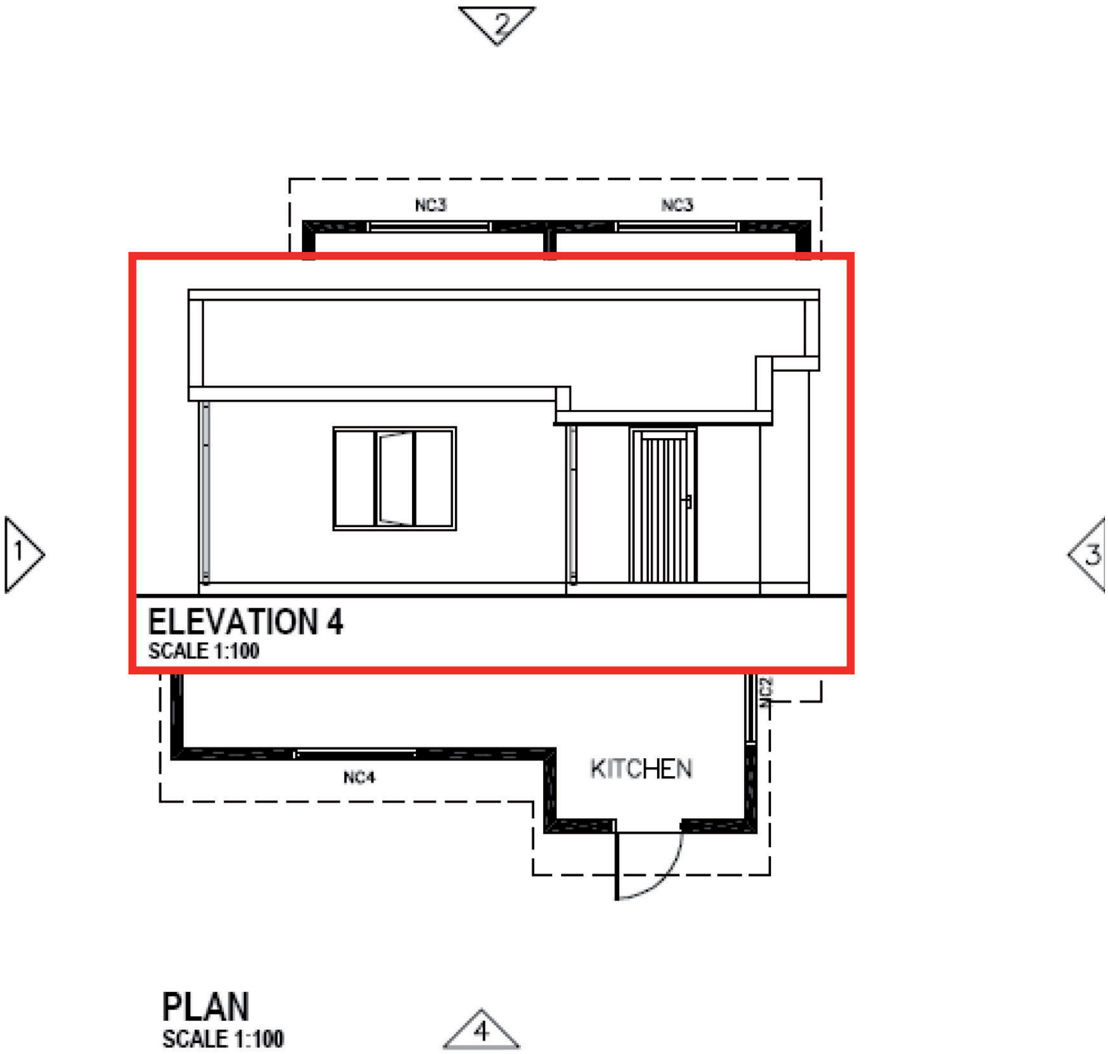
Feature	Symbol	Colour	Number in house
Windows		Red	
Doors (semi-circle shows in which direction the door will open)		Blue	__ external doors __ internal doors

The dotted line on the plan represents the extent of the roof.

Builders will need far more complex drawings when building the house. For example they will need drawings that show them where to put the various electrical fittings such as lights, plugs and switches. This is shown on the plan below:

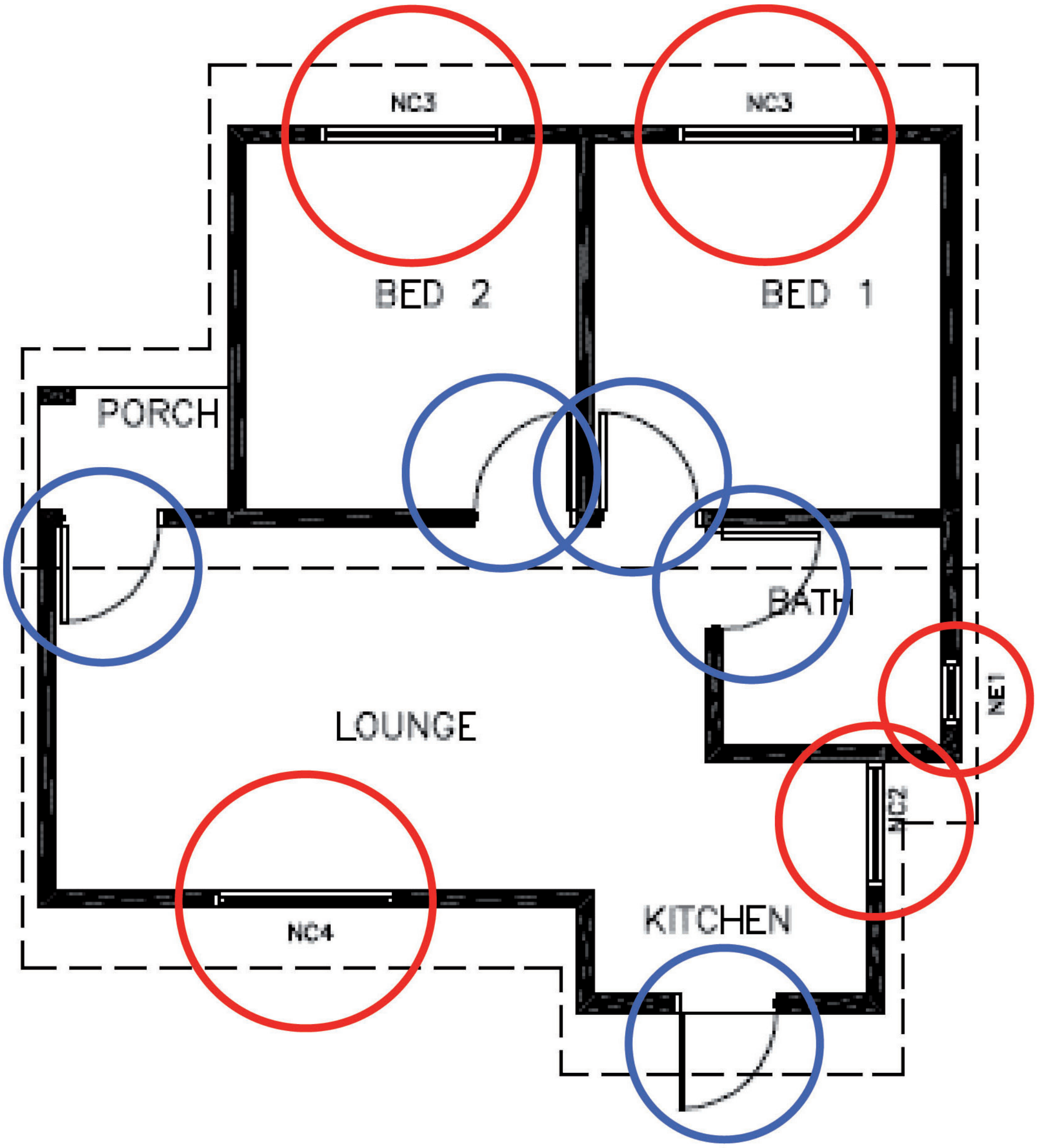


As much as a floor plan or footprint can show us where the rooms are in relation to each other and where the doors and windows are it cannot show us how tall each of the windows is or how far above the ground it is or how high the walls are or whether or not the house has a flat or pitched roof. To show this the plans also have elevation drawings of each side of the house. The elevation of the back of the house is shown in the diagram below.



Solution

There are 5 windows, 4 interior doors and 2 exterior doors.



Activity

INDIVIDUAL

PROJECT

1. Using a suitable scale, make a floor plan of your own house using the symbols and conventions discussed above.

SPACE, SHAPE AND MEASUREMENT

Lesson 53

House plans II

Learning Outcomes and Assessment Standards

Learning Outcome 3
Space, Shape and Measurement
The learner is able to measure using appropriate instruments, to estimate and calculate physical quantities, and to interpret, describe and represent properties of and relationships between 2-dimensional shapes and 3-dimensional objects in a variety of orientations and positions.
Assessment Standard
AS 11.3.1
Solve problems in 2-dimensional and 3-dimensional contexts by:
- Estimating, measuring and calculating values which involve:
- Lengths and distances.
- Perimeters and areas of polygons.
- Volumes of right prisms and right circular cylinders.
- Surface areas of right prisms and right circular cylinders.
- Making adjustments to calculated values to accommodate measurement errors and inaccuracies due to rounding.

AS 11.3.2
Convert units of measurement between different scales and systems using conversion tables provided.
AS 11.3.3
Use and interpret scale drawings of plans to:
- estimate and calculate values according to scale.

Overview

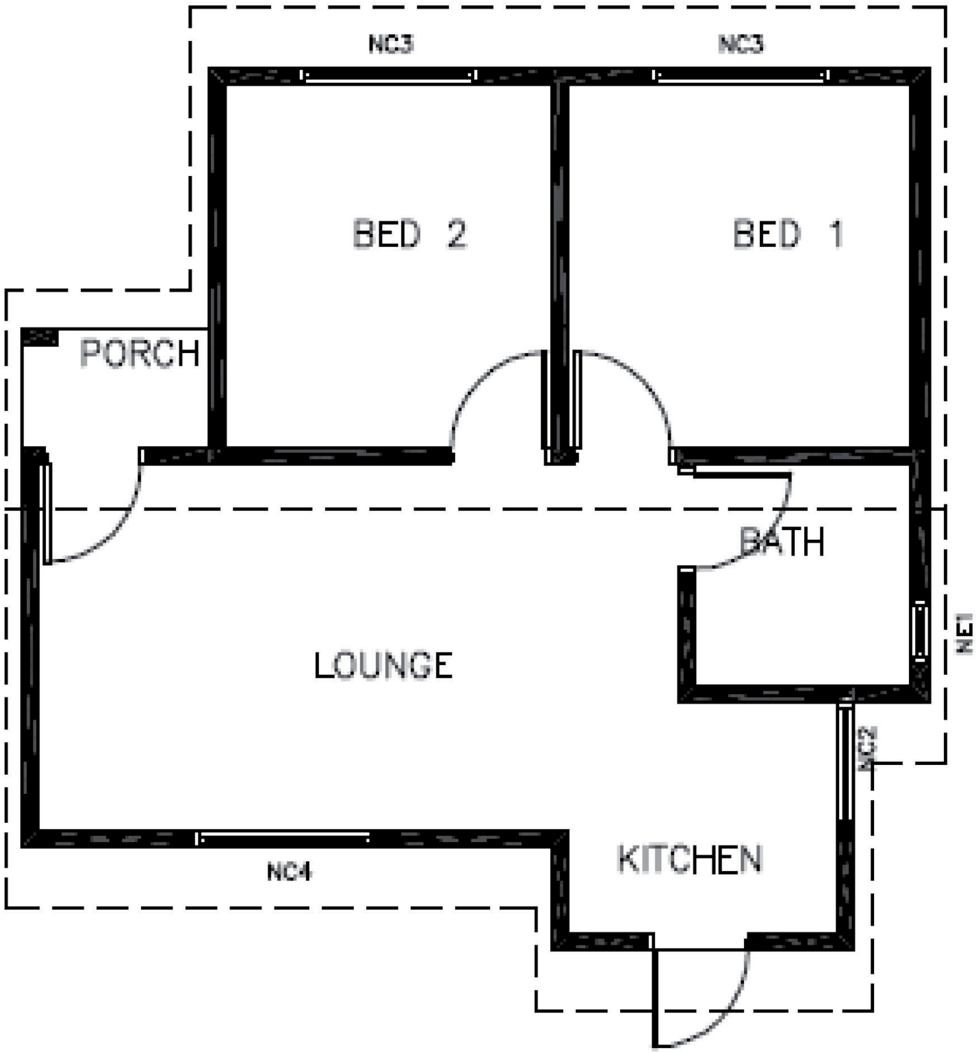
In the previous lesson we looked at a house plan and discussed the advantages and disadvantages of this type of model. In this lesson we will work with the same house plan and use it to plan the amount and kind of furniture that we can put into the house. We can do this because a house plan is drawn to scale.

Lesson

LC DVD

Methods and worked examples

For today's lesson we will enlarge the scale of the house plan to 1 : 50. This you will find on the next page



Worked example

Use the dimensions given below and make scale models of typical pieces of furniture given below. Use the same scale to make the furniture as the house plan namely 1:50.

Typical Bed sizes:

- King size: 195 cm × 203 cm
- Queen size: 152 cm × 203 cm
- Double: 138 cm × 203 cm; 138 cm × 190 cm
- Twin: 100 cm × 203 cm
- Single: 90 cm × 190 cm

Other bedroom furniture sizes:

- Wardrobe: 90 cm × 60 cm
- Dresser: 85 cm × 45 cm; 165 cm × 45 cm
- Bedside table: 38 cm × 48 cm
- Chair: 48 cm × 50 cm

Make as many different arrangements of furniture in the following rooms as possible:

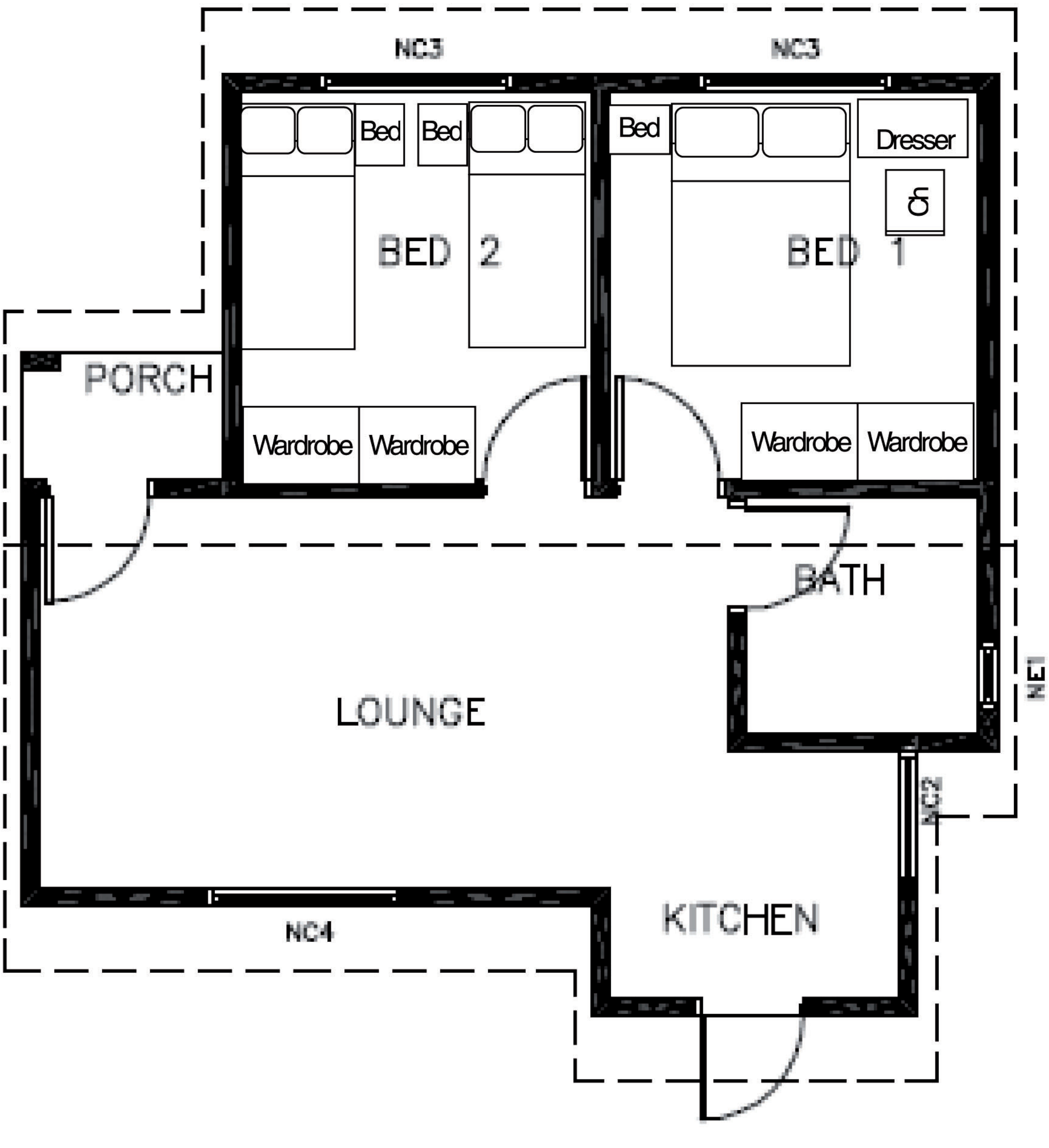
We want to put the following furniture into the main bedroom (bed 1):

- a double bed;
- two wardrobes;
- a bedside table or two; and
- a dresser and chair.

We want to put the following furniture into the second bedroom (bed 1):

- two single beds;
- two wardrobes; and
- a bedside table or two.

Solution



PAIRS

Activity

1. Use the following dimensions of furniture to furnish the lounge area. Try and create as many different arrangements as possible. Give the advantages and disadvantages of each arrangement.

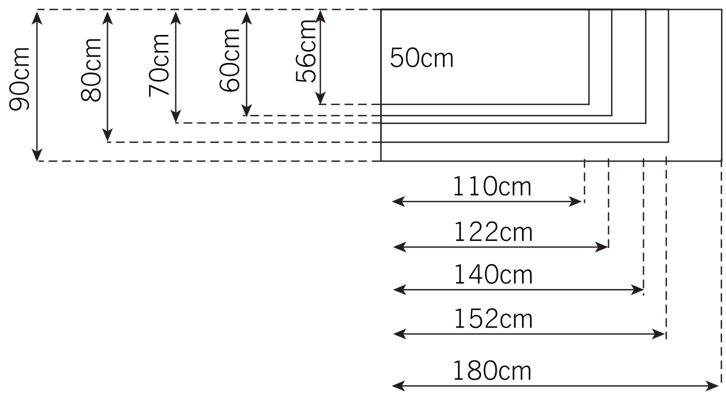
   Chairs:

   - Desk chair: 48 cm × 45 cm
   - Lounge/sofa: width typically varies from 142 cm to 152 cm – determined as follows: 60 cm per person plus 10 cm to 15 cm for each armrest, with a full size sofa typically being 230 cm wide depth typically varies from 46 cm to 56 cm.

   Tables:

   - Bedside: 38 cm × 48 cm
   - Coffee (rectangular): 45 cm to 60 cm × 90 cm to 152 cm*
   - Coffee (round): 90 cm to 110 cm diameter
   - Writing table: 92 cm to 102 cm × 50 cm to 60 cm*
   - Rectangular table: see below

   * Note rectangular tables are typically manufactured with the ratios shown below



2. Use the dimensions of bedroom furniture given in the worked example to furnish the bedrooms in a different way. Try and create as many different arrangements as possible. Give the advantages and disadvantages of each arrangement.

SPACE, SHAPE AND MEASUREMENT

House plans III

Lesson 54

Learning Outcomes and Assessment Standards

Learning Outcome 3
Space, Shape and Measurement
The learner is able to measure using appropriate instruments, to estimate and calculate physical quantities, and to interpret, describe and represent properties of and relationships between 2-dimensional shapes and 3-dimensional objects in a variety of orientations and positions.
Assessment Standard
AS 11.3.1
Solve problems in 2-dimensional and 3-dimensional contexts by:
- Estimating, measuring and calculating values which involve:
- Lengths and distances.
- Perimeters and areas of polygons.
- Volumes of right prisms and right circular cylinders.
- Surface areas of right prisms and right circular cylinders.
- Making adjustments to calculated values to accommodate measurement errors and inaccuracies due to rounding.

AS 11.3.2
Convert units of measurement between different scales and systems using conversion tables provided.
AS 11.3.3
Use and interpret scale drawings of plans to:
- Estimate and calculate values according to scale.

Overview

In this lesson we will continue working with the house plans that we have been using in the previous two lessons. We will explore how to use the plans in making budgeting decisions regarding things like wallpaper, tiles and paint.

Lesson

Methods and worked examples

A common mistake made when determining the amount of wallpaper needed to cover a wall is to calculate the area of the wall. Wallpaper is, however, sold in rolls which means that you firstly have to buy more than just the area of the wall and secondly, the process of wallpapering the wall which involves avoiding joints in the wrong places and ensuring that the patterns of the consecutive strips line up will results in wasted off-cuts.

It is the same when carpeting a floor and when tiling a wall or floor. Calculating the area of a wall is, however, useful when painting a wall as a spread rate per m^2 is usually quoted on the tin.

Mathematical Literacy is not about learning how to wallpaper a wall, but what we do want you to learn as you become mathematically literate is that there are situations in day to day life where the formula that seems most tempting is not completely appropriate, as is the case with the wall paper. The way in which the number of rolls of wallpaper needed to do a job is determined is by using a table that somebody else has already developed for us.

Below is an example of such a table:

	Distance around the room																	
	30’ 9 m	34’ 10 m	38’ 12 m	42’ 13 m	46’ 14 m	50’ 15 m	54’ 16 m	58’ 17 m	62’ 18 m	66’ 19 m	70’ 21 m	74’ 22 m	78’ 23 m	82’ 24 m	86’ 26 m	90’ 27 m	94’ 28 m	98’ 30 m
’ 6” – 2,30 m	4	5	5	6	6	7	7	8	8	9	9	10	10	11	12	12	13	13
- 8’ – 2,45 m	5	5	6	6	7	7	8	8	9	9	10	10	11	11	12	13	13	14
3’ 6” – 2,60 m	5	5	6	7	7	8	9	9	10	10	11	12	12	13	14	14	15	15
- 9’ – 2,75 m	5	5	6	7	7	8	9	9	10	10	11	12	12	16	14	14	15	15
9’ 6” – 2,90 m	6	6	7	7	8	9	9	10	10	11	12	12	13	14	14	15	15	16
- 10’ – 3,05 m	6	6	7	8	8	9	10	10	11	12	12	13	14	14	15	16	16	17
10’ 6” – 3,20 m	6	7	8	8	9	10	10	11	12	13	13	14	15	16	16	17	18	19

Note the following:

- Along the top (horizontal axis) the table lists the distance (perimeter) around the room;
- Along the vertical axis the table lists the height of the room; and
- The dimensions are given in both metric units – metres and centimetres – and imperial units – feet and inches.

If the perimeter of the room that you wanted to wall paper was $15\frac{1}{2}$ m, we should be safe and choose 16m and if the height of the room was $2\frac{1}{2}$ m then we would choose 2,45 to 2,60m. The table then suggests that we would need 9 rolls of wall paper (shown in green above).

INDIVIDUAL

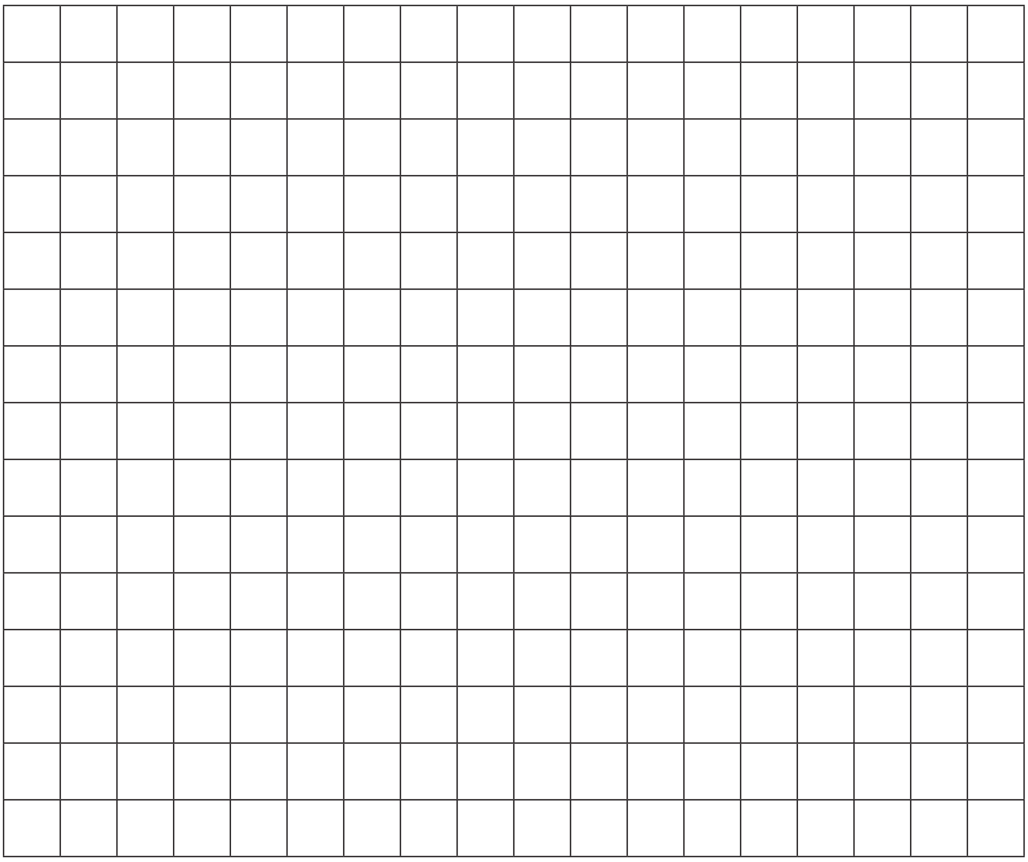
FORMATIVE ASSESSMENT

Worked example

(a) Determine the amount of wallpaper needed to wallpaper the lounge (excluding the kitchen) of the house that we have been working with over the past two lessons. The height of the walls is given as 3 m.

(b) Calculate the wastage of wallpaper, if it is given that a roll of wallpaper has the dimensions: 10 m long and 0,53 m wide.

(c) Using the (1 : 50 scale) grid provided (below) determine how many boxes of tiles you would need to tile the lounge and kitchen floors using a tile of 40 cm by 40 cm. Each box of tiles contains 20 tiles.



Solution

(a) The perimeter of the lounge = 29,2 cm × 50 cm
= 1 460 cm
= 14,6 m

The height of the room is 3 m so according to the table we need 9 rolls of wallpaper. Answer in red in the table above.

(b) Area of wallpaper = 10 m × 0,53 m
= 5,3 m^2

Therefore 9 rolls has an area = 5,3 m^2 × 9
= 45 m^2

The actual area of the wall = Area of the wall − 4 × doors − 1 × window
= (14,55 m × 3 m) − 4(0,75 m × 2 m) − (1,2 m × 1,5 m)
= 35,85 m^2

Number of rolls of wallpaper = 35,85 m^2 ÷ 5,3 m^2
= 6,7 rolls
= 7 rolls

This means we are buying two rolls more than we need.

(c) Tiling problem:

Number of tiles needed ≈ 130 tiles

INDIVIDUAL

SELF ASSESSMENT

Activity

1. Using the grid provided, determine the number of boxes of tiles you would need to tile:
   - 1.1 bedroom 1 in white tile; and
   - 1.2 bedroom 2 in grey tiles;
2. Use the wallpaper chart to determine how many rolls of wallpaper you would need to wallpaper bedroom 1. Take the height of the room as 3 m.

SPACE, SHAPE AND MEASUREMENT

Lesson 55

The impact of rounding

Learning Outcomes and Assessment Standards

Learning Outcome 3
Space, Shape and Measurement
The learner is able to measure using appropriate instruments, to estimate and calculate physical quantities, and to interpret, describe and represent properties of and relationships between 2-dimensional shapes and 3-dimensional objects in a variety of orientations and positions.
Assessment Standard
AS 11.3.1
Solve problems in 2-dimensional and 3-dimensional contexts by:
- Estimating, measuring and calculating values which involve:
- Lengths and distances.
- Perimeters and areas of polygons.
- Volumes of right prisms and right circular cylinders.
- Surface areas of right prisms and right circular cylinders.
- Making adjustments to calculated values to accommodate measurement errors and inaccuracies due to rounding.

AS 11.3.2
Convert units of measurement between different scales and systems using conversion tables provided.

Overview

In this lesson we will continue with our study of space, shape and measurement and turn our attention to an interesting problem that arises when working with measurements and dimensions: rounding and in particular the impact that rounding can have on a problem/situation.

Lesson

LC DVD

Methods and worked examples

Below is a mixing guide which is published by a local cement company:

The guide deals with the cement and sand requirements for a wall built with a particular kind of block. In the top section it discusses the cement and sand requirements for a wall using 1 000 blocks and in the lower part it deals with the requirements for a 10 m^2 wall.

We will show in the worked example below that if we use these directions quite literally we will end up with two different amounts of cement for a particular wall. We are not suggesting that the cement company is wrong in its recommendations or that builders who follow these will build weaker or stronger walls – we are simply saying that to be mathematically literate is to have a sense of how to deal with the ambiguity that arises from directions such as these.

Worked example

Two builders want to build a wall of about 240 m^2 and they agreed that this would require approximately 3 000 blocks. Use the information in the guide to determine the amount of cement needed in two ways: i.e. use the top section for the first calculation and the bottom section for the second calculation.

Solution

Using the information from the top half of the information block – namely that you would need 9 bags of cement for every 1 000 blocks, you would use 3 × 9 = 27 bags of cement and 3 × 2 = 6 m^3 of sand.

Using the information from the bottom half of the information block – namely that you would need 2 bags of cement for every 125 blocks, you would use 2 × 3000 ÷ 125 = 48 bags of cement and 3 000 ÷ 125 × 0,25 = 6 m^3 of sand.

Notice that the same amount of sand was required but the amount of cement required based on each calculation differs greatly. An explanation for this follows:

When we look at the table with its headings, it gives the quantities required. It does not, however, tell us how to mix the cement and sand i.e. in what ratio we must mix the cement and sand— that is done elsewhere in the brochure (see below). We read that cement and sand must be mixed in the ratio 1 to 6. This means that 1 bucket of cement must be mixed with 6 buckets of sand.

Although from the buying guide it appears as if the mixing ratio is 9 to 2 – this is not the case since the units are not the same (bags and square-metres). The guide is only recommending how much of each quantity should be bought so that you have enough to build your wall and it is not saying that you should necessarily mix the two quantities you buy together.

Notice that the guide also states that there can be up to 20% wastage when building with cement.

Having established all of the above we can see how the ambiguity arises as follows:

We can build a wall of 80 m^2 with 1 000 blocks and this requires 9 bags of cement and 2 m^3 of sand. This is a very large wall – more than enough to build the outside wall of the house we discussed in the previous lessons. Not all builders will be building walls using so many blocks. To help builders building smaller walls the cement company has made a suggestion regarding the amount of materials to buy for a 10m^2 wall which requires approximately 125 blocks. 10 m^2 is of the 80 m^2 wall and will therefore require of 9 bags of cement. This comes to 1,125 bags of cement. Since it is not possible to buy 0,125 of a bag of cement, the guide rounds up the amount and recommends that the builder buys 2 bags of cement. This explains why when we used this guide to calculate the amount of cement for a wall with 3 000 blocks we got so many bags – the amount that we used included a large rounding-up.

INDIVIDUAL

Additional example 1

Using both guides calculate the number of bags of cement needed to build a wall with 250 blocks. Compare your two answers.

FORMATIVE ASSESSMENT

Solution

1. This amount of blocks is somewhere between the two recommendations.

   Using the information from the top half of the information block – namely that you need 9 bags of cement for every 1 000 blocks, would give: 9 ÷ 4 = 2,25 bags of cement and you would round-up and buy 3 bags.

   Using the information from the bottom half of the information block – namely that you need 2 bags of cement for every 125 blocks, would give: 2 × 250 ÷ 125 = 4 bags of cement.

This time the amounts seem to be more in line with each other.

Note: It is important that you can make sense of instructions – a skill that relies on us (a) having some sense of how the table was developed and (b) having some sense of the variables – such as wastage and others.

The problem of rounding and the errors it can cause can arise in many other situations, for example in cookery books where there are often conversion tables and those tables can lead to disasters if you don't interpret then correctly.

Additional example 2

2. In a volume table in a cookery book the table says that 15ml is equal to 1 fluid ounce. It also says that 1 litre is equal to 35 fluid ounces. Using the first conversion, calculate how many ml in 35 fluid ounces.

INDIVIDUAL

FORMATIVE ASSESSMENT

Solution

2. 15 ml × 35 fluid ounces = 525 ml which is a little more than a half a litre. This is way different to the second conversion. This again emphasises that we must use tables appropriately – the smaller conversion rates being more appropriate for small amounts and the larger values for larger amounts.

Note: One tablespoon is equal to 15 ml and a fluid ounce is a little more than a tablespoon but not quite 2 so on this occasion the people developing the cookery book rounded down.

Activity

INDIVIDUAL

SELF ASSESSMENT

1. The webpage below shows an extract from an advertisement for a shipping company:

   A standard container of type 40' × 9'6'' is said to have the following specifications:

   - A cubic capacity of 2 705,1ft^3 or 76,6m^3; and
   - A maximum payload of 57 288 pounds or 26 040 kg

   Answer the following questions related to the above. Answers must be accurate to three decimal places:

   1.1 According to the SI conversion rate there are 2,539999…cm to 1 inch. Check whether you come up with the same values as the advertisement i.e. that 2 705 ft^3 = 76,6m^3 by doing the following:

   (a) Round off the conversion factor to one decimal place. If there are 12 inches to 1 foot and using this rounded off value, calculate how many cubic feet there are in 76,6 m^3.

   (b) Round off the conversion factor to two decimal places and repeated the process.

   (c) Which answer was closer to the advertisement answer? Explain why you think it is the closer of the two.

(d) What percentage error is there in the answer you found in (a) and the answer you found in (b)?

1.2 According to the SI conversion rate there are 0,03527 ounces to 1g. Check whether you come up with the same values as the advertisement i.e. that 57 288 pounds = 26 040 kg by doing the following. Answers to three decimal places.

(a) Round off the conversion factor to two decimal places. If there are 16 ounces to 1 pound and using this rounded off value, calculate how many pounds there are in 26 040 kg.

(b) Round off the conversion factor to three decimal places and repeated the process.

(c) Round off the conversion factor to four decimal places and repeated the process.

(d) Use the conversion factor to five decimal places and repeated the process.

(e) Which answer was closest to the advertisement answer? Explain why you think it is the closest of them all.

(f) What percentage error is there in the answer you found in (a) and the advertisement answer?

SPACE, SHAPE AND MEASUREMENT

The impact of small values

Lesson 56

Learning Outcomes and Assessment Standards

Learning Outcome 3
Space, Shape and Measurement
The learner is able to measure using appropriate instruments, to estimate and calculate physical quantities, and to interpret, describe and represent properties of and relationships between 2-dimensional shapes and 3-dimensional objects in a variety of orientations and positions.
Assessment Standard
AS 11.3.1
Solve problems in 2-dimensional and 3-dimensional contexts by:

- Estimating, measuring and calculating values which involve:
  - Lengths and distances.
  - Perimeters and areas of polygons.
  - Volumes of right prisms and right circular cylinders.
  - Surface areas of right prisms and right circular cylinders.
  - Making adjustments to calculated values to accommodate measurement errors and inaccuracies due to rounding.

AS 11.3.2
Convert units of measurement between different scales and systems using conversion tables provided.

Overview

In this lesson we will continue with our study of space, shape and measurement and turn our attention to an interesting problem that arises when working with measurements and dimensions: the impact of small values. In many of the introductions to these lessons we have made the point that by being mathematically literate you would be better prepared to participate in and contribute to world in which you live.

In this lesson we explore a situation in which a tiny change in habit by many of us could make a huge impact on the world – in particular in global warming.

Lesson

Methods and worked examples

In this example we will be looking at cellphone chargers and their impact on the environment.

Everybody who has a cell phone needs to charge the battery and to do so we plug the charger into an electricity source, draw electricity until the phone is charged and then we unplug the phone and carry on using it.

This is where the trouble starts. Although most of us unplug the phone from the charger, we leave the charger plugged into the electricity socket and as long as the charger remains plugged into the socket it is using electricity. It might be a small amount but it is electricity nonetheless and all of those small amounts of electricity can add up to a large amount and that amount, in turn, can have a huge impact on the environment.

Read the following article before you look at the worked example:

Nokia becomes the first phone maker to add energy saving alerts to mobiles
May 10, 2007

Espoo, Finland - Nokia has launched the first mobile phones to include alerts encouraging people to unplug the charger once the battery is full, a move that could save enough electricity to power 85 000 homes a year.

Kirsi Sormunen, Vice-President of Environmental Affairs at Nokia said, "Around two-thirds of the energy used by a mobile phone is lost when it is unplugged after charging but the charger itself is left in a live socket. We want to reduce this waste and are working on reducing to an absolute minimum the amount of energy our chargers use. The new alerts also play an important role, encouraging people to help us in this goal by unplugging their chargers."

www.nokia.com

You will need the following formulae and information to be able to work through the worked example. You have worked with both of these formulae in previous lessons.

Electricity consumption by an appliance (kWh)

= power rating of the appliance (in kiloWatts) × time used (hours)

Cost of using an appliance (Rand) = electricity consumption (kWh) × rate (R/kWh)

Impact of electricity consumption:

The production of 1kWh of electricity:

- Consumes 0,49 kg of coal (coal is a fossil fuel and is a dwindling resource);
- Consumes 1,2 litres of water (another valuable resource); and
- Produces 966 g of carbon dioxide which contributes to global warming.

INDIVIDUAL

FORMATIVE ASSESSMENT

Worked examples

Assume that:

- The power rating of a cell phone charger that is charging a cell phone is 4,5 Watts;
- The power rating of a cell phone charger in standby mode it is 1,5 W; and
- The cost of electricity is taken as 48 c/kWh

Calculate the amount of electricity used by a cell phone charger in one year as well as the cost if:

(a) The cell phone is charged for 3 hours per day; and

CLASS

Worked example b

(b) The charger is left on standby for the rest of each day.

Now consider the following:

(c) If there are approximately 36,2 million cell phone users in South Africa and all of them leave their cell phone chargers plugged in all day, calculate the total amount of electricity consumed by these cell phone chargers in standby mode.

(d) If it is given that South Africans use 218 120 GWh of electricity per year and that 17% of that represents domestic consumption, calculate the fraction of the household consumption this wasted electricity represents.

(e) If it is given that 90% of South Africa's electricity is produced from coal, calculate the impact on the environment of the cell phone chargers on standby.

Solution

(a) Rating of charger is 4,5 W ÷ 1 000 = 0,0045 kW

Consumption = 0,0045 kW × 3 hours

= 0,0135 kWh of electricity

Cost = 0,0135 kWh × R0,48/kWh

= R0,00648

This is just over half a cent per day.

In a year = 12 × 30 × R0,005

= R1,80

(b) Rating in standby mode = 0,0015 kW

Consumption = 0,0015 kW × 21 hours

= 0,0315 kWh of electricity

Cost = 0,0315 kWh × R0,48/kWh

= R0,01512

This is roughly one and a half cents per day.

In a year = 12 × 30 × R0,015

= R5,40

From the above you can see that the charger left on standby costs approximately three times as much as it costs to charge your phone.

(c) Consumption = 36,2 mil × 0,0315 kWh × 365 days

= 416 209 500 kWh

(d) Domestic consumption = 17% × 218 120 GWh

= 37 080,4 GWh

1GWh = 1000 000 kWh

$\frac{416\ 209\ 500\text{kWh}}{37\ 080\ 400\ 000\text{ kWh}} \approx 1{,}12\%$

(e) Amount of coal produced electricity = 90% × 416 209 500 kWh

= 374 588 550kWh

Coal consumption = 0,49 kg/kWh × 374 588 550 kWh

= 183 548 tons of coal

Water consumption = 1,2 litres of water/kWh × 374 588 550 kWh

= 449 506 260 litres of water; and

Carbon dioxide produced = 966 g/kWh × 374 588 550 kWh

= 361 852 tons of carbon dioxide

Note: Cellphone chargers are not the only electrical appliances that are on standby. Other examples include: stoves; televisions; video machines; computers and printers to name but a few.

Small numbers can become very large numbers very quickly and although it may be tempting to want to disregard these numbers as "unimportant" they can have a huge impact!

INDIVIDUAL

FORMATIVE ASSESSMENT

Activity

1. Use the information supplied in the following table to answer the following questions:

Appliance	Maximum Stand-by Power (watts)	Minimum Stand-by Power (watts)	Average Stand-by Power (watts)
Mobile phone charger	3	1	1,5
Kitchen Oven	18	6	11,1
Personal Computer	27	1	6,9
Television	22	1	4,5
Video Recorder (VCR)	30	1	7,2
Hi-fi Stereo	24	1	5,5
Doorbell	1,5	1	1,1

1.1 On average which appliance wastes the most electricity per hour while on standby?

1.2 Calculate the average wastage of electricity if the kitchen oven is left on standby for $22\frac{1}{2}$ hours every day for a year.

1.3 What is the cost per year of the wasted electricity?

1.4 What is the cost per year to the environment i.e. how much coal and water is used and how much carbon dioxide is manufacture?

1.5 Calculate the wastage of electricity if a particular video recorder has a maximum standby rating and it is left on standby for approximately 16 hours everyday for a year.

1.6 What is the cost per year of this wasted electricity?

1.7 What is the cost per year to the environment i.e. how much coal and water is used and how much carbon dioxide is manufacture?

2 If South Africans use 37 080 GWh household electricity per year and as much 12% is spent on standby electricity what is the impact as a nation in terms of:

2.1 Money that has been wasted; and

2.2 The environment and resources

SPACE, SHAPE AND MEASUREMENT: VOLUME I

Lesson 57

Learning Outcomes and Assessment Standards

Learning Outcome 3
Space, Shape and Measurement
The learner is able to measure using appropriate instruments, to estimate and calculate physical quantities, and to interpret, describe and represent properties of and relationships between 2-dimensional shapes and 3-dimensional objects in a variety of orientations and positions.
Assessment Standard
AS 11.3.1
Solve problems in 2-dimensional and 3-dimensional contexts by:
- Estimating, measuring and calculating values which involve:
- Lengths and distances.
- Perimeters and areas of polygons.
- Volumes of right prisms and right circular cylinders.
- Surface areas of right prisms and right circular cylinders.
- Making adjustments to calculated values to accommodate measurement errors and inaccuracies due to rounding.

AS 11.3.2
Convert units of measurement between different scales and systems using conversion tables provided.

Overview

In this lesson we will continue with our study of space, shape and measurement and focus on volume. We will not be calculating volume in this lesson but you will need a good understanding of volume to be able to make sense of the ideas that underlie this lesson.

Lesson

Methods and worked examples

The *Arrive Alive* campaign in South Africa strives to make people aware of how you can cause hurt in other people's lives if your drive a car while under the influence of alcohol.

What we want to do in this lesson is to have a look at the mathematics of blood alcohol levels so that you have a sense of how drinking impacts on different people in different ways.

There are two legal ways of determining how sober or not a person is (measured in terms of blood alcohol level):

- Blood tests – which involve a sample of blood; and
- Breathalyzer tests – which involve breathing into a machine.

In the case of the blood test, 0,05 g of alcohol per 100 ml of blood is considered to be the legal limit. This means that if the alcohol concentration is greater than this, the driver of a car is considered to be "over the limit". We summarise that as 0,05 g/100ml.

When you are stopped in a road block and the police take a sample of blood, it is sent to the laboratories where they determine the concentration of alcohol in your blood. This is measured in grams per 100 ml – weight and volume.

The first thing that we need to understand is how many grams of alcohol you add to your system every time you enjoy "a drink". All alcoholic beverages have their alcohol content clearly marked on the container. It is expressed as a percentage of the volume. A few beverages are summarised in the table below:

Beverage	Alcohol content	Volume of single drink
Beer	5% per volume	340 ml
Light beer	4,5% per volume	340 ml
Wine	12% per volume	125 ml
Spirits (vodka, brandy, whisky)	40% per volume	25 ml

In order to be able to investigate this situation we need the following information:

Firstly, we need to know that although water has a density of 1 g/ml which means that 1 ml of water weighs 1 g and 100 ml of water weighs 100 g, alcohol has a density of 0,79 g/ml which means that 1ml of alcohol weighs 0,79 g.

Secondly we need a way of calculating a person's blood alcohol level is related to the amount of alcohol that they drink. A Swede, Erik Widmark, developed a formula many years ago to estimate a person's blood alcohol level (BAL). The formula is known as the *Widmark Formula* and it is published on the *Arrive Alive* website.

There are a very large number of factors that impact on how and how quickly the alcohol a person drinks converts into alcohol in your blood. These factors include:

- Whether or not you have eaten before drinking;
- The kinds of food that you have eaten; and
- What percentage of your body weight is made up of fat – in this case women are at a slight disadvantage because more of a woman's body weight is made up of fat than is the case for a man.

Widmark determined that the blood alcohol level (BAL) is given by the following formula:

$$BAL = \frac{A}{p \times r \times 10}$$

Where: A = amount of alcohol consumed in grams

p = mass of the person

r = Widmark factor – 0,7 for males and 0,6 for females.

INDIVIDUAL

FORMATIVE ASSESSMENT

Worked example 1

(1) Calculate the typical alcohol content of a single drink in grams for the beverages in the table above.

(2) Determine the blood alcohol levels of a man with the weight of 75 kg who has had two drinks and a woman with a weight of 65 kg who has had 2 drinks.

Solution

(1) Beer:

Volume = 340 ml

Alcohol = 5% per volume

$5\% \times 340$ ml = 17 ml

Mass of alcohol in a can of beer = 17 ml × 0,79 g/ml = 13,43 g

Drinking a regular 340 ml can of beer with an alcohol content of 5% per volume introduces 13,43 g of alcohol into your system

Light beer:

$$\text{Volume} = 340 \text{ ml}$$
$$\text{Alcohol} = 4{,}5\% \text{ per volume}$$
$$5\% \times 340\text{ml} = 15{,}3 \text{ ml}$$
$$\text{Mass of alcohol in a can of beer} = 15{,}3 \text{ ml} \times 0{,}79 \text{ g/ml} = 12{,}087 \text{ g}$$

This is not much less alcohol than a regular beer. The advantage of the light beer lies in its reduced carbohydrate content – carbohydrates being one of the contributors to weight gain. Light beer is great if you are trying to lose weight but not that different to regular beer in terms of its impact on your alcohol content.

Wine:

$$\text{Volume} = 125 \text{ ml per glass}$$
$$\text{Alcohol} = 12\% \text{ per volume}$$
$$12\% \times 125 \text{ ml} = 15 \text{ ml}$$
$$\text{Mass of alcohol in a glass of wine} = 15\text{ml} \times 0{,}79 \text{ g/ml} = 11{,}85 \text{ g}$$

Spirits:

$$\text{Volume} = 25\text{ml per tot}$$
$$\text{Alcohol} = 40\% \text{ per volume}$$
$$40\% \times 25\text{ml} = 10\text{ml}$$
$$\text{Mass of alcohol in a tot of spirits} = 10\text{ml} \times 0{,}79 \text{ g/ml} = 7{,}9 \text{ g}$$

Surprisingly the beverage with the lowest alcohol concentration, beer has the largest number of grams of alcohol per drink. And the beverage with the highest alcohol concentration, spirits has the lowest number of grams of alcohol per drink.

For the remainder of this lesson we will regard a single drink (a can of beer; a glass of wine or a shot of spirits) as having an alcohol content of 10g.

Worked example 2

INDIVIDUAL

FORMATIVE ASSESSMENT

(2) Man:

$$A = 2 \times 10 \text{ g}$$
$$= 20 \text{ g}$$
$$p = 75 \text{ kg}$$
$$r = 0{,}7 \text{ for males}$$
$$BAL = \frac{20}{75 \times 0{,}7 \times 10} = 0{,}038 \text{ g/100 ml}$$

Woman:

$$A = 2 \times 10 \text{ g}$$
$$= 20 \text{ g}$$
$$p = 65 \text{ kg}$$
$$r = 0{,}6 \text{ for females}$$
$$BAL = \frac{20}{65 \times 0{,}6 \times 10} = 0{,}051 \text{ g/100ml}$$

Note the following:

- The male is under the limit and the female is over the limit.
- Weight has an impact on the number of drinks you can drink before you are over the limit.
- Gender also has an impact on the number of drinks you can drink before you are over the limit.

Activity

INDIVIDUAL

SELF ASSESSMENT

1 The following article appeared in The Mercury:

Heavyweight fight for light drink

November 26, 2003

A brainwave at a braai and an outdated piece of liquor legislation could emerge as the best marketing tools Durban entrepreneur Jean-Marc Tostee has ever laid his hands on.

Tostee, the creator of the fashion brand "Bad Girl", which boasts annual turnover of R100 million, has developed a range of low alcohol spirit coolers which authorities say is too soft.

The story goes like this: Tostee was at a family braai on a hot Durban Sunday recently when he noticed that the men were quaffing back their low alcohol 2.5% beers while the women were sharing their 5% alcohol-based coolers.

Tostee thought it odd that the men, who generally prefer beer, could chose from a variety of light products on the market, while the only option available to women who often drank spirit coolers, were drinks with double the alcohol content.

So Tostee created his own version of what he believed women would drink, especially those who fall within the Bad Girl fashion market.

He developed a range of dry lemon, vanilla lime and cranberry flavoured vodka-based spirit coolers in pink and blue colours, with a 2.5% alcohol content – the equivalent of one sixth of a glass of wine.

He called the cooler "Bad Girl Angel Ice" with less alcohol and kilojules and made the bottles 275 ml versus the standard 330 ml "because the girls in my market have small, dainty hands and can't clutch a larger bottle".

Tostee fitted them with screw-tops adorned with pink angels, the trademark logo.

Tostee got government approval for the range and had Bulmers, a Johannesburg based bottling company, produce 720 000 bottles of Bad Girl "Angel Ice".

The first order in the national distribution effort, last week, was whipped off the shelves, says Tostee.

The entrepreneur has registered trademarks in the European Union, the USA, Australia, Brazil and Japan and has geared up for much bigger volumes in South Africa.

But that's when government officials informed him that the next batch of Angel Ice had to be increased to at least 4% alcohol volume because that is the minimum legislated requirement for a vodka-based spirit cooler.

"I told them that was ridiculous and that there is no way my market, having been primed by Bad Girl to drink something less alcoholic, will now drink something stiffer."

Read the article and answer the following questions:

1.1 Calculate the alcohol content of a single drink (340 ml) in grams of a low alcohol beer.

1.2 Calculate the alcohol content of a single drink (330 ml) in grams of a vodka based cooler.

1.3 Calculate the alcohol content of a single drink (275 ml) in grams of Tootsie's new low alcohol drink.

1.4 Use your answer for 1.1 to determine the blood alcohol level of a man with the weight of 75 kg who has had four light beers. Should he be driving home?

1.5 Use your answer for 1.3 to determine the blood alcohol level of a woman with the weight of 60 kg who has had four of Tootsie's new low alcohol drink. Should she be driving home?

Lesson 58

SPACE, SHAPE AND MEASUREMENT: VOLUME II

Learning Outcomes and Assessment Standards

Learning Outcome 3
Space, Shape and Measurement
The learner is able to measure using appropriate instruments, to estimate and calculate physical quantities, and to interpret, describe and represent properties of and relationships between 2-dimensional shapes and 3-dimensional objects in a variety of orientations and positions.
Assessment Standard
AS 11.3.1
Solve problems in 2-dimensional and 3-dimensional contexts by:
- Estimating, measuring and calculating values which involve:
- Lengths and distances.
- Perimeters and areas of polygons.
- Volumes of right prisms and right circular cylinders.
- Surface areas of right prisms and right circular cylinders.
- Making adjustments to calculated values to accommodate measurement errors and inaccuracies due to rounding.

AS 11.3.2
Convert units of measurement between different scales and systems using conversion tables provided.

Overview

In this lesson we will continue with the problem that we started in the previous lesson namely calculating blood alcohol levels which is a problem based on an understanding of volume.

DVD

Lesson

Methods and worked examples

Don't drink and drive! One of the appeals of the *Arrive Alive* campaign.

This is a very important issue as it's not only the damage that you may cause to yourself if you cause an accident because you are under the influence of alcohol, but also the innocent people you involve in the accident.

As much as we are exploring this topic and using mathematical knowledge and skills to understand the issues, it is also a lifestyle issue. If you develop a better understanding of the situation in general then mathematical literacy will, once again, have played a role in helping you to engage with and make sense of the world in which you live.

In the previous lesson we showed how you can use the Widmark Formula (given below) to determine a person's blood alcohol level (BAL) – a measure of how sober you are.

Widmark (BAL) formula:

$$BAL = \frac{A}{p \times r \times 10}$$

Where: A = amount of alcohol consumed in grams

p = mass of the person

r = Widmark factor – 0,7 for males and 0,6 for females.

Blood alcohol levels are measured in grams of alcohol per 100 ml of blood. We saw last time how to calculate the number of grams of alcohol in a typical drink and how to use the Widmark formula to determine a person's BAL. In today lesson we are going to address three more questions around these issues.

INDIVIDUAL

FORMATIVE ASSESSMENT

Worked example 1

(1) How many drinks can the man who weighs 75 kg have before he is over the limit?

Worked example 2

(2) Determine how heavy a person should be in order to be able to drink two drinks and still be under the legal limit.

Worked example 3

(3) How long will it take for a person who is over the limit before they can once again drive? For this question you need to know the rate at which the body eliminates alcohol. Research suggests that the body eliminates alcohol at a constant rate of approximately 0,015 g/hour. Our example is as follows: A 95 kg man goes out drinking and enjoys 6 beers. How long will it take after he has stopped drinking for his blood level to reach the legal limit?

Solution

(1)

$$A = ?$$
$$p = 75 \text{ kg}$$
$$r = 0{,}7 \text{ for males}$$
$$BAL = 0{,}05 \text{ g/100ml}$$
$$0{,}05 \text{ g/100ml} = \frac{A}{75 \times 0{,}7 \times 10}$$
$$0{,}05 \times 75 \times 0{,}7 \times 10 = A$$
$$A = 26{,}25$$
$$\text{Number of drinks} = \frac{26{,}25}{10}$$
$$= 2{,}625 \text{ drinks}$$

(2) Man:

$$\text{Bal} = 0{,}05 \text{ g/100ml}$$
$$\text{A} = 2 \text{ drinks} \times 10 \text{ g/drink}$$
$$= 20 \text{ g}$$
$$\text{p} = ?$$
$$\text{r} = 0{,}7 \text{ for males}$$
$$0{,}05 \text{ g/100 ml} = \frac{20}{p \times 0{,}7 \times 10}$$
$$0{,}05 = 207\,p$$
$$7\,p = 400$$
$$p = 57{,}14 \text{ kg}$$

The man must have a mass greater than 58 kg

Woman:

$$\begin{aligned}
Bal &= 0{,}05 \text{ g/100 ml} \\
A &= 2 \text{ drinks} \times 10 \text{ g/drink} \\
&= 20 \text{ g} \\
p &= ? \\
r &= 0{,}6 \text{ for females} \\
0{,}05 \text{ g/100 ml} &= \frac{20}{p \times 0{,}6 \times 10} \\
0{,}05 &= 206\,p \\
6p &= 400 \\
p &= 66{,}7 \text{ kg}
\end{aligned}$$

The woman must have a mass greater than 67 kg

Note the following:

- Once again this calculation highlights that gender has an impact on the number of drinks you can drink before you are over the limit.

(3) The first thing we need to determine is his BAL using the Widmark formula:

$$\begin{aligned}
A &= 6 \times 10 = 60 \text{ g} \\
p &= 95 \text{ kg} \\
r &= 0{,}7 \text{ for males} \\
BAL &= ? \\
BAL &= \frac{60}{95 \times 0{,}7 \times 10} \\
BAL &= 0{,}101 \text{ g/100 ml}
\end{aligned}$$

To be legal again he must lose:

$$0{,}101 \text{ g} - 0{,}05 \text{ g} = 0{,}051 \text{ g}$$

$$\frac{0{,}051 \text{ g}}{0{,}015 \text{ g/hour}} = 3{,}4 \text{ hours}$$

INDIVIDUAL

SELF ASSESSMENT

Activity

1 How many drinks can the following people have before they are over the limit:

1.1 A girl of mass 57 kg if she is drinking the low alcohol drink mentioned in the previous lesson which has an alcohol mass of 5,4g per drink.

1.2 A boy of mass 94 kg if he is drinking a low alcohol beer mentioned in the previous lesson which has an alcohol mass of 6,7 g per drink.

2 Determine how heavy a man and a woman should be in order to be able to drink three drinks and still be under the legal limit. Assume the alcohol content of a drink to be 10 g.

3 How long will it take for the following people who have stopped drinking to regain the legal BAL i.e. before they can drive again? Assume the alcohol content of a drink to be 10 g.

3.1 A 58 kg female who has had 4 drinks.

3.2 A 75 kg male who has had 6 drinks.

SPACE, SHAPE AND MEASUREMENT

Reflection

Lesson 59

Learning Outcomes and Assessment Standards

Learning Outcome 3
Space, Shape and Measurement
The learner is able to measure using appropriate instruments, to estimate and calculate physical quantities, and to interpret, describe and represent properties of and relationships between 2-dimensional shapes and 3-dimensional objects in a variety of orientations and positions.
Assessment Standard
AS 11.3.1
Solve problems in 2-dimensional and 3-dimensional contexts by:

- Estimating, measuring and calculating values which involve:
- Lengths and distances.
- Perimeters and areas of polygons.
- Volumes of right prisms and right circular cylinders.
- Surface areas of right prisms and right circular cylinders.
- Making adjustments to calculated values to accommodate measurement errors and inaccuracies due to rounding.

AS 11.3.2
Convert units of measurement between different scales and systems using conversion tables provided.
AS 11.3.3
Use and interpret scale drawings of plans to:

- Estimate and calculate values according to scale.

AS 11.3.4
Use grids and maps, and compass directions, in order to:

- Determine locations.
- Describe relative positions.

Overview

In this lesson we summarise the key knowledge, skills and values that we have covered over the last several lessons on space, shape and measurement.

The work we have covered includes:

- using maps to plan a journey;
- using timetables to decide on which train to take;
- using house plans to solve problems;
- exploring the impact of rounding and small values; and
- calculating blood alcohol levels based on the volume of alcohol that a person consumes.

Lesson

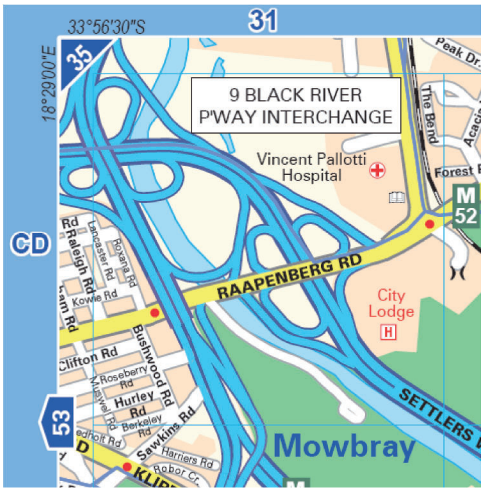
Methods and worked examples

Using maps:

The smart way of finding a place on a map is to look for the street name in the index. The index will give you a grid reference for example Raapenberg road has a grid reference of CD 51 on page 54. See below:

Raap St	Bellville South ..	**40**	CA	49
Raapenberg Rd ..	Mowbray	**54**	CD	31
Raapkraal Cr.	Kirstenhof	**103**	CU	27

The next step is to locate the street on the map using the grid references.



The arrow (circled in pink) on the side of the page enable you to follow the road in a particular direction by telling you which page to turn to.

If the trip takes you to another suburb it is useful to look at the front page of the street guide which has a general map of the city with page numbers allocated to the relevant parts of the map. This can assist in helping you to determine the relative positions of the two places of interest.

Different maps are needed for different purposes – a map is a representation and we choose maps for a particular purpose – the map on the front page gives us a sense of what the Cape Peninsula looks like but cannot show the detail of the individual roads, the inside pages can show the roads but not the relative positions of the locations we are interested in.

The key thing that makes these maps different is that they have different scales.

The scale of the street guide is 1 : 20 000. This means that every unit on the map corresponds to 20 0000 of those units on the ground. For example a distance of 95 cm on the map represents 0,2 km on the ground. The calculation follows:

Map	Ground
95cm	95 × 20 000 cm

= 1 900 000 cm

100cm = 1 m

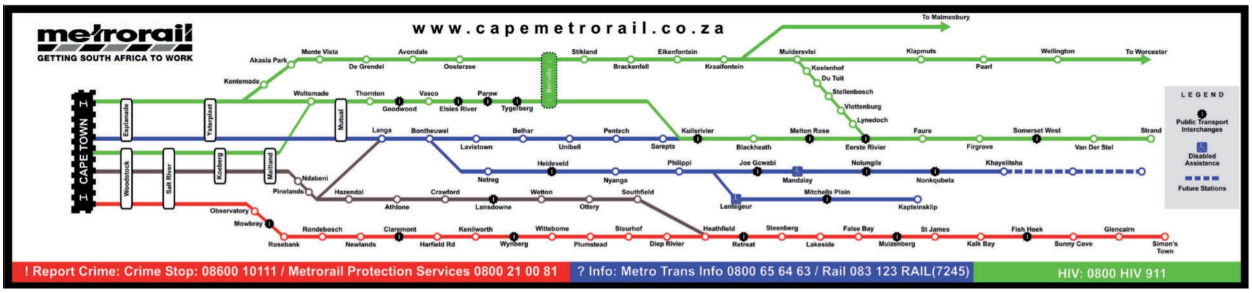
= 19 000 m

1 000m = 1 km

= 19 km

Using Train timetables:

In the train route-plan supplied by the railways, the stations are not arranged according to the geographic location of the station but rather according to the sequence of the stations. This route-plan can be used to determine a way of getting from a station on one line to a station on another line.

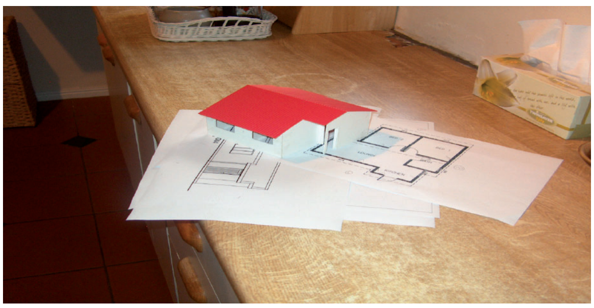


Once again we have different representations for different purposes. The train route plan helps us to know which trains to take, but it can not tell us at what times to catch the train or how long the journeys will take. We need a train time tables in order to do this.

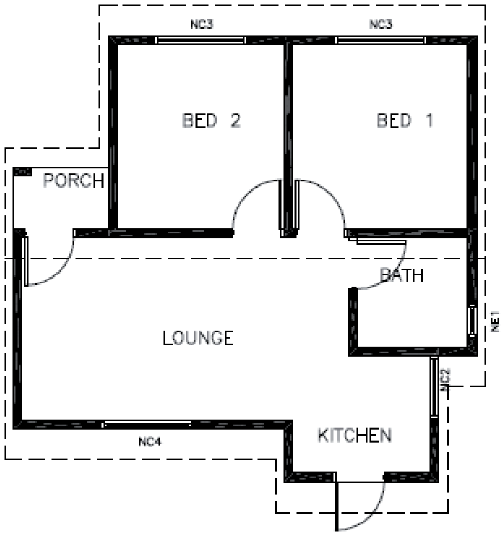
Using house plans:

PHILIPPI		07:38	..	07:51	07:51	08:01
NYANGA		07:43	..	07:56	07:56	08:06
HEIDEVELD		07:47	..	08:00	08:00	08:10
NETREG		07:51	..	08:04	08:04	08:14
BONTHEUWEL	A	**07:54**	**08:04**	**08:07**	**08:07**	**08:17**
	D	**07:55**	**08:05**	**08:08**	**08:08**	**08:18**
LANGA		07:58	08:08	08:11	08:11	08:21
MUTUAL		08:02	08:12	08:15	..	..
YSTERPLAAT		08:08	08:18	08:21	..	..
PAARDENEILAND		..	..	..	..	..
ESPLANADE		08:13	08:23	08:26	..	..
PINELANDS		..	..	..	08:16	08:26
NDABENI		..	..	..	08:18	08:28
MAITLAND		..	..	..	08:21	08:31
KOEBERG RD		..	..	..	08:23	08:33
SALT RIVER		..	..	..	08:26	08:36
WOODSTOCK		..	..	..	08:29	08:39
CAPE TOWN		**08:18**	**08:28**	**08:31**	**08:33**	**08:43**

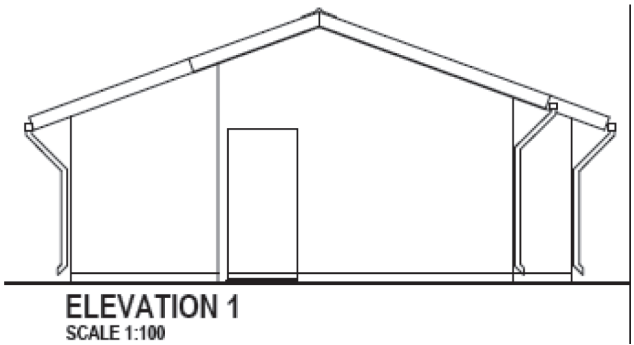
House or building plans are representations of a home. See below:



Architects use certain conventions in drawing plans and that we needed to know about these in order to be able to read and interpret plans. Among these we identified the symbol for windows (red circle), and the one for doors (blue circle). The door symbol shows us the position of the door and the direction in which it opens. Building plans have scales just like maps.

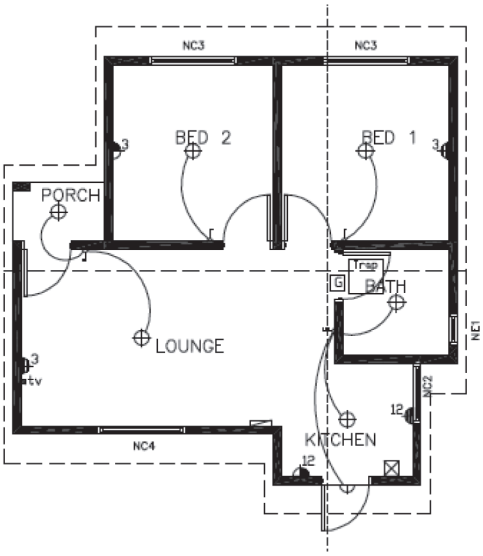


Plans also have elevations (see below) which help us to get a sense of what the house will look like when you look at it from a particular side.



The key thing that we learnt in the case of house plans is that you need to use the two different plans together to get a sense of the detail – although the top view or floor plan can show you the relative positions of the rooms, doors and windows, it cannot show you how high above the ground the windows are or how tall they are – for that you need the elevation.

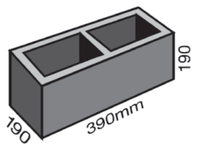
This highlights, once again, the existence of different versions of the plan for different purposes. Another example of this is the existence of the electrical drawing below:



A set of house plans is used by a builder to determine all of the materials needed to build the house, including the number of bags of cement.

The role of rounding:

Rounding has a large impact on calculations that we do. An example of this is in the table below:

BLOCK SIZE	FOR 1 000 BLOCKS		
	WALL SIZE m^2	No. BAGS SUREBUILD	m^3 SAND
	80	9	2,00
	FOR $10m^2$ WALL		
	NUMBER OF BLOCKS	No. BAGS SUREBUILD	m^3 SAND
	125	2,00	0,25

In this case rounding produced a table that gave rise to different answers to the same problem.

The challenge for the mathematically literate person is to be able (a) to deal with this ambiguity and (b) to understand how to use a table such as this one most effectively.

The impact of small numbers:

To highlight the impact of small numbers we explored the wastage of electricity by cell phone chargers. We showed that the wastage created by leaving our cell phone chargers plugged into the wall when they are not actively charging the phone accounted for approximately 1% of all domestic electricity usage.

Much more than simply wasting electricity we also showed that the impact on the environment was huge. We showed that leaving cell phone charges plugged in on standby mode contributed to a wastage of 183 000 tons of coal; 449 million litres of water; and the production of 361 852 tons of carbon dioxide – the gas that contributes to global warming.

Calculating Blood Alcohol Levels (BAL):

As an application of volume and rate in our lives we explored the blood alcohol levels – a measure of how much a person has had to drink and whether or not the person is fit to drive a car.

We established that there exists a formula that can be used to determine the blood alcohol level of a person who has been drinking. The formula, called the Widmark formula and published on the Arrive Alive website, is given below:

$$BAL = \frac{A}{p \times r \times 10}$$

Where: A = amount of alcohol consumed in grams

p = mass of the person

r = Widmark factor – 0,7 for males and 0,6 for females.

We showed that there exist a relationship between blood alcohol level, the number of drinks that you drink, your weight and your gender.

We hoped that by understanding these concepts you would be more responsible about drinking and driving.

INDIVIDUAL

SUMMATIVE ASSESSMENT

Activity

1. A map has a scale of 1 : 20 000.
   1.1 Determine the distance on the ground if the distance on the map is 56,5 cm.
   1.2 If the measurement on the map was inaccurate by 0,75 cm, determine the percentage error on the journey.
2. How long was your train trip if you caught a train at 17:54 and arrived at your destination at 19:02.
3. How many boxes of tiles do you need to tile a rectangular room with dimensions 2,6 m by 3,25 m. The dimensions of the tile are 50 cm by 50 cm. You can do this by calculation or scale drawing. Each box contains 20 tiles.
4. The spreading rate of a particular brand of paint is 9m^2 per litre. Calculate how many litres of paint you require if you want to paint the room in question 3. The height of the walls is 2,6 m. Ignore doors and windows.
5. Using the Widmark formula, determine whether a man weighing 82 kg is legally able to drive home after he has had 3 drinks. Assume that the mass of alcohol per drink is 10 g.

MANAGING YOUR ENERGY

Input and output

Lesson 60

Learning Outcomes and Assessment Standards

Learning Outcome 1
Number and Operations in Context
The learner is able to use knowledge of numbers and their relationships to investigate a range of different contexts which include financial aspects of personal, business and national issues.
Assessment Standard
AS 11.1.1
In a variety of contexts, find ways to explore and analyse situations that are numerically based, by:

- Estimating efficiently.
- Working with formulae by hand and with a calculator.
- Showing awareness of the significance of digits.
- Checking statements and results by doing relevant calculations.

(The range of problem types includes percentage, ratio, rate and proportion.)
AS 11.1.2
Relate calculated answers correctly and appropriately to the problem situation by:

- Interpreting answers in terms of the context.
- Reworking a problem if the first answer is not sensible or if the initial conditions change.
- Interpreting calculated answers logically in relation to the problem, and communicating processes and results.

Learning Outcome 2
Functional relationships
The learner is able to recognise, interpret, describe and represent various functional relationships to solve problems in real and simulated contexts.
Assessment Standard
AS 11.2.1
Work with numerical data and formulae in a variety of real-life situations, including:

- Finding break-even points involving linear functions by solving simultaneous equations.

(Types of relationships to be dealt with include linear and inverse proportion relationships.)
Learning Outcome 3
Space, Shape and Measurement
The learner is able to measure using appropriate instruments, to estimate and calculate physical quantities, and to interpret, describe and represent properties of and relationships between 2-dimensional shapes and 3-dimensional objects in a variety of orientations and positions.
AS 11.3.1
Solve problems in 2-dimensional and 3-dimensional contexts by:

- Estimating, measuring and calculating values which involve:
- Lengths and distances.
- Perimeters and areas of polygons.
- Volumes of right prisms and right circular cylinders.
- Surface areas of right prisms and right circular cylinders.
- Making adjustments to calculated values to accommodate measurement errors and inaccuracies due to rounding.

AS 11.3.2
Convert units of measurement between different scales and systems using conversion tables provided
Learning Outcome 4
Data Handling
The learner is able to collect, summarise, display and analyse data and to apply knowledge of statistics and probability to communicate, justify, predict and critically interrogate findings and draw conclusions
Note: Assessment Standard 11.4.4. is the focal Assessment Standard of LO4. The other LO4 Assessment Standards serve to develop the skills that will enable learners to achieve this one.
Assessment Standard
AS11.4.4
Critically interpret data and representations thereof (with awareness of sources of error and bias), in order to draw conclusions on problems investigated and make predictions and to critique other interpretations.

AS 11.4.1
Investigate a problem on issues such as those related to:
- Social, environmental and political factors.
- People's opinions.
- Human rights and inclusivity by:
- Collecting or finding data by appropriate methods (e.g. interviews, questionnaires, the use of data bases) suited to the purpose of drawing conclusions to the questions.
- Using appropriate statistical methods.
- Selecting a representative sample from a population with due sensitivity to issues relating to bias.
- Comparing data from different sources and samples.

AS 11.4.2
Select, justify and use a variety of methods to summarise and display data in statistical charts and graphs inclusive of:
- Tallies.
- Tables.
- Pie charts.
- Single and compound bar graphs.
- Line and broken-line graphs.

AS 11.4.3
Understand that data can be summarised and compared in different ways by calculating, and using measures of central tendency and spread (distribution), for more than one set of data inclusive of the:
- Mean.
- Median.
- Mode.
- Range.

Overview

This is the last lesson of the Mathematical Literacy Grade 11 series and draws on the knowledge, skills and values that we developed through all of the Learning Outcomes of Mathematical Literacy. Throughout our series of lessons, we have impressed on you that Mathematical Literacy is a powerful subject that prepares you to participate in and contribute to the world in which you live. It is a subject that helps you to engage with and make sense of the numbers that you face on a day to day basis.

This lesson is entitled: managing your energy input and output.

Lesson

The act of living requires energy. Although it is obvious that you are using a lot of energy when you go for a fast run it might not be as obvious that your body even requires energy to sleep. Your body uses energy every minute of every day. Because your body cannot store all of the energy that it needs for a life time, you have to put energy into the body and you do this by eating.

Methods and worked examples

This brings us to an important set of equations which we used in the finance lessons:

- income $>$ expenses money to save
- income $=$ expenses break even
- income $<$ expenses need to borrow

The same equations apply to your relationship with energy:

- input > output energy to save
- input = output break even
- input < output need to borrow

Phrased differently when your input exceeds your output and you have energy to save that saving translates into weight gain or when your input equals your output and you are breaking even; your weight remains stable or when your input is less than your output; you need to borrow energy to survive and because this energy come from your fat reserves you will lose weight.

- input > output energy to save gain weight
- input = output break even weight is constant
- input < output need to borrow lose weight

In exactly the same way that it is important to manage your finances correctly, it is important to manage your energy to maintain a healthy body. You should have a sense of:

- both the sources energy and the amounts (inputs);
- how you use/burn energy and the amounts (outputs).

Worked example

Energy input:

All of the food stuffs that you buy have an energy rating on them. A typical nutrition information table is shown below. This example comes from a packet of wine gums. Although these sweets contain no fat, they do contain a certain amount of energy. If you eat the whole packet which is 25 g, you will have an input of 341 kJ. Is you ate one half of the packet you would have half of the energy input.

TYPICAL NUTRITIONAL INFORMATION		
	per 100 g	per 25 g serving
Energy	13651,1 kJ	341,3 kJ
Protein	8,6 g	2,2 g
Total fat	0 g	0 g
Saturated fat	0 g	0 g
Carbohydrates	77,5 g	19,4 g
Sugar	57,6 g	14,4 g
Sodium	75 mg	19 mg

Below is the nutritional information for a carton of juice:

TYPICAL NUTRITIONAL INFORMATION (serving size: 250 ml)			
	per 100 ml	per 250 ml serving	*%RDA per 40g serving
Energy	236 kJ	590 kJ	
Protein	0,2 g	0,5 g	0,8%
Carbohydrates	14,2 g	35,5 g	
Total fat	0 g	0 g	
Saturated fat	0 g	0 g	
Trans fatty acids	0 g	0 g	
Total dietary fibre	0,2 g	0,5 g	
Sodium	3 mg	8 mg	
Vitamin C	24 mg	60 mg	80%

Notice that in addition to the energy; protein; fat; carbohydrates; sugar; and sodium mentioned on the wine gums label it also mentions what are called micro nutrients such as Vitamin C.

There are strict guidelines about what information the manufacturers of food need to show on the packaging. The details are not that important but what is important is that by studying these nutritional information labels you can get a sense of your energy input.

INDIVIDUAL

FORMATIVE ASSESSMENT

Complete your own energy table

The first step in managing your energy input and output is a data collection activity. Below is an example of a data collection sheet. Your serving size will mostly be an estimate.

Day	Food	Serving size	Energy per serving	Total energy
Monday	Porridge	1 cup (250 ml)		
	Milk	Glass		

For some of the food that you eat you will not find the nutritional value table on the box – for example when you cook a bowl of macaroni cheese you will use ingredients from many boxes and it will be hard to estimate the energy content. It might help to consult a modern recipe book as many modern cookbooks list the nutritional values for a recipe.

The longer the period over which you collect your data, the better sense you will have of your typical energy input.

Having collected the data of your energy consumption you will need to organise and summarise it. You will want to know how much energy you consumed on each day of the period for which you collected the data and possibly an average or mean value for each of the days. A graph might help you to detect trends – such as greater energy intake over weekends.

Energy output:

Scientists have developed data on typical energy consumption for a wide range of activities. This list gives the typical energy consumption by the body for one hour worth of each activity.

Energy consumption in kJ per hour for a number of day to day activities			
Arguing with somebody	440	Sitting (mental activity, computer work, concentrating)	460
Dancing (active)	1 675	Sitting working (operating a machine)	335
Driving a car	500	Skateboarding	1 150
Eating a meal	585	Sleeping	250
Exercise, some sweating	1 045	Sport (badminton, volleyball, tennis, karate)	1 425
Gardening (lawn mowing)	1 235	Sport (cycling, hiking, rock climbing)	1 610
Gardening (weeding)	960	Sport (rowing)	2 815
Grocery shopping	375	Sport (running)	2 930
House work (sweeping, making beds etc)	395	Sport (softball, netball, soccer, free-style swimming)	1 090

Housework (cooking)	425	Sport (squash, football, hockey, basketball, rugby)	1 925
Housework (scrubbing, vacuuming)	940	Standing (quietly in line)	420
Housework (washing a car)	970	Walking (comfortable pace)	545
Playing in the garden (throwing the ball for the dog etc)	440	Walking (fast)	1 780
Showering	1090	Walking (the dog)	1 800
Sitting (reading, writing, watching TV)	315	Wrestling	755
Sitting (talking)	500		

At the end of this worksheet is a table that you can use to collect your energy output data. We suggest that each activity is done for at least half an hour to make the analysis easier. The longer the period over which you collect the data the better sense you will have of your typical energy consumption.

Having collected the data for a period of time, you will be ready to analyse it.

Finally having collected the data on your energy input and consumption you are able to compare your input and consumption from day to day and to see which of the three equations applies. But much more than just having a sense of whether or not you consume more energy than you use or the other way around you should also have a sense of how you could change your habits. From deciding to eat less of one foodstuff and more of another or to doing more of a particular activity in preference to another.

You are the only person who can make a difference in your life – and by looking at these concepts we have given you a tool with which to do so.

Mathematical Literacy is about so much more than doing calculations – its about gaining control over your life and the world in which you live by making sense of and taking control over the numbers that define the world in which you live.

Activity

INDIVIDUAL

PROJECT

1. Use tables given in the lesson above and collect data on your input of energy and your output of energy. Draw appropriate graphs to help you analyse your data. Interpret your findings in the light of the three equations given. Discuss with evidence whether you think you need to change your habits.

ANSWERS AND ASSESSMENT

Lesson 1

1.1 68% boys
1.2 32% girls
2 85% full
3 First test = 76,7%; Second test = 62,5%
She did better in the first test.
4 83% brought their money, so they went on the outing.
5.1 60% full
5.2 80% full
5.3 The outside area is fuller percentage-wise, but there are more people to be served in the inside area.
5.4 77% full
6.1 71% were under 18
6.2 69% were under 18
6.3 70% were under 18
6.4 87% full
6.5 97% full
6.6 Own answer

Lesson 2

1 R35.25
2 R6.99
3.1 Siswe gets R6,330 and Belulwe gets R3,692.50.
3.2 Siswe gets more added to his salary as his salary is larger than Belulwe's salary.
4 R215,460
5 R4.59
6.1 Rebecca earns R5,843.75 and Ishmael earns R9,450.
6.2 Rebecca gets R343.75 more and Ishmael gets R450 more. Rebecca received a bigger percentage increase than Ishmael, but will get less money added to her salary than he will.
7 26

Lesson 3

1.1 R233.45
1.2 R133.33
1.3 R172.75
2.1 ≈ 6kg
2.2 79kg
3 2440
4.1 Johan : R275; Nomakhazi: R375; Zahrah: R210
4.2 Johan : R5,225; Nomakhazi: R7,125; Zahrah: R3,990
4.3 Nomakhazi
5.1 Footloose: R552.50; Run Wear: R540
5.2 Run Wear

Lesson 4

1 13,5%
2 14%
3 10,6%
4.1 7,8%
4.2 63,5%
4.3 69,7%

4.4 109,4%
4.5 229%
4.6 Vryheid
4.7 Johannesburg with 2,863,399 people.
4.8 112,1%

5 % change = -36,8%. This means a discount of 36,8%.

6 % change = -28,2%. This means he lost 36,8% of his flock.

7.1 6,7%
7.2 6%

Lesson 5

1 R199,00.57
2 R105.26
3 4,58kl
4 200 cattle
5 R62.50
6 R350
7 1,428km
8 R845.56
9.1 40,5m
9.2 $28m^2$
9.3 7m by 4m
10.1 R224.25
10.2 R250
11 68,5%
12 R6,51
13 32,5%

Lesson 6

1.1 Two possible solutions:
Khutso : Vuyani = 12 : 8 or Vuyani : Khutso = 8 : 12
1.2 Khutso : Vuyani = 3 : 2 or Vuyani : Khutso = 2 : 3
1.3 R40
2.1 21 : 945
2.2 1 : 45
2.3 25 : 1,050 = 1 : 42
2.4 School B. There are fewer learners per teacher.
3.1 3 : 5
3.2 2,5 litres
3.3 4 litres
4.1 For every 2 units of vinegar you must add 3 units of salad oil.
4.2 30ml
4.3 500ml
5.1 105 boys are born for every 100 girls that are born. More boys than girls are born.
5.2 21 : 20
5.3 5,250
5.4 No. Men have a lower-life expectancy than women. So the ratio would show that there are more women than men.

Lesson 7

1 Soso: 48 apples; Zileka: 60 apples
2 330 boys.
3 Jason: R100; Lee: R50
4 Concentrate: 375ml; Water: 1 125ml
5 Ismail: R600; Darshan: R350; Zahrah: R250

6 Gravel ≈ 3,429kl; Sand ≈ 1,714kl; Cement ≈ 0,857kl
7 Samson: R2 500; Edward: R750

Lesson 8

1.1 1,200 g chicken; 500 g mushrooms; 6 tomatoes; 1 cup stock; 60 ml flour
1.2 800 g chicken; 330 g mushrooms; 4 tomatoes; $\frac{2}{3}$ cup stock; 40 ml flour
2.1 100 000 cm
2.2 1 km
2.3 12 cm
3.1

bars	1	2	3	4	5	6	7	8
cost	2,50	5	7,50	10	12,50	15	17,50	20

3.2 R30
4. 4,71 m
5. 7,8 hours
6. 15 minutes
7. 9 packets (you need to round up)
8. 68,4 litres

Lesson 9

1.1 100 people
1.2 40 people
2.1 2 tablets
2.2 4 tablets
3. 4 pieces
4. 12 cm
5. $4\frac{1}{2}$ sweets.

Lesson 10

1. R370
2.1 The packet of four muffins.
2.2 It would not be economical if you wanted only one muffin and the rest of the muffins went stale.
3. 219.8 c/kg
4. You: rate = R7.50/hr; Friend: rate = R7.77/hr – better rate
5. 10,4 km/litre
6.1 R249.20
6.2 $1,123.60
7. 500 g: rate = 3,6 c/g — more economical; 250 g: rate = 4,6 c/g

Lesson 11

1 R63,86/day
2.1 No. Her speed varied throughout the journey. At times she drove faster than at other times and at one point she stopped for 10 minutes.
2.2 150 minutes = 2,5 hours
2.3 She stopped.
2.4 90 km/h
3.1 Petrol consumption of a car is not constant. A car uses more petrol for town driving because of all the stops and starts. Petrol consumption is better on the open road.
3.2 ≈ 74,9 litres
4.1 No. On some days they could consume less than 2,5 litres and on other days they could consume more.

4.2 It is useful to know how much milk to buy and it helps to budget.
4.3 R358.50
5.1 Tips vary daily so you do not always earn exactly the same amount every day.
5.2 R660

Lesson 12

1. 31 drops/ml
2. 42 drops/ml
3. 3,6 hours
4. 6,7 hours

Lesson 13

1.1

Years	BMI
two	16,6
three	16
four	15,6
five	15,4
six	15,4

1.2 2–3 years : –0,6 BMI/year
3–4 years : –0,4 BMI/year
4–5 years : –0,2 BMIyear
5–6 years : 0 *BMI/year*

1.3 The BMI values are decreasing and the amount by which they are decreasing is getting less each year.

1.4

Years	BMI
six	15,4
seven	15,5
eight	15,8
nine	16,2
10	16,6
11	17,2
12	17,8

1.5 6–7 years : 0,1 BMI/year
7–8 years : 0,3 BMI/year
8–9 years : 0,4 BMI/year
9–10 years : 0,4 BMI/year
10–11 years : 0,6 BMI/year
11–12 years : 0,6 BMI/year

1.6 The BMI values are increasing and the amount by which they are decreasing is getting more each year.
1.7 An average rate would not show the trend over 20 years. You would not know that the BMI actually decreases till about six years of age and then it increases again.
2.1 September 1998
2.2 April 2005
2.3 From March 1998 to September1998 there was a steady, rapid increase in the prime interest rate. From September 1998 to December 1999 there was a sharp decrease in the prime interest rate. From December 1999 to June 2001 there was a decrease in the prime interest rate, but

it was not as dramatic as the previous decrease. From June 2001 to the September of the same year there was a steeper decrease than the previous interval. From September 2001 to September 2002 there was a steady rise in prime interest rate.

2.4 No. The prime interest rate has varied dramatically over the 15-year period. An average change in rate per year would not show that there have been increases and decreases over that period. It would be better to describe the trends in the graph rather than use the average change in rate per year.

Lesson 14

1.1 An income and expenditure statement is a record of the actual income and expenses for a given period of time.

1.2 A budget shows the projected income and expenses for a given period of time and is often based on previous income and expenditure statements.

1.3 Amounts that remain constant from one month to the next, e.g. salary and rent.

1.4 Amounts that vary from one month to the next, e.g. groceries and haircuts.

1.5 Expenses that have to be paid, e.g. water, electricity, education.

1.6 Expenses that you can cut down on, e.g. telephone bills, entertainment.

2.1 No solution given.

2.2 No solution given.

Lesson 15

1.1

Expenses		**Income**	
Raw materials • chips • packets	R15,50 × 6 = R93 R3,75 × 2 = R7,50	Sales	R1 × 6 × 25= R150
Rental	–		
Storage	–		
Transport	R5 × 2 = R10		
TOTAL:	R110,50	TOTAL:	R150

1.2 He must be careful not to buy too many chips and then be unable to sell them all in a day.

2

Expenses		**Income**	
Raw materials • flour • salt • sugar • instant yeast • butter • water • oil	 R11,49 × 2 = R22,98 R1,85 R5,95 R1,29 × 2 = R2,58 R11,49 – R7,99 × 2 = R15,98	Sales	R1,30 × 2 × 36= R93.60
Rental	–		
Electricity	Ignore this for now		
Transport	R5 × 2 = R10		
TOTAL :	R70,53	TOTAL :	R93,60

Profit = R93.60 – R70.53 = R23.07

Lesson 16

1.1 (a) R7 (b) R12 (c) R0
(d) R12 (e) R26 (f) R26

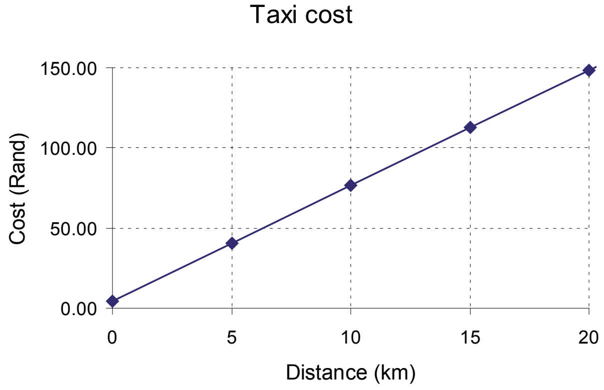
1.2 (a) R200 (b) Cost for open-shade parking = R73
Amount saved = R127

2.1 R148.60

2.2

Distance (km)	0	5	10	15	20	25	30
Cost (rand)	R4,60	R40,60	R76,60	R112,60	R148,60	R184,60	R220,60

2.3



Lesson 17

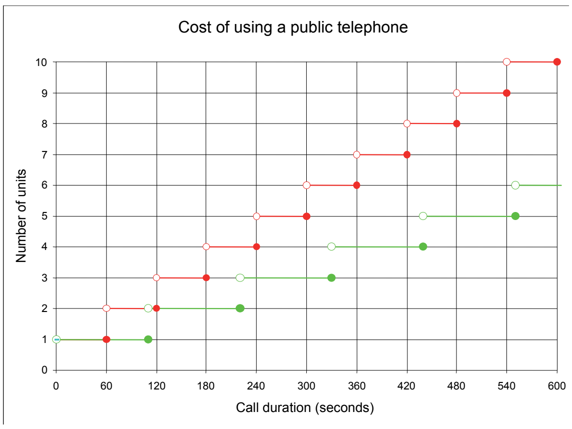
1.1 (a) R1 (b) R5 (c) R2.50 (d) R1

1.2 (a) R4.50 (b) R10 (c) R6.50 (d) R6.50

1.3 Some suggested answers:

- It is much cheaper to phone to your friend's landline.
- It is much cheaper to phone in Callmore time when phoning a landline.
- It is cheaper to phone at Rate 2 times when phoning a cellular phone, but it is still more expensive than a landline call.
- Callmore time is an hour longer per day than Rate 2 time for cellular phones.

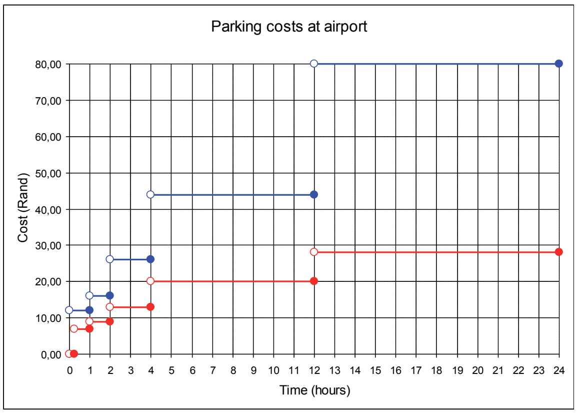
2.1



2.2 The standard time graph.

2.3 The unit time interval for standard time is much less than the unit time interval for Callmore time. This means that the cost for standard time increases at a faster rate than the Callmore time. On the graph this shows as a steeper gradient.

2.4 See graph. Cost = R1.50



3.1
3.2
3.3 It makes it very easy to compare the two different options.
3.4 R24
3.5 (a) R88 (b) R240

Lesson 18

1.1 (a) R5.40 (b) R37.71
1.2 (c) R125.92 (d) R161.56
1.3 (a) R237.70 (b) R309.64
1.4 Some suggested answers:
- To encourage people to use less water. The rate you pay is higher the more water you use.
- To help poorer people.

Lesson 19

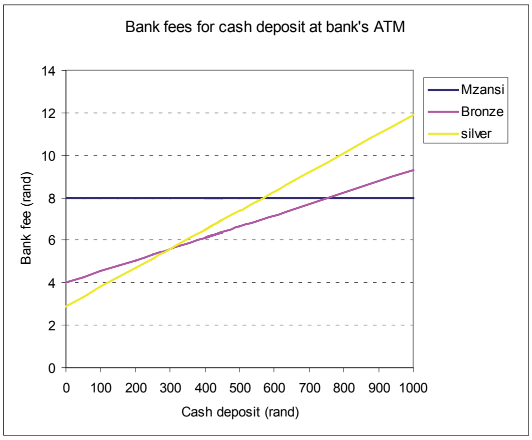
No solution given.

Lesson 20

1.1 (a) ≈ R122 (b) R220
1.2

Amount deposited	Mzansi	Bronze	Silver
R50	R8	R4.20	R3.35
R100	R8	R4.40	R3.80
R150	R8	R4.60	R4.25
R200	R8	R4.80	R4.70
R250	R8	R5	R5.15
R500	R8	R6	R 7.40
R1,000	R8	R8	R11.90

1.3 The graphs are as follows:



1.4 (a) ≈ R755 (b) ≈ R567 (c) ≈ R297

Lesson 21
See lessons 1 to 13 for activities.

Lesson 22
See lessons 14 to 20 for activities.

Lesson 23
See lessons 14 to 20

Lesson 24
1.1 Buhle
1.2 Buhle's uncle
1.3 R30
2. The bank
2.1 Delani
2.2 R26,54

Lesson 25
1.1 R375
1.2 13%
1.3 R5 555,50
1.4 R1 702,50
1.5 R1 031,25

Lesson 26
1.1 (a) 1,75% (b) ≈ 0,15% (c) R454,07
1.2 (a) 2,75% (b) ≈ 0,23%
(c) 1 362,04 (d) 2,8%
2.1 No solution given

Lesson 27
1.1 11 Months
1.2 R1 783,16

Lesson 28
1.1 (a) ≈ 4%
1.2 (b) ≈ 6%
2.1 The percentage change in the monthly repayment is almost double that of the percentage change in interest rate.

Lesson 29
2. R0,90
3. (A) 19 MONTHS (B) 14 MONTHS

3.1 R10899, 80
3.2 R10665.54
4.1 9%
4.2 6.66%

Lesson 30

1. (a) R96,21; R4,81 (b) R316,95 ; R3,96
2.

Number of recipes	Number of packets of cookies	Unit cost per packet	Profit per packet
1	4	R9,24	R2,76
3	12	R5,38	R6,62
5	20	R4,81	R7,19
10	40	R4,32	R7,68
20	80	R3,96	R8,04

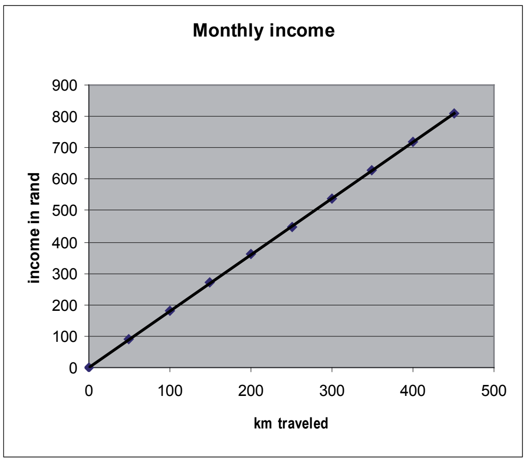
4. Some suggested answers:
The biggest difference in the unit cost per packet is going from making 1 recipe to making 3 recipes. It would certainly be more worth your while making at least 3 recipes.
After 5 recipes the decrease in unit cost is becoming less significant.

Lesson 31

1.1 Income = R1,80/km × n
1.2

Number of km traveled	Income
0	R0
50	R90
100	R180
150	R270
200	R360
250	R450
300	R540
350	R630
400	R720

1.3

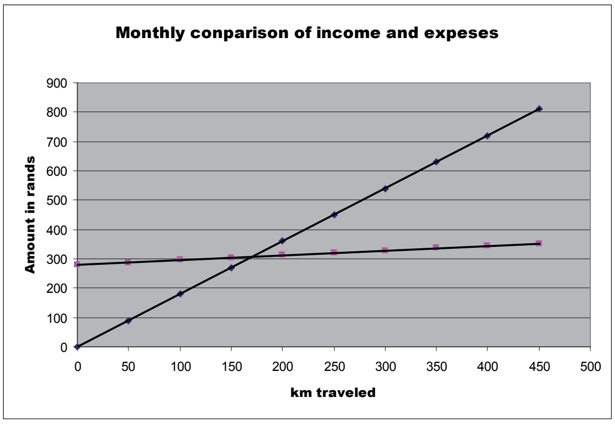


1.4 Monthly cost = R380 + R0,16/km × n

1.5

Number of km traveled	Cost
0	R280
50	R288
100	R296
150	R304
200	R312
250	R320
300	R328
350	R336
400	R344

1.6



1.7 Break even point:
- (a) Between R150 and R200
- (b) Between R150 and R175. See graph.
- (c) ≈ R171km

Lesson 32

No solution required.

Lesson 33

1.1 25 000

1.2 Over the internet.

1.3 Some suggested answers are

(A) Do you buy facial products? (yes/no).
Do you use cologne? (yes/no).

(B) Do you use moisturiser? Please tick. (once a day–twice a day–occasionally–never).
How much time do you spend grooming? (1 hour everyday – 1 hour everyday – 1 hour in a week – don't really bother at all)

(C) Why do you spend time grooming? How much money do you spend on your appearance per month?

Lesson 34

1.1 Some suggested answers are:

Article 1:
- People coming to the clinic could be asked to fill in a questionnaire which would establish personal details;
- Using available information to make comparisons.

Article 2:
- Students could have been asked to fill in a questionnaire.

Article 3:
- Readership could be found by looking at the sale figures for the different newspapers i.e. using available information;
- Customers could be asked to fill in a survey.

1.2 Some suggested answers are:

Article 1:
- To survey schools would require stratified random sampling;
- Treatment clinics in all socio-economic areas as well as rural and urban clinics must be included in the sample.

Article 2:
- All types of schools need to be included in the survey i.e. single sexed schools and co-educational schools;
- Schools from all socio-economic groups must be considered;
- Stratified random sampling must occur; and
- Both rural and urban schools should be considered.

Article 3:
- Different socio economic groups must be targeted;
- Different physical areas need to be included.

Lesson 35

1.1 No solutions required.

2.

Time	FEMALE	MALE	TOTAL
$t \leq 35$		6	6
$35 < t \leq 40$		16	16
$40 < t \leq 45$	2	22	24
$45 < t \leq 50$	11	51	62
$50 < t \leq 55$	10	38	48
$55 < t \leq 60$	28	43	71
$60 < t \leq 65$	27	23	50
$65 < t \leq 70$	20	14	34
$70 < t \leq 75$	4	6	10
$75 < t \leq 80$	15	7	22
$80 < t \leq 85$	11	9	20
$85 < t \leq 90$	6	1	7
$90 < t \leq 95$	1		1
TOTAL	135	236	371

Lesson 36

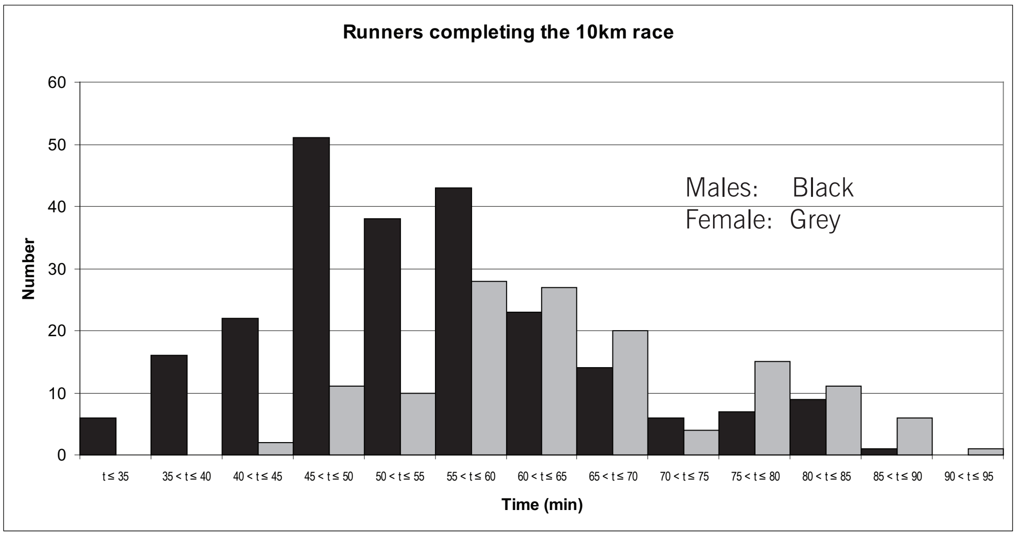
1. Mean = 54,3 min. RANE's time is slightly faster than the mean (average) time
2. Median = 52,5 min. RANE's time places him in the slowest half of the field.
3. MODE 47,5 min. RANE's time is slower than the most frequently clocked time.

    2.1 Mean: 58 min.

    2.2 Median: 57,5 min

    2.3 Mode: 57,5 min.

Lesson 37



Lesson 38

1.1 No solution given.

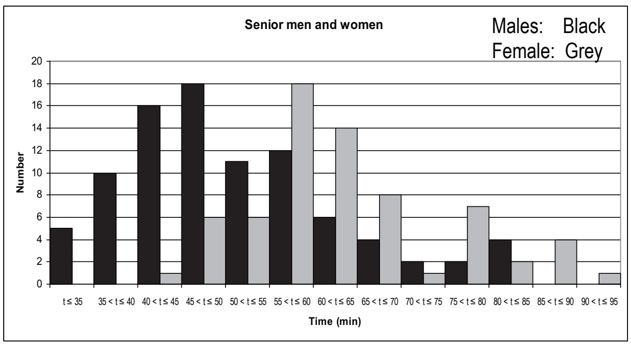
1.2

Time	Female	Male	Total
30< t ≤ 35	0	5	5
35< t ≤ 40	0	10	10
40< t ≤ 45	1	16	17
45< t ≤ 50	6	18	24
50< t ≤ 55	6	11	17
55< t ≤ 60	18	12	30
60< t ≤ 65	14	6	20
65< t ≤70	8	4	12
70< t ≤ 75	1	2	3
75< t ≤ 80	7	2	9
80< t ≤ 85	2	4	6
85< t ≤ 90	4	0	4
90< t ≤ 95	1	0	1
Total	68	5	158

1.3 (a) F = 63 ; M = 48 and field = 58

(b) F = 63,5 ; M = 51,3 and field = 56,5

1.4



1.5 When compared to the **senior runners** only: The girl's time of 59 min, places her in the top half of the female field as her time is faster than the senior female median (63 min) but is in the slower half of the

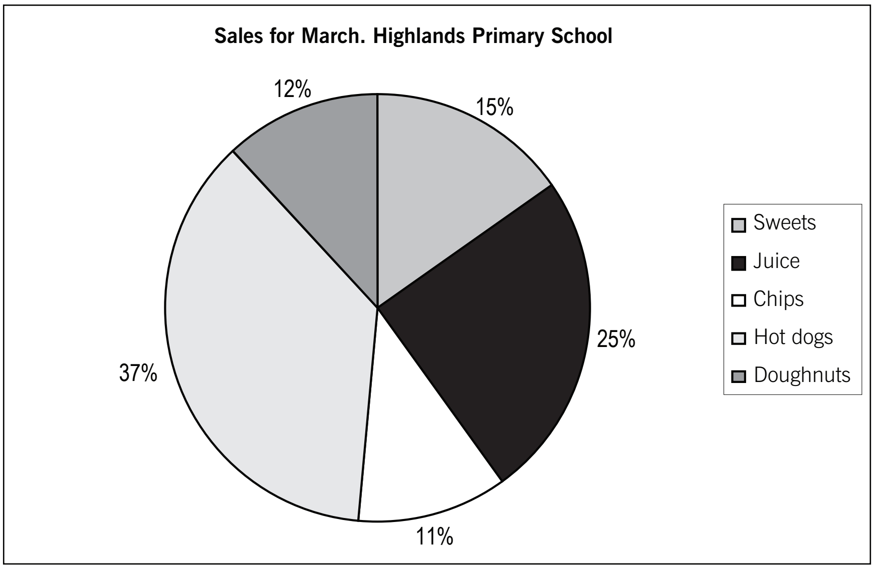
senior field as her time is slower than the senior median (58 min). She is faster than the senior female mean (63,5 min) and slower than the senior mean (56,5 min). The boy's time of 54min, places him in the bottom half of the senior male field as his time is slower than the senior male median (48 min) but he is in the top half of the whole field as his time is faster than the senior median (58 min). He is slower then the senior male mean (51,3 min) but faster than the senior mean (56,5 min).

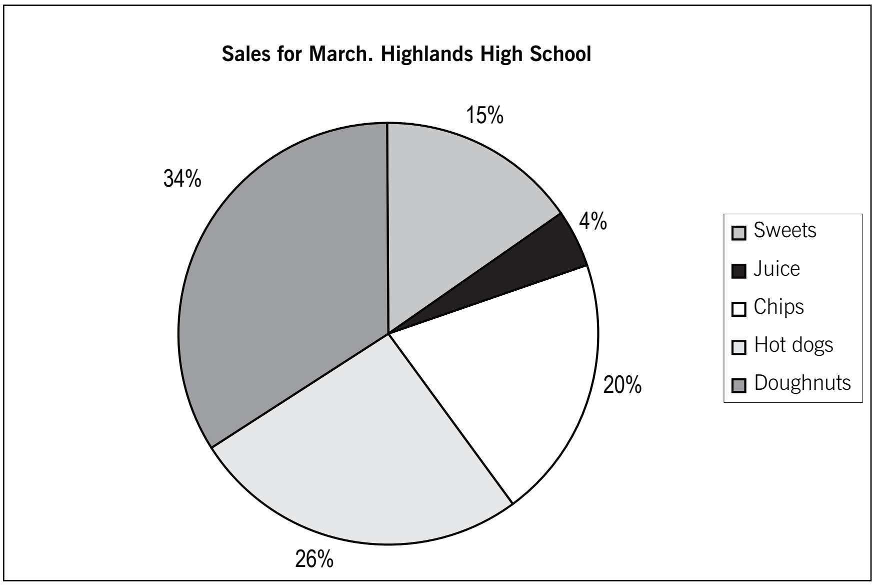
Lesson 39

1.1

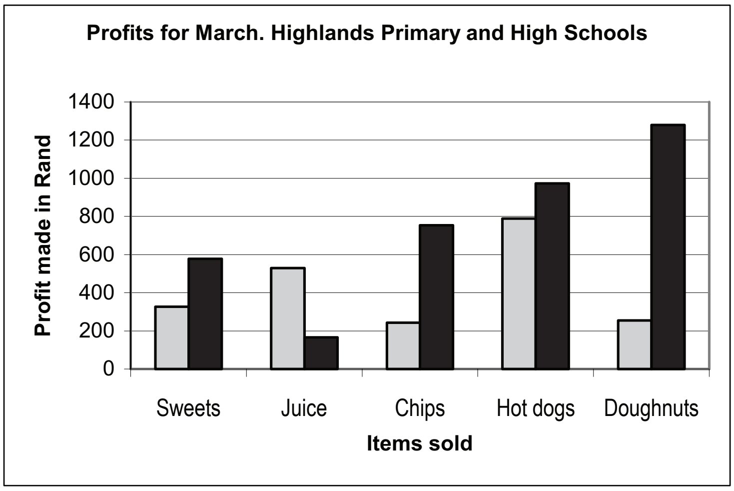
Profit made on the following items	Primary School	High School
Sweets	15%	15%
Juice	25%	4%
Chips	11%	20%
Hot dogs	37%	26%
Doughnuts	12%	34%

1.2





1.3



Primary School: grey
High School: black

Lesson 40

1 It gives the impression that the average house price in 2006 is almost five times the price in 2004 and two and a half times since 2005.

2 By starting the vertical axes at R350 000. The bars do not represent the actual values.

3

Lesson 41

No solutions provided.

Lesson 42

No solutions provided.

Lesson 43

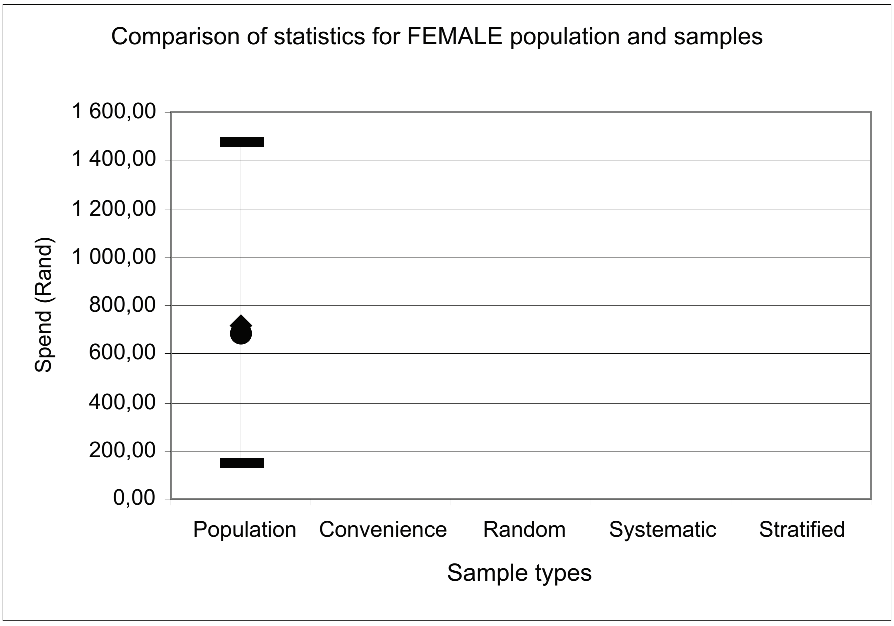
Only those values that are available are supplied – the remaining values will vary from learner to learner depending on their samples in lessons 41 and 42.

1 Females

1.1

	Population	Convenience	Random	Systematic	Stratified
Minimum	R146,00				
Mean	R682,16				
Median	R720,00				
Maximum	R1 480,00				

1.2



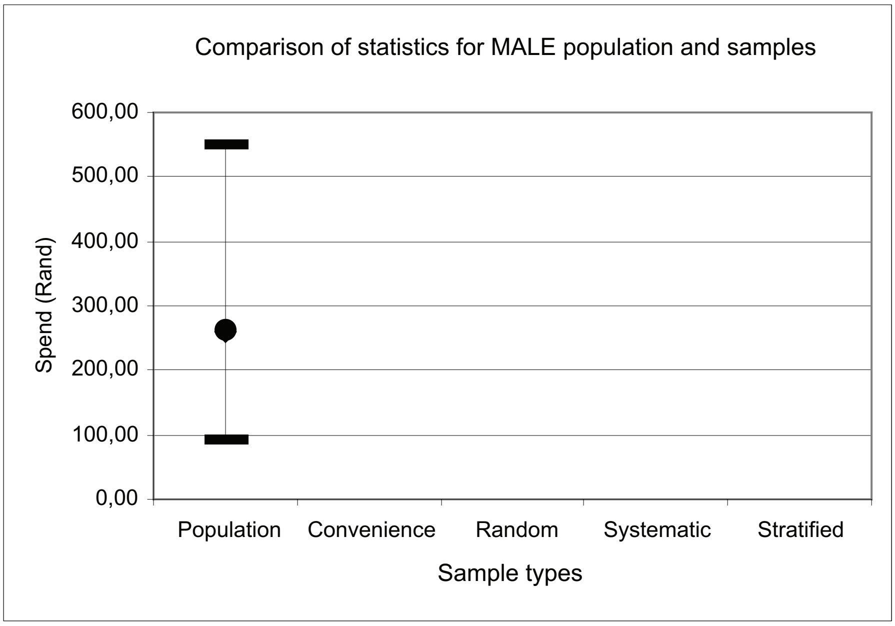
1.3 Learners will almost certainly find that although the means and medians of the samples are pretty similar to the means and median of the population, the ranges will vary a great deal from one sample to the next.

2 Males

2.1

	Population	Convenience	Random	Systematic	Stratified
Minimum	R94,00				
Mean	R262,85				
Median	R261,00				
Maximum	R552,00				

2.2



2.3 Learners will almost certainly find that although the means and medians of the samples are pretty similar to the means and median of the population, the ranges will vary a great deal from one sample to the next.

Lesson 44

1.1 “the risk”.

1.2 70%–90% effective means that if young adults have the flu injection then between 70 to 90 out of 100 of these adults would not get flu in a year.

1.3 Yes. In a young person the effectiveness of the flu injection is high. (or any reasonable answer)

2.1 “the chance”

2.2 A 40% chance is close to 50% (as likely to happen as not) which is quite a high increase.

2.3 1,4% chance means that in 1 000 pregnancies, 14 pregnancies could result in twins.

2.4 14 pregnancies in every 1000 would become: $14 + 40\%$ of $14 \approx 19$ pregnancies out of 1 000 pregnancies.

2.5 When one looks at the actual numbers 40% does not appear so alarming as it is only 5 more in every 1 000! That is not to say that the chance of having twins isn’t greater.

Lesson 45

No solutions needed.

Lesson 46

1.1

Log Table Group E							
Team	**MP**	**W**	**D**	**L**	**GF**	**GA**	**Pts**
Italy	1	1	-	-	2	-	3
Ghana	1	-	-	1	-	2	0
USA	1	-	-	1	-	3	0
Czech	1	1	-	-	3	-	3

1.2 Yes, they do. Since each team still has two games to play each team can potentially earn 6 points so each team can theoretically still come first or second in the group.

1.3

17-Jun-06	Kaiserlauten	ITA:USA	1:1
17-Jun-06	Cologne	CZE:GHA	0:2

Log Table Group E							
Team	**MP**	**W**	**D**	**L**	**GF**	**GA**	**Pts**
Italy	2	1	1	-	3	1	4
Ghana	2	1	-	1	2	2	3
USA	2	-	1	1	1	4	1
Czech	2	1	-	1	3	2	3

1.4 Each country must still play one more game and the maximum number of points they can get from that game is 3. If the USA wins it can catch up to Italy if Italy does not win. Each country will then have 4 points. However this means that Czechoslovakia must have beaten Italy which makes Czechoslovakia top of the group with 6 points and Italy and the USA second. If Italy beats Czechoslovakia then it will be top of the group with 7 points. Czechoslovakia will have only three points and cannot come second no matter what happens in the match against

Ghana and the USA. If the USA wins they will have 4 points and will be second and if Ghana wins they will have 6 points and they will be second. If Italy draws with Czechoslovakia they will have 5 points and Czechoslovakia 4. Then if Ghana beat the USA, Ghana will have 6 points and be top of the group with Italy second.

1.5

22-Jun-06	Hamburg	CZE:ITA	0:2	
22-Jun-06	Nuremberg	USA:GHA	2:1	
Log Table Group E				
Team				**PtsGAGFLDWM**P
Italy				715–123
Ghana				3432–13
USA				4531113
Czech				3432–13

Italy top of the group and USA second.

Lesson 47

No solutions given

Lesson 48

1.1 CH 31, Rondebosch

1.2 CG 34, Athlone

1.3 From Ranelagh go towards Avenue de Mist and turn right into Avenue de Mist, left into Milner Road, right into Kromboom Road, over the highway, railway and Jan Smuts drive, then turn left into Tennant Road right into Louevale Road and finally right into Raglan Rd.

2 No solution provided.

Lesson 49

1.1 Distance on ground = 2,72 km which will take him about half an hour.

2.1 0,1 km or 100 m

2.2 ≈ 4%

2.3 0,75 km

Lesson 50

1. Take the green route (Bellville, Wellington, Stellenbosch, Strand) from Kuils River towards Eerste River. At Eerste River take the train to Stellenbosch.

1.2 Three alternatives. Route 1: Take the blue route (Langa, Bellville, Kapteinsklip, Khayelitsha) as far as Kuils River and then change to the green route (Bellville, Wellington, Stellenbosch, Strand) to Eerste River and then on to Stellenbosch. Route 2: Take the green route (Bellville, Wellington, Stellenbosch, Strand) to Eerste River and then on to Stellenbosch. Route 3: Take the green route (Bellville, Wellington, Stellenbosch, Strand) to Muldersvlei and then on to Stellenbosch.

1.3 Take the green route (Bellville, Wellington, Stellenbosch, Strand) from Stellenbosch towards Eerste River. At Eerste River take the train to Kuils River and then change to take the blue route (Langa, Bellville, Kapteinsklip, Khayelitsha) as far as Bontheuwel. Change to take the train to Nyanga

1.4 Take the red route (Wynberg, Simonstown) to Cape Town and then you can pick one of the 3 routes from question 1.2.

Lesson 51
No solution given.

Lesson 52
No solution given.

Lesson 53
No solutions given.

Lesson 54
1.1 3 boxes;
1.2 3 boxes;
7 rolls

Lesson 55
1.1 (a) (a) 2 837,037 ft^3
(b) (b) 2 705,103 ft^3
(c) (c) Answer (b).
(d) (d) ≈ 5%
1.2 (a) 65 100 pounds
(b) 56 962,500 pounds
(c) 57 450,750 pounds
(d) 57 401,925 pounds
(e) Answer (d).
(f) ≈ 13%

Lesson 56
1.1 An oven.
1.2 ≈ 148 kWh
1.3 R79,96
1.4 Coal ≈ 73 kg; water ≈ 178 litres; and CO_2 ≈ 143 kg
1.5 ≈ 175 kWh
1.6 R84,10
1.7 Coal ≈ 86 kg; water ≈ 210 litres; and CO_2 ≈ 169 kg
2.1 ≈ R14 332 million
2.2 Water used ≈ 53 395 million kl
Coal used ≈ 21 804 804 000 kg
CO_2 emissions ≈ 37 821 600 000 kg.

Lesson 57
No solution given.

Lesson 58
No solution given.

Lesson 59
No solution given.

Lesson 60
No solution given

TIPS FOR TEACHERS

Lesson 1

1. It is important to emphasise that, when the learners are changing a fraction into a percentage, they find equivalent fractions.
2. A percentage is a fraction with a denominator of 100.
3. A percentage is written as a whole number with a percentage symbol, namely %, but it is a fraction. For example, 25% means $\frac{25}{100}$.

Lesson 2

1. One of the most important skills in solving problems related to percentage is knowing what the whole is in the problem you are working with — you need to impress this on your learners.
2. When adding a percentage of an amount to an amount, what you are really doing is:
   - i. Dividing the whole into 100 equal parts to determine what 1% is;
   - ii. Determining the amount that corresponds to the percentage of the amount that we want to add by "counting" the desired number of percents; and
   - iii. Adding the amount that corresponds to the desired percentage of the amount to the amount.

Lesson 3

1. Subtracting an amount from an amount is decreasing an amount by a percentage of the whole amount, whereas when you add an amount to an amount, you are increasing an amount by a percentage of the whole amount. The process is similar.
2. When subtracting a percentage of an amount from an amount, what you are doing is:
   - Dividing the whole into 100 equal parts to determine what 1% is;
   - Determining the amount that corresponds to the percentage of the amount we want to subtract by "counting" the desired number of percents; and
   - Subtracting the amount that corresponds to the desired percentage of the amount from the amount.

Lesson 4

1. In the inflation example when we changed $\frac{650}{5 000}$ to a percentage, 13%, we found an equivalent fraction with a denominator of 100, i.e. $\frac{650}{5 000} = \frac{13}{100} = 13\%$
2. When we calculate percentage change it is important to know that:
   - 2.1. The original amount is the whole or 100%;
   - 2.2. The difference between the final and the original is what was added to or subtracted from the whole; and
   - 2.3. The percentage change that you calculate is the percentage by which the original amount increased/decreased.
3. This process is the opposite to the calculation we did in lessons 2 and 3 when we knew the percentage change and first had to calculate the amount to be added or subtracted (depending on the situation) and then added/subtracted it to/from the whole. In this calculation we know the amount added/subtracted to/from the whole and need to express this as a percentage of the original amount.

Lesson 5

1. The key to these types of problems is to understand where the whole or the 100% part of the problem is.
2. A common error learners make with these problems is to take the final amount as the 100%.
   For instance, in the problem with the man who went on a diet. They find 15% of the 80kg and then add it to the 80kg. This gives an answer of 92kg. However, when one checks the answer by finding 15% of 92kg and subtracting it from 92kg, you get 78,2kg — not 80kg.

Lesson 6

1. It is important learners understand that a ratio has no units and that you can chose which unit you want to work with. However, the units for both values must be the same, i.e. if you use a cup for the one quantity, you must use a cup for the other quantity as well.
2. Writing a ratio in its simplest form means that the terms of the ratio have no common factor. As long as the terms of a ratio have a common factor the ratio is not in its simplest form.
3. Ratios do not give actual quantities.
4. Note the following:

	gravel	**sand**	**cement**
Low strength	6	3	1
Medium strength	4	2	1

- For low-strength concrete the number of bags in the mixture is:
  $6 + 3 + 1 = 10$ bags.
- The cement is therefore 110 of the mixture, or 10%.
- For medium-strength concrete the number of bags in the mixture is:
  $4 + 2 + 1 = 7$ bags.
- The cement is therefore 17 of the mixture, or 14,3%.
- This shows that the medium-strength concrete has a greater percentage of cement present, hence the name medium strength.

Lesson 7

The key to all these types of problems is to establish how many equal parts the whole or mixture has been divided into. This is achieved by adding the elements of the ratio together and dividing the total into the whole.

Lesson 8

1. Note that if any one quantity increases as another quantity increases, it does not necessarily mean that the quantities are in direct proportion. For the quantities to be in direct proportion the ratio formed by the two quantities must be constant.
2. It is important to relate the word problem to a table and to a graph because these are methods that can be used to solve direct proportion problems. Encourage your learners to see the link between the different methods.
3. A graph of a direct proportion problem will always be a straight line graph with the gradient representing the proportionality constant. The graph goes through the origin.

Lesson 9

1. Note that if any one quantity decreases as another quantity increases, it does not necessarily mean that the quantities are inversely proportional. For the quantities to be inversely proportional, the product of the two quantities must be constant.
2. In the lesson on direct proportion we used two methods for testing direct proportion. For consistency, we will show you two methods for testing inverse proportion. The first is highlighted in the example with the string and working out length and width. That is, if one quantity increases as the other decreases and the product of the two quantities is constant, the quantities are inversely proportional. The second method is to compare their ratios.
   We will use the example of the barrel of water. 0,5 litre bottles fill 400 bottles.
   Expressed as an inverse ratio 0,5 : 1400 = 1 : 0,005
   5 litre bottles fill 40 bottles. Expressed as an inverse ratio
   5 : 140 = 1 : 0,005
   The ratios are equal so the two quantities are inversely proportional.
3. It is important to relate the word problem to a table and to a graph because these are all methods that can be used to solve inversely proportional problems. Encourage your learners to see the link between the different methods.
4. A graph of an inversely proportional problem will always be a hyperbola with the general equation $y = kx$; where k is the constant product.

Lesson 10

1. Units are vital when dealing with rates. The units are determined by the context of the problem.
2. Context is important in rate problems because in some cases you want the lower rate (see activity seven) and in other cases you want the higher rate (see activity four). Your choice is determined by the context. If in activity seven the learner worked out the rate as grams per cent, you would choose the higher of the two rates to get the better value for your money. This problem highlights the need for units and the need for context.

Lesson 11

1. A common mistake that learners make is to think the average speed is the speed at which the runner ran the whole race. That is they think that the runner ran at a constant speed equal to the average speed. The runner, however, did not.
2. The average speed or rate gives an overall picture for the event and not the actual details.

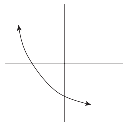
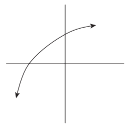
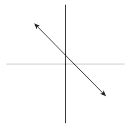
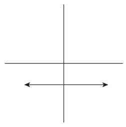
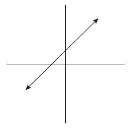
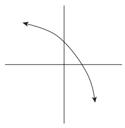
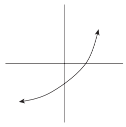
Lesson 12

1. In the worked example on constant rate, the rate does not depend on the interval we look at. When the drip is set up the way it is, exactly 2,5ml will be delivered every minute. After 10 minutes 25ml will have been delivered and after 20 minutes 50ml. Every minute is like every other minute.
2. If you plot a graph of a constant rate problem the graph will be a straight line. The gradient of the graph represents the rate.

Lesson 13

1. When we look at rate we are interested in two aspects:
   - First, how the output value changes — the baby's weight in our example; and
   - Second, how the rate of change varies — the growth rate in our example.
   -

The different possibilities are illustrated in the table and graph below:

		Output value		
		Decreasing	**Constant**	**Increasing**
Rate of change of the output value	**Decreasing**			
	Constant			
	Increasing			

2. For varying rate situations it is important to talk about trends rather than average rate.

Lesson 14

The focus of this lesson is to define vocabulary for the financial section. Learners must be able to classify items into income and expenses; whether they are fixed or varying items and whether expenses are high or low priority.

Lesson 15

It is worthwhile discussing ways to bring down the cost price of items to maximise profit. Suggestions include: buying in bulk; decreasing transport costs; hiring someone to help; and finding a cheaper supplier.

Lesson 16

1. It is important for the learners to understand that for a constant function the value being determined is fixed for a particular interval. However, for a linear function the values being determined change at a constant rate all the time and this leads to a straight line graph.
2. Notice that the linear function graph does not necessarily go through the origin (see lesson on direct proportion). The value where the graph cuts the cost axes represents the basic service fee before you have travelled any distance, i.e. when distance = 0 km.
3. The graph of the constant function example used in this lesson will be addressed in Lesson 17.

Lesson 17

1. There are three different kinds of rounding. Rounding off where we round to the nearest whole number. Rounding down where we round down to the previous whole number and rounding up where we round up to the next whole number. The Telkom telephone tariff is an excellent example of where we need to round up. Telkom does not charge per part units, so you need to round up to the next unit. Rounding down would mean that your call was cut short.
2. The graph of a constant function also is a linear function as it is a straight line. The gradient, however, is zero.

Lesson 18

1. Although step functions and piecewise functions are in principle the same, i.e. a new function for each interval, it is important to note that for the step function the adjacent functions do not continue on from one another. In other words, there is a "jump". This is shown in the graph by the use of closed and open circles. In the piecewise function the adjacent functions follow on from one another.
2. Three different water tariff tables were given in this lesson. They are slightly different and these differences might need to be pointed out to the learners. The tariff table in the actual lesson shows that the kilolitre interval follows on from one step to the other using a + sign, i.e. step 1 — 0 to 6kl ; step 2 — 6+ to 12kl. The Johannesbug tariff table does not show this continuation, but seems to jump by 1kl, i.e. step 1 — first 6kl ; step 2 — 7kl to 10kl. This table still means, however, that the intervals are continuous. The eThekweni tariff table is more complicated than the others because you have to include a fixed charge before you calculate your rate per litre.

Lesson 19

1. While the Life Line consumption rate is greater and hence the slope of the graph is steeper, there is no basic fee with this option so the graph starts out below the graph of the Two Part option – at some point the graphs must intersect – the break even point.
2. The graph of the Two Part Flat option starts above the graph of the Life Line option – this is because of the fixed monthly fee (service charge) that is charge irrespective of the consumption.
3. Both the graph and the table approach to determining the break-even point give us good approximations. However, neither approach gives us the exact value. To determine the exact value of the break even point we could use equations. Students should realise that although a limitation of these methods is that they cannot give the exact value; they can show a trend more clearly than an equation would. In both the electricity and the banking contexts, the amount on which the fee is determined will vary from month to month and from transaction to transaction so it is probably more useful to get a sense of the trend than to know the exact value.

Lesson 20

1. This lesson looks at all three ways of comparing options and finding the break-even point:
   - Filling in a table of values;
   - Drawing a graph of the two functions; and
   - Solving (simultaneous) equations.
2. The learners must be comfortable with all three methods as each has advantages and disadvantages, depending on the purpose — see the comments for the previous lesson in this regard.

Lesson 21

The lessons summarised here are:

- Percentage — Lessons 1 to 5
- Ratio/proportion — Lessons 6 to 9
- Rate — Lessons 10 to 13

Lesson 22

The lessons covered in this lesson are:

- Income and expenses — Lesson 14
- Calculating unit cost — Lesson 15
- Linear and constant functions — Lessons 16 to 18

- Break-even points — Lessons 19 and 20

Lesson 23

1.1 This example covers all the types of functions covered so far:
- Constant functions;
- Linear;
- Step; and
- Piecewise

1.2 Learners need to apply all the skills learnt so far to understand this situation. It is a good example of how the sections which the learners have covered can be integrated.

Lesson 24

1.1 It is important for learners to realise that interest is a fee — a fee paid for providing a service. When a shops sells packets of chips it “marks up” the price that it paid to determine the price the customer must pay. This “mark-up” is its “profit” — its reward for providing the service. When a money-lender lends money to a borrower he/she also expects a “reward” for the service he/she provides. His or her reward comes in the form of a fee called interest.

1.2 The main focus of this lesson is to highlight the different situations where interest is involved. If you borrow money, you will pay interest, but if you invest money in a bank, you will be paid interest. Learners need to understand that in some situations they are the borrower, i.e. when they take out a loan, and in other situations they are the lender, i.e. when they deposit money into a bank.

Lesson 25

The simple interest formula is not the main focus of this lesson. It is more important for learners to realise that in simple interest situations, interest is determined once-off for the period of the loan and that the loan and the interest are both paid back together.

Lesson 26

1. An understanding of the effect of compound interest is vital for good management of personal finances.
2. A valuable exercise would be to look at various types of savings accounts and to discuss why some accounts have a higher interest rate than others and the advantages and disadvantages of different accounts.
3. Although there is a formula for calculating compound interest, it is, however, more important for learners to understand how compound interest works and the effect compound interest has on loan and investment situations — this understanding is developed through paper and pencil analyses, such as those used in this lesson.

Lesson 27

There are formulae that can be used to make all sorts of calculations regarding loans, the repayment amounts, interest and so on, but they are too complex for us to work with. It is better for learners to understand the principles of how loans and investments work than to work with these complex formulae — and this is better done by the paper and pencil method shown in this lesson.

Lesson 28

This lesson serves to highlight the effect a small change in interest rate has on the repayment of loans. The larger the loan, the greater the effect of a small percentage increase in the interest rate on the actual repayments. It might be helpful to show your learners information pamphlets from different banks that show how the interest rate affects monthly repayments on housing loans.

Lesson 29

This lesson provides a very important insight into managing debit and serves to illustrate how by paying a little more back every month, you can make a huge difference in the length of time it takes you to pay back a loan and hence the actual amount you pay.
Some hidden formula calculators can be used to do calculations involving number of months you need to repay your loan and the monthly repayments. Encourage your students to find these on the internet and experiment with them.

Lesson 30

1. This is a very important lesson for the any self managing person. It is relevant for the management of both one's personal finances as well as the viability of any business venture.
2. This example highlights the need to read and interpret information in shops with regard to unit costs. The students should be encouraged to visit supermarkets and look for the information labels on the shelves.
3. What is not pointed out in this lesson is the comparison of unit cost between different brands of the same ingredient. This could be a task that you could set for your students.

Lesson 31

1. This example serves to illustrate how break even points can be determined by three different methods. Students will require the following skills in order to be able to do this:
   - Write equations/formulae to represent linear functions. Words can be used instead of symbols in the formulae.
   - Fill in a table of values.
   - Draw a graph to represent the function by either plotting the points from the table or by other means.
   - Read off values from a graph.
   - Solve simultaneous linear equations.
2. Calculating the break even point for a situation helps you to make a more informed decision.

Lesson 32

1. This introduction to data handling serves to make students aware how often they are exposed to data and someone's interpretation of that data. Hopefully by the end of the data handling section they will be more familiar with the process of data handling and therefore be more critical of the statistics that they are presented with.
2. Although we will look at each stage of the data handling separately in the next few lessons, we need to understand that we cannot see each stage in isolation.

Lesson 33

The aim of this lesson is to emphasise that the way in which you ask a question is crucial in determining the answer you will get.

Lesson 34

1. To be able to critique an article effectively, it is very important to understand the effect of sampling on the outcomes of the data collected. Students should always ask the question: Is the sample a reasonable representation of the population?
   The answer to this will help with determining the validity of the article.
2. Stats South Africa is a wonderful resource of all kinds of data and is readily available on the internet.
3. This lesson will hopefully encourage the students, as contributing citizens, to fill in surveys honestly and be as helpful as possible when people are trying to gather information.

Lesson 35

1. Encourage students to use excel spreadsheets if computers are available.
2. Emphasise the importance of working accurately when organising data and to fill in the tally tables correctly and efficiently.
3. www.statssa.gov.za is a good source of data for extra activities.

Lesson 36

Calculating mean, median and mode is only one way of summarising data. In Grade 12 we will look at other ways. The mean or average is the most commonly used of the three statistics.

Lesson 37

For the graphs to be an effective way of representing the data, they must be clearly and accurately labeled.

Lesson 38

1. This stage of the data handling cycle possibly requires the most attention. We are bombarded all the time with tables, graphs and diagrams of already organised and summarised data and we need to be able to analyse the results critically. The previous seven lessons serve to give the students an understanding of how data is collected and organised and should hopefully equip them to make more informed opinions about statistically based articles.
2. Encourage your students to ask themselves questions like:
   - What sample of the population was taken?
   - How big was the sample size in comparison to the population size?
   - What methods of data collection were used?
   - What statistical measures were used?
   - Is the representation misleading or is it valid?
   - What scales have been used on the axes?
   - Under what conditions were experiments done?

Lesson 39

1. These graphs can be drawn using excel spread sheets. Students should be encouraged to use this technology if available.
2. It should be emphasised that graphs need to be labelled carefully.

Lesson 40

1. Encourage the students to find articles in newspapers and magazines that have used misleading representations.
2. The term histogram is used for a bar graph that represents continuous data i.e. there are no gaps between the bars.

Lesson 41

1. The activities require careful and accurate work, although they are repetitive, they lend themselves to group work.

2. Discuss with the students how these methods of sampling could introduce bias and when these different techniques of sampling may be useful and when they might not be.

Lesson 42

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Lesson 43

1. The activities require careful and accurate work, although they are repetitive, they lend themselves to group work.
2. It would be a good idea to have students compare their findings across the class and, in so doing, to develop a sense of just how different samples can be in terms of their characteristics.
3. Discuss with the students how these methods of sampling could introduce bias and when these different techniques of sampling may be useful and when they might not be.

Lesson 44

1. Expressions of likelihood can only be used to predict the trend over a period of time and cannot be used to predict the outcome of any single event.
2. It is important to distinguish the difference between the two ways of making predictions about the future. We can either make predictions based on the observations of a large number of actual events (experimental probability) or we can make predictions based on the structure of the situations (theoretical probability).
3. When the weather report predicts a 10% chance of rain, what the weather office is really saying is that based on the many years of information available to them, on 70% of the days when the atmospheric conditions were the same as those expected tomorrow, it rained. In other words they are not saying it will rain 70% of tomorrow, or even in 70% of the region. In fact they are not even saying it will rain tomorrow.

Lesson 45

1. The ability to determine the theoretical probability for an event is a very important for people who run games of chance like the lottery.
2. It is important that your students understand that the theoretical probability gives you the likelihood or probability of an event in the long term i.e. it cannot tell you the outcome of the next event. For example: the probability of getting a head when you toss a coin is 1 in 2. This does not mean that if your first toss was a tail then the next toss would have to be a head. Rather, it means that if you tossed the coin a large number of times then the ratio of heads to tails would tend towards 1 to 1 or the ratio of heads to tosses would towards 1 to 2.
4. We expect the experimental probability to tend towards the theoretical probability as the number of events gets very large.

Lesson 46

This lesson shows how data can be analysed and then how different scenarios can be put forward. It does not, however, enable one to predict what is going to happen in the next match or who is going to be top of the group.

Lesson 47

1. The activity in this lesson could be set as a project.

2. The purpose of the activity is to encourage students to read articles in the light of what they have learnt in order to critically evaluate what they are reading.

Lesson 48

1. This is a very practical lesson and it would benefit your students if you could source maps with indexes for your local area.
2. Encourage your students to verbalise directions.

Lesson 49

1. Different maps have different scales depending on how detailed the map needs to be.
2. Measuring distances with string is not a very accurate method of measuring but it does give you a good approximation of the distance that you want to determine. Always accept a range of correct answers if you have asked your students to determine a distance by measuring with a piece of string.
3. It is important that your students get a "feel" for distances. During class activities it would be worthwhile to encourage your students to estimate the distance before measuring it.

Lesson 50

1. Try source route maps for your area.
2. This lesson highlights how different maps/representations are useful for different kinds of information.

Lesson 51

1. Try source timetables in your area.
2. Emphasise the need to work backwards in the case where you need to be at a certain destination at a certain time.
3. This lesson highlights the important skill of reading timetables.

Lesson 52

1. A floor plan of a house is another example of a mathematical model as it is a representation of the real situation.
2. This lesson, once again, highlighted the existence of different models (floor plans) for a variety of different needs.
3. We use models to solve problems and the house plan provides us with a means of actually building the house.

Lesson 53

1. This lesson focuses on the use of plans to make decisions or to solve problems.
2. Models and scale drawings are powerful tools to use in order to solve problems

Lesson 54

1. This lesson shows very clearly how a model (the floor plan) can be used to solve problems.
2. An important point that is made in this lesson is that it is not always appropriate to find the area of the wall or floor when tiling, wallpapering and carpeting. It is necessary to solve this problem in other ways.
3. Many do-it-yourself shops have charts, tables and diagrams available to assist you in determining the quantities required to do a job yourself. Encourage your students to bring as many different types of chart etc. to class. The mathematically literate person must be able to use these different tools available with confidence.
4. You can also emphasise the use of skills learnt in the lessons on functions namely substituting into a given formula.

Lesson 55

1. This lesson is all about making sense of instructions – a skill that relies on us (a) having some sense of how the table was developed and (b) having some sense of the variables – such as wastage and others.
2. The mathematical technique that underlies the problem which we were dealing with in the lesson is rounding – rounding up, rounding down or rounding off.
   In the case of the cement instructions the manufacturers rounded up thinking that some extra would be better than too little whereas in the case of the cookery book they rounded down.

Lesson 56

1. As much as today's lesson is about measurement, calculations, conversions and about working with numbers; the real message to take away is that small numbers can become very large very easily and although it may be tempting to want to disregard these numbers as "unimportant" they can have a huge impact!
2. The topic of this lesson is a springboard for a discussion on the responsibility of a concerned and critical citizen. It highlights the importance of being mathematically literate in order to have an understanding of the impact of our lives on the world around us.

Lesson 57

1. This lesson does not deal with actual calculations of volumes but serves to show how volumes are used in a particular context.
2. This lesson also returns once again to the concept of a rate and highlights the importance of always specifying units.
3. You can also emphasise the use of skills learnt in the lessons on functions namely substituting into a given formula.
4. This lesson also gives opportunity for discussion on social responsibility.

Lesson 58

1. This lesson again does not deal with actual calculations of volumes but serves to show how volumes are used in a particular context.
2. This lesson also returns once again to the concept of a rate and highlights the importance of always specifying units.
3. You can also emphasise the use of skills learnt in the lessons on functions namely substituting into a given formula.
4. It must be emphasised that these are approximate values only they help us to teach students about responsible drinking.

Lesson 59

1. This lesson highlights the skills needed in the Space, Shape and Measurement Learning Outcome. Many of these skills were also dealt with in the other Learning Outcomes namely Number and Operations in Context (LO1) and Functional Relationships (LO2). The Space, Shape and Measurement Learning Outcome provides many contexts in which to use the skills developed in LO1 and LO2.

Lesson 60

1. This lesson is a good example of an activity that integrates all the learning outcomes and as such would be a good activity to use as a project for assessment.

Energy consumption recording grid

ACTIVITY	kJ/h	Total time	Total kJ	0h00	1h00	2h00	3h00	4h00	5h00	6h00	7h00	8h00	9h00	10h00	11h00	12h00	13h00	14h00	15h00	16h00	17h00	18h00	19h00	20h00	21h00	22h00	23h00
Arguing with somebody	440																										
Dancing (active)	1675																										
Driving a car	500																										
Eating a meal	585																										
Exercise, some sweating	1045																										
Gardening (lawn mowing)	1235																										
Gardening (weeding)	960																										
Grocery shopping	375																										
Housework (light)	395																										
Housework (heavy)	940																										
Playing in the garden	440																										
Showering	1090																										
Sitting (reading, writing, TV)	315																										
Sitting (talking, concentrating)	500																										
Sleeping	250																										
Sport (light)	1425																										
Sport (medium)	1610																										
Sport (heavy)	2930																										
Standing still	420																										
Walking (comfortable pace)	545																										
Walking (fast)	1780																										
Working sitting (using a machine)	335																										
Working standing and walking	850																										
Working (hard manual labour)	1020